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Q-Level Subnearring Of Q-Intuitionistic L-Fuzzy Subnearrings

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ABSTRACT

In this paper, we study some of the properties of Q-level subnearring of Q-intuitionistic L-fuzzy subnearring of a nearring and prove some results on these. 2000 AMS SUBJECT CLASSIFICATION: 03F55 , 08A72 , 20N25.

SUMMARY:

Let A be a Q-intuitionistic L-fuzzy subnearring of a nearring R. Then for α and β in L such that $\alpha \leq \alpha A(e, q)$ and $\beta \geq \nu A(e, q)$, $A_{(\alpha, \beta)}$ is a Q-level subnearring of R and let (R, +, .) be a nearring and A be a Q-intuitionistic L-fuzzy subset of R such that $A_{(\alpha, \beta)}$ be a Q-level subnearring of R. If α and β in L satisfying $\alpha \leq \alpha A(e, q)$ and $\beta \geq \nu A(e, q)$, then A is a Q-intuitionistic L-fuzzy subnearring of R. Also the homomorphic image of a Q-level subnearring of a Q-intuitionistic L-fuzzy subnearring of a nearring R is a Q-level subnearring of a Q- intuitionistic L-fuzzy subnearring of a nearring R'.

Keywords : (Q,L)-fuzzy subset, Q-intuitionistic L-fuzzy subset, Q-intuitionistic L-fuzzy subnearring, Q-level subset.

INTRODUCTION.

After the introdution of fuzzy sets by L.A.Zadeh[16], several researchers explored on the generalization of the notion of fuzzy set. The concept of intuitionistic L-fuzzy subset was introduced by K.T.Atanassov[2], as a generalization of the notion of fuzzy set. Azriel Rosenfeld[3] defined a fuzzy groups. Asok Kumer Ray[1] defined a product of fuzzy subgroups and A.Solairaju and R.Nagarajan[14] have introduced and defined a new algebraic structure called Q-fuzzy subgroups. We introduce the concept of Q-level subnearring of Q-intuitionistic L-fuzzy subnearring of a nearring and established some results.

1.PRELIMINARIES:

1.1 Definition: Let X be a non-empty set and L = (L, ≤) be a lattice with least element 0 and greatest element 1 and Q be a non-empty set. A (Q, L)-fuzzy subset A of X is a function

$A : XxQ \rightarrow L$.

1.2 Definition: Let (L, ≤) be a complete lattice with an involutive order reversing operation $N : L \rightarrow L$ and Q be a non-empty set. A Q-intuitionistic L-fuzzy subset (QILFS) A in X is defined as an object of the form A={< (x, q), μA(x, q), νA(x, q) > / x in X and q in Q },

where $\mu A : XxQ \rightarrow L$ and $\nu A : XxQ \rightarrow L$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively and for every $x \in X$ and q in Q satisfying $\mu A(x, q) \leq N(\nu A(x, q))$.

1.3 Definition: Let (R, +, .) be a nearring. A Q-intuitionistic L-fuzzy subset A of R is said to be a Q-intuitionistic L-fuzzy subnearring(QILFSNR) of R if it satisfies the following axioms:

(i) $\mu A(x-y, q) \geq \mu A(x, q) \wedge \mu A(y, q)$

(ii) $\mu A(xy, q) \geq \mu A(x, q) \wedge \mu A(y, q)$

(iii) $\nu A(x-y, q) \leq \nu A(x, q) \vee \nu A(y, q)$

(iv) $\nu A(xy, q) \leq \nu A(x, q) \vee \nu A(y, q)$, for all x and y in R and q in Q.

1.4 Definition: Let X and X′ be any two sets. Let $f : X \rightarrow X'$ be any function and A be a Q-intuitionistic L-fuzzy subset in X, V be a Q-intuitionistic L-fuzzy subset in f(X) = X′, defined by $\mu V(y, q) = \sup_{x \in f^{-1}(y)} \mu A(x, q)$ and $\nu V(y, q) = \inf_{x \in f^{-1}(y)} \nu A(x, q)$, for all x in X and y in X′. A is called a preimage of V under f and is denoted by f-1(V).

1.5 Definition: Let A be a Q-intuitionistic L-fuzzy subset of X. For α and β in L, a Q-level subset of A corresponding to α, β is the set $A_{(\alpha, \beta)} = \{ x \in X : \mu A(x, q) \geq \alpha$ and $\nu A(x, q) \leq \beta \}$.

2.- Q-LEVEL SUBNEARRING OF Q-INTUITIONISTIC L-FUZZY SUBNEARRINGS OF R

2.1 Theorem: Let A be a Q-intuitionistic L-fuzzy subnearring of a nearring R. Then for α and β in L such that $\alpha \leq \mu A(e, q)$ and $\beta \geq \nu A(e, q)$, A(α, β) is a Q-level subnearring of R.

Proof : For all x and y in A(α, β), we have, $\mu A(x, q) \geq \alpha$ and $\nu A(x, q) \leq \beta$ and $\mu A(y, q) \geq \alpha$ and $\nu A(y, q) \leq \beta$. Now, $\mu A(x - y, q) \geq \mu A(x, q) \wedge \mu A(y, q) \geq \alpha \wedge \alpha = \alpha$, which implies that,

$\mu A(x-y, q) \geq \alpha$. And, $\mu A(xy, q) \geq \mu A(x, q) \wedge \mu A(y, q) \geq \alpha \wedge \alpha = \alpha$, which implies that, $\mu A(xy, q) \geq \alpha$. And also, $\nu A(x-y, q) \leq \nu A(x, q) \vee \nu A(y, q) \leq \beta \vee \beta = \beta$, which implies that, $\nu A(x - y, q) \leq \beta$. And, $\nu A(xy, q) \leq \nu A(x, q) \vee \nu A(y, q) \leq \beta \vee \beta = \beta$, which implies that, $\nu A(xy, q) \leq \beta$. Therefore, $\mu A(x-y, q) \geq \alpha$ and $\nu A(x-y, q) \leq \beta$ and $\mu A(xy, q) \geq \alpha$ and $\nu A(xy, q) \leq \beta$. We get, x – y and xy in $A_{(\alpha, \beta)}$. Hence $A_{(\alpha, \beta)}$ is a Q-level subring of the nearring R.

2.2 Theorem: Let A be a Q-intuitionistic L-fuzzy subnearring of a nearring R. Then two Q-level subnearrings $A_{(\alpha 1, \beta 1)}$, $A_{(\alpha 2, \beta 2)}$ and α_1, α_2, β_1, β_2 in L and $\alpha 1 \leq \mu A(e, q)$, $\alpha 2 \leq \mu A(e, q)$ and $\beta 1 \geq \nu A(e, q)$, $\beta 2 \geq \nu A(e, q)$ with $\alpha 2 < \alpha 1$ and $\beta 1 < \beta 2$ of A are equal if and only if there is no x in R such that $\alpha 1 > \mu A(x, q) > \alpha 2$ and $\beta 1 < \nu A(x, q) < \beta 2$.

Proof : Assume that $A_{(\alpha 1, \beta 1)} = A_{(\alpha 2, \beta 2)}$. Suppose there exists x in R such that $\alpha 1 > \mu A(x, q) > \alpha 2$ and $\beta 1 < \nu A(x, q) < \beta 2$. Then $A_{(\alpha 1, \beta 1)} \subseteq A_{(\alpha 2, \beta 2)}$ implies x belongs to $A_{(\alpha 2, \beta 2)}$, but not in

$A_{(\alpha 1,\ \beta 1)}$. This is contradiction to $A_{(\alpha 1,\ \beta 1)} = A_{(\alpha 2,\ \beta 2)}$.

Therefore there is no x in R such that $\alpha 1 > \mu A(x, q) > \alpha 2$ and $\beta 1 < \nu A(x, q) < \beta 2$. Conversely, if there is no x in R such that $\alpha 1 > \mu A(x, q) > \alpha 2$ and $\beta 1 < \nu A(x, q) < \beta 2$. Then $A_{(\alpha 1,\ \beta 1)} = A_{(\alpha 2,\ \beta 2)}$.

2.3 Theorem: Let (R, +, .) be a nearring and A be a Q-intuitionistic L-fuzzy subset of R such that $A(\alpha, \beta)$ be a Q-level subnearring of R. If α and β in L satisfying $\alpha \leq \mu A(e, q)$ and $\beta \geq \nu A(e, q)$, then A is a Q-intuitionistic L-fuzzy subnearring of R.

Proof: Let (R, +, .) be a nearring and x, y in R. Let $\mu A(x, q) = \alpha 1$ and $\mu A(y, q) = \alpha 2$, $\nu A(x, q) = \beta 1$ and $\nu A(y, q) = \beta 2$.

Case (i): If $\alpha 1 < \alpha 2$ and $\beta 1 > \beta 2$, then x, y $\in A_{(\alpha 1,\ \beta 1)}$. As $A_{(\alpha 2,\ \beta 2)}$ is a Q-level subnearring of R, we get x − y and xy in . Now, $\mu A(x - y, q) \geq \alpha 1 = \alpha 1 \wedge \alpha 2 = \mu A(x, q) \wedge \mu A(y, q)$, which implies that $\mu A(x-y, q) \geq \mu A(x, q) \wedge \mu A(y, q)$, for all x and y in R. Also, $\mu A(xy, q) \geq \alpha 1 = \alpha 1 \wedge \alpha 2 = \mu A(x, q) \wedge \mu A(y, q)$, which implies that $\mu A(xy, q) \geq \mu A(x, q) \wedge \mu A(y, q)$, for all x and y in R. And, $\nu A(x-y, q) \leq \beta 1 = \beta 1 \vee \beta 2 = \nu A(x, q) \vee \nu A(y, q)$, which implies that $\nu A(x-y, q) \leq \nu A(x, q) \vee \nu A(y, q)$, for all x and y in R. Also, $\nu A(xy, q) \leq \beta 1 = \beta 1 \vee \beta 2 = \nu A(x, q) \vee \nu A(y, q)$, which implies that $\nu A(xy, q) \leq \nu A(x, q) \vee \nu A(y, q)$, for all x and y in R.

Case (ii): If $\alpha 1 < \alpha 2$ and $\beta 1 < \beta 2$, then x, y $\in A_{(\alpha 1,\ \beta 1)}$. As $A_{(\alpha 2,\ \beta 2)}$ is a Q-level subnearring of R, we have x – y and xy in $A_{(\alpha 1,\ \beta 1)}$. Now, $\mu A(x-y, q) \geq \alpha 1 = \alpha 1 \wedge \alpha 2 = \mu A(x, q) \wedge \mu A(y, q)$, which implies that $\mu A(x-y, q) \geq \mu A(x, q) \wedge \mu A(y, q)$, for all x and y in R. Also, $\mu A(xy, q) \geq \alpha 1 = \alpha 1 \wedge \alpha 2 = \mu A(x, q) \wedge \mu A(y, q)$, which implies that $\mu A(xy, q) \geq \mu A(x, q) \wedge \mu A(y, q)$, for all x and y in R. And, $\nu A(x-y, q) \leq \beta 2 = \beta 2 \vee \beta 1 = \nu A(y, q) \vee \nu A(x, q)$, which implies that $\nu A(x-y, q) \leq \nu A(x, q) \vee \nu A(y, q)$, for all x and y in R. Also, $\nu A(xy, q) \leq \beta 2 = \beta 2 \vee \beta 1 = \nu A(y, q) \vee \nu A(x, q)$, which implies that $\nu A(xy, q) \leq \nu A(x, q) \vee \nu A(y, q)$, for all x and y in R.

Case (iii): If $\alpha 1 > \alpha 2$ and $\beta 1 > \beta 2$, then x, y $\in A_{(\alpha 1,\ \beta 1)}$. As $A_{(\alpha 2,\ \beta 2)}$ is a Q-level subnearring of R, we have x –y and xy $\in A_{(\alpha 1,\ \beta 1)}$. Now, $\mu A(x-y, q) \geq \alpha 2 = \alpha 2 \wedge \alpha 1 = \mu A(y, q) \wedge \mu A(x, q)$, which implies that $\mu A(x-y, q) \geq \mu A(x, q) \wedge \mu A(y, q)$, for all x and y in R. Also, $\mu A(xy, q) \geq \alpha 2 = \alpha 2 \wedge \alpha 1 = \mu A(y, q) \wedge \mu A(x, q)$, which implies that $\mu A(xy, q) \geq \mu A(x, q) \wedge \mu A(y, q)$, for all x and y in R. And, $A(x-y, q) \leq \beta 1 = \beta 1 \vee \beta 2 = \nu A(x, q) \vee \nu A(y, q)$, which implies that $\nu A(x-y, q) \leq \nu A(x, q) \vee \nu A(y, q)$, for all x and y in R. Also, $\nu A(xy, q) \leq \beta 1 = \beta 1 \vee \beta 2 = \nu A(x, q) \vee \nu A(y, q)$, which implies that $\nu A(xy, q) \leq \nu A(x, q) \vee \nu A(y, q)$, for all x and y in R.

Case (iv): If $\alpha 1 > \alpha 2$ and $\beta 1 < \beta 2$, then x, y $\in A_{(\alpha 1,\ \beta 1)}$. As $A_{(\alpha 2,\ \beta 2)}$ is a Q-level subnearring of R, we have x–y and xy in $A_{(\alpha 1,\ \beta 1)}$. Now, $\mu A(x-y, q) \geq \alpha 2 = \alpha 2 \wedge \alpha 1 = \mu A(y, q) \wedge \mu A(x, q)$, which implies that $\mu A(x-y, q) \geq \mu A(x, q) \wedge \mu A(y, q)$, for all x and y in R.

Also, $\mu A(xy, q) \geq \alpha 2 = \alpha 2 \wedge \alpha 1 = \mu A(y, q) \wedge \mu A(x, q)$, which implies that $\mu A(xy, q) \geq \mu A(x, q) \wedge \mu A(y, q)$, for all x and y in R. And, $\nu A(x-y, q) \leq \beta_2 = \beta_2 \vee \beta_1 = \nu A(y, q) \vee \nu A(x, q)$, which implies that $\nu A(x-y, q) \leq \nu A(x, q) \vee \nu A(y, q)$, for all x and y in R. Also, $\nu A(xy, q) \leq \beta_2 = \beta_2 \vee \beta_1 = \nu A(y) \vee \nu A(x)$, which implies that $\nu A(xy, q) \leq \nu A(x, q) \vee \nu A(y, q)$, for all x and y in R.

Case (v): If $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$. It is trivial.

In all the cases, A is a Q-intuitionistic L-fuzzy subnearring of the nearring R.

2.4 Theorem: Let A be a Q-intuitionistic L-fuzzy subnearring of a nearring R. If any two Q-level subnearrings of A belongs to R, then their intersection is also Q-level subnearring of A in R.

Proof : Let α_1, α_2, β_1, $\beta_2 \in$ L and $\alpha_1 \leq \mu A(e, q)$, $\alpha_2 \leq \mu A(e, q)$ and $\beta_1 \geq \nu A(e, q)$, $\beta_2 \geq \nu A(e, q)$.

Case (i): If $\alpha 1 < \mu A(x, q) < \alpha 2$ and $\beta 1 > \nu A(x, q) > \beta 2$, then $A_{(\alpha 1,\ \beta 1)} \subseteq A_{(\alpha 2,\ \beta 2)}$.

Therefore, $A_{(\alpha 1,\ \beta 1)} \cap A_{(\alpha 2,\ \beta 2)} = A_{(\alpha 2,\ \beta 2)}$, but $A_{(\alpha 1,\ \beta 1)}$ is a Q-level subnearring of A.

Case (ii): If $\alpha_1 > \mu A(x, q) > \alpha_2$ and $\beta_1 < \nu A(x, q) < \beta_2$, then $A_{(\alpha 1,\ \beta 1)} \subseteq A_{(\alpha 2,\ \beta 2)}$.

Therefore, $A_{(\alpha 1,\ \beta 1)} \cap A_{(\alpha 2,\ \beta 2)} = A_{(\alpha 1,\ \beta 1)}$, but $A_{(\alpha 1,\ \beta 1)}$ is a Q-level subnearring of A.

Case (iii): If $\alpha_1 < \mu A(x, q) < \alpha_2$ and $\beta_1 < \nu A(x, q) < \beta_2$, then $A_{(\alpha 2,\ \beta 1)} \subseteq A_{(\alpha 1,\ \beta 2)}$.

Therefore, $A_{(\alpha 2,\ \beta 1)} \cap A_{(\alpha 1,\ \beta 2)} = A_{(\alpha 2,\ \beta 1)}$, but $A_{(\alpha 2,\ \beta 1)}$ is a Q-level subnearring of A.

Case (iv): If $\alpha_1 > \mu_A(x, q) > \alpha_2$ and $\beta_1 > \nu_A(x, q) > \beta_2$, then $A_{(\alpha_1, \beta_2)} \subseteq A_{(\alpha_2, \beta_1)}$.

Therefore, $A_{(\alpha_1, \beta_2)} \cap A_{(\alpha_2, \beta_1)} = A_{(\alpha_1, \beta_2)}$, but $A_{(\alpha_1, \beta_2)}$ is a Q-level subnearring of A.

Case (v): If $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$, then $A_{(\alpha_1, \beta_1)} = A_{(\alpha_2, \beta_2)}$.

In all cases, intersection of any two Q-level subnearrings is a Q-level subnearring of A.

2.5 Theorem: Let A be a Q-intuitionistic L-fuzzy subnearring of a nearring R. If α_i, $\beta_j \in$L, $\alpha_i \leq \mu_A(e, q)$ and $\beta_j \geq \nu_A(e, q)$ and $A_{(\alpha_i, \beta_j)}$, i, j$\in$I is a collection of Q-level subnearrings of A, then their intersection is also a Q-level subnearring of A.

Proof: It is trivial.

2.6 Theorem: Let A be a Q-intuitionistic L-fuzzy subnearring of a nearring R. If any two Q-level subnearrings of A belongs to R, then their union is also a Q-level subnearring of A in R.

Proof : Let α_1, α_2, β_1, $\beta_2 \in$L and $\alpha_1 \leq \mu_A(e, q)$, $\alpha_2 \leq \mu_A(e, q)$ and $\beta_1 \geq \nu_A(e, q)$, $\beta_2 \geq \nu_A(e, q)$.

Case (i): If $\alpha_1 < \mu_A(x, q) < \alpha_2$ and $\beta_1 > \nu_A(x, q) > \beta_2$, then $A_{(\alpha_2, \beta_2)} \subseteq A_{(\alpha_1, \beta_1)}$.

Therefore, $A_{(\alpha_1, \beta_1)} \cup A_{(\alpha_2, \beta_2)} = A_{(\alpha_1, \beta_1)}$, but $A_{(\alpha_1, \beta_1)}$ is a Q-level subnearring of A.

Case (ii): If $\alpha_1 > \mu_A(x, q) > \alpha_2$ and $\beta_1 < \nu_A(x, q) < \beta_2$, then $A_{(\alpha_1, \beta_1)} \subseteq A_{(\alpha_2, \beta_2)}$.

Therefore, $A_{(\alpha_1, \beta_1)} \cup A_{(\alpha_2, \beta_2)} = A_{(\alpha_2, \beta_2)}$, but $A_{(\alpha_2, \beta_2)}$ is a Q-level subnearring of A.

Case (iii): If $\alpha_1 < \mu_A(x, q) < \alpha_2$ and $\beta_1 < \nu_A(x, q) < \beta_2$, then $A_{(\alpha_2, \beta_1)} \subseteq A_{(\alpha_1, \beta_2)}$.

Therefore, $A_{(\alpha_2, \beta_1)} \cup A_{(\alpha_1, \beta_2)} = A_{(\alpha_1, \beta_2)}$, but $A_{(\alpha_1, \beta_2)}$ is a Q-level subnearring of A.

Case (iv): If $\alpha_1 > \mu_A(x, q) > \alpha_2$ and $\beta_1 > \nu_A(x, q) > \beta_2$, then $A_{(\alpha_1, \beta_2)}$

$\subseteq A_{(\alpha_2,\beta_1)}$.

Therefore, $A_{(\alpha_1,\beta_2)} \cup A_{(\alpha_2,\beta_1)} = A_{(\alpha_2,\beta_1)}$, but $A_{(\alpha_2,\beta_1)}$ is a Q-level subnearring of A.

Case (v): If $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$, then $A_{(\alpha_1,\beta_1)} = A_{(\alpha_2,\beta_2)}$.

In all cases, union of any two Q-level subnearrings is a Q-level subnearring of A.

2.7 Theorem: Let A be a Q-intuitionistic L-fuzzy subnearring of a nearring R. If α_i, $\beta_j \in L$, $\alpha_i \le \mu_A(e, q)$ and $\beta_j \ge \nu_A(e, q)$ and $A_{(\alpha_i,\beta_j)}$, i, j∈I is a collection of Q-level subnearrings of A,

then their union is also a Q-level subnearring of A.

Proof: It is trivial.

2.8 Theorem: Any subnearring H of a nearring R can be realized as a Q-level subnearring of some Q-intuitionistic L-fuzzy subnearring of R.

Proof: Let A be the Q-intuitionistic L-fuzzy subset of a nearring R defined by

$\mu_A(x, q) = \alpha$ if $x \in H$, $0 < \alpha \le 1$

0 if $x \notin H$, and

$\nu_A(x, q) = \beta$ if $x \in H$, $0 < \beta \le 1$

0 if $x \notin H$,

and $\alpha + \beta \le 1$, where H is subnearring of a nearring R.

We claim that A is a Q-intuitionistic L-fuzzy subring of a nearring R.

Let x and y in R. If x and y in H, then x–y and xy in H, since H is a subnearring of R, we have $\mu_A(x-y, q) = \alpha$, $\mu_A(x, q) = \alpha$, $\mu_A(y, q) = \alpha$, $\mu_A(xy, q) = \alpha$ and $\nu_A(x-y, q) = \beta$, $\nu_A(x, q) = \beta$, $\nu_A(y, q) = \beta$, $\nu_A(xy, q) = \beta$. So, $\mu_A(x-y, q) \ge \mu_A(x, q) \wedge \mu_A(y, q)$ and $\mu_A(xy, q) \ge \mu_A(x, q) \wedge \mu_A(y, q)$. Also, $\nu_A(x-y, q) \le \nu_A(x, q) \vee \nu_A(y, q)$ and $\nu_A(xy, q) \le \nu_A(x, q) \vee \nu_A(y, q)$. If x, y $\notin$ H, then x–y and xy may or may not belong to H. Clearly $\mu_A(x-y, q) \ge \mu_A(x, q) \wedge \mu_A(y, q)$, $\mu_A(xy, q) \ge \mu_A(x, q) \wedge \mu_A(y, q)$ and $\nu_A(x-y, q) \le \nu_A(x, q) \vee \nu_A(y, q)$ and $\nu_A(xy, q) \le \nu_A(x, q) \vee \nu_A(y, q)$. Hence, A is a Q-intuitionistic L-fuzzy subnearring of R.

2.9 Theorem: The homomorphic image of a Q-level subnearring of a Q-intuitionistic L-fuzzy subnearring of a nearring R is a Q-level subnearring of a Q- intuitionistic L-fuzzy subnearring of a nearring R^I.

Proof: Let (R, +, .) and (R^I , +, .) be any two nearrings and f : R → R^I be a homomorphism. That is, f(x + y) = f(x) + f(y) and f(xy) = f(x)f(y), for all x and y in R. Let A be a Q-intuitionistic L-fuzzy subnearring of a nearring R and V be the homomorphic image of A under f. Clearly V is a Q-intuitionistic L-fuzzy subnearring of a nearring R^I. Let x and y in R and q in Q, implies f(x) and f(y) in R^I and $A_{(\alpha,\beta)}$ be a Q-level subnearring of A. That is, $\mu_A(x, q) \ge \alpha$ and $\nu_A(x, q) \le \beta$; $\mu_A(y, q) \ge \alpha$ and $\nu_A(y, q) \le \beta$; $\mu_A(x-y, q) \ge \alpha$, $\mu_A(xy, q) \ge \alpha$ and $\nu_A(x-y, q) \le \beta$, $\nu_A(xy, q) \le \beta$. We have to prove that f ($A_{(\alpha,\beta)}$) is a Q-level subnearring of V. Now, μ_V(f(x), q) ≥ $\mu_A(x, q) \ge \alpha$, which implies that μ_V(f(x), q) ≥ α; and μ_V(f(y), q) ≥ $\mu_A(y, q) \ge \alpha$, which implies that μ_V(f(y), q) ≥ α and μ_V(f(x)– f(y), q) = μ_V(f(x– y), q) ≥ μ_A(x–y, q) ≥ α, which implies that μ_V(f(x)– f(y), q) ≥ α. Also, μ_V(f(x)f(y), q) = μ_V(f(xy), q) ≥ μ_A(xy, q) ≥ α, which implies that μ_V(f(x)f(y), q) ≥ α. And, ν_V(f(x), q) ≤ ν_A(x, q) ≤ β, which implies that ν_V(f(x), q) ≤ β; ν_V(f(y), q) ≤ ν_A(y, q) ≤ β, which implies that ν_V(f(y), q) ≤ β and ν_V(f(x)– f(y), q) = ν_V(f(x–y), q) ≤ ν_A(x–y, q) ≤ β, which implies that ν_V(f(x)– f(y), q) ≤ β. Also, ν_V(f(x)f(y), q) = ν_V(f(xy), q) ≤ ν_A(xy, q) ≤ β, which implies that ν_V(f(x)f(y), q) ≤ β. Therefore, μ_V(f(x)–f(y), q) ≥ α,

ν_V(f(x)– f(y), q) ≤ β, μ_V(f(x)f(y), q) ≥ α and ν_V(f(x)f(y), q) ≤ β. Hence f ($A_{(\alpha,\beta)}$) is a Q-level subnearring of a Q-intuitionistic L-fuzzy subnearring V of R^I.

2.10 Theorem: The homomorphic pre-image of a Q-level subnearring of a Q-intuitionistic

L-fuzzy subnearring of a nearring R^I is a Q-level subnearring of a Q-intuitionistic L-fuzzy subnearring of a nearring R.

Proof: Let (R, +, .) and (R^I , +, .) be any two nearrings and f : R → R^I be a homomorphism. That is, f(x + y) = f(x) + f(y) and f(xy) = f(x)f(y), for all x and y in R. Let V be a Q-intuitionistic L-fuzzy subnearring of a nearring R^I and A be a homomorphic pre-image of V under f. Clearly A is a Q-intuitionistic L-fuzzy subnearring of the nearring R. Let f(x) and f(y) in R^I, implies x and y in R and q in Q. Let f($A_{(\alpha,\beta)}$) is a Q-level subnearring of V. That is, μ_V(f(x), q) ≥ α and ν_V(f(x), q) ≤ β; μ_V(f(y), q) ≥ α and ν_V(f(y), q) ≤ β; μ_V(f(x)–f(y), q) ≥ α, μ_V(f(x)f(y), q) ≥ α and ν_V(f(x)–f(y), q) ≤ β, ν_V(f(x)f(y), q) ≤ β. We have to prove that $A_{(\alpha,\beta)}$ is a Q-level subnearring of A. Now, $\mu_A(x, q) = \mu_V$(f(x), q) ≥ α, implies that $\mu_A(x, q) \ge \alpha$; $\mu_A(y, q) = \mu_V$(f(y), q) ≥ α, implies that $\mu_A(y, q) \ge \alpha$ and μ_A(x–y, q) = μ_V(f(x–y), q) = μ_V(f(x)–f(y), q) ≥ α, which implies that $\mu_A(x-y, q) \ge \alpha$. Also, μ_A(xy, q) = μ_V(f(xy), q) = μ_V(f(x)f(y), q) ≥ α, which implies that $\mu_A(xy, q) \ge \alpha$. And, $\nu_A(x, q) = \nu_V$(f(x), q) ≤ β, implies that $\nu_A(x, q) \le \beta$; $\nu_A(y, q) = \nu_V$(f(y), q) ≤ β, implies that $\nu_A(y, q) \le \beta$ and ν_A(x–y, q) = ν_V(f(x–y), q) = ν_V(f(x)–f(y), q) ≤ β, which implies that $\nu_A(x-y, q) \le \beta$. And, ν_A(xy, q) = ν_V(f(xy), q) = ν_V(f(x)f(y), q) ≤ β, which implies that $\nu_A(xy, q) \le \beta$. Therefore, μ_V(f(x)–f(y), q) ≥ α, ν_V(f(x)– f(y), q) ≤ β, μ_V(f(x)f(y), q) ≥ α and ν_V(f(x)f(y), q) ≤ β. Hence, $A_{(\alpha,\beta)}$ is a Q-level subnearring of a Q-intuitionistic L-fuzzy subnearring A of R.

2.11 Theorem: The anti-homomorphic image of a Q-level subnearring of a Q-intuitionistic

L-fuzzy subnearring of a nearring R is a Q-level subnearring of a Q-intuitionistic L-fuzzy subnearring of a nearring R^I.

Proof: Let (R, +, .) and (R^I , +, .) be any two nearrings and f : R → R^I be an anti-homomorphism. That is, f(x + y) = f(y) + f(x) and f(xy) = f(y)f(x), for all x and y in R. Let A be a Q-intuitionistic L-fuzzy subnearring of R and V be the anti-homomorphic image of A under f. Clearly V is a Q-intuitionistic L-fuzzy subnearring of R^I. Let x and y in R and q in Q, implies f(x) and f(y) in R^I. Let $A_{(\alpha,\beta)}$ be a Q-level subnearring of A. That is, $\mu_A(x, q) \ge \alpha$ and $\nu_A(x, q) \le \beta$; $\mu_A(y, q) \ge \alpha$ and $\nu_A(y, q) \le \beta$. And, μ_A((-y) + x, q) ≥ α , μ_A(yx, q) ≥ α and ν_A((-y) + x, q) ≤ β, ν_A(yx, q) ≤ β. We have to prove that f ($A_{(\alpha,\beta)}$) is a Q-level subnearring of V. Now, μ_V(f(x), q) ≥ $\mu_A(x, q) \ge \alpha$, which implies that μ_V(f(x), q) ≥ α; and, μ_V(f(y), q) ≥ $\mu_A(y, q) \ge \alpha$, which implies that μ_V(f(y), q) ≥ α. Now, μ_V(f(x)–f(y), q)= μ_V(f(x)+f(-y), q) = μ_V(f [(-y) + x], q) ≥ μ_A[(-y) + x, q] ≥ α, which implies that, μ_V(f(x)– f(y), q) ≥ α. Also, μ_V(f(x)f(y), q) = μ_V(f(yx), q) ≥ μ_A(yx, q) ≥ α, which implies that μ_V(f(x) f(y), q) ≥ α. And, ν_V(f(x), q) ≤ $\nu_A(x, q) \le \beta$, which implies that ν_V(f(x), q) ≤ β and ν_V(f(y), q) ≤ $\nu_A(y, q) \le \beta$, which implies that ν_V(f(y), q) ≤ β. Now, ν_V(f(x)–f(y), q) =ν_V(f(x)+f(-y), q) = ν_V(f [(-y) + x], q) ≤ ν_A[(-y) + x, q] ≤ β, which implies that ν_V(f(x)–f(y), q) ≤ β. Also,

ν_V(f(x) f(y), q) = ν_V(f(yx), q) ≤ ν_A(y x, q) ≤ β, which implies that ν_V(f(x)f(y), q) ≤ β. Therefore, μ_V(f(x)–f(y), q) ≥ α , ν_V(f(x)–f(y), q) ≤ β and μ_V(f(x)f(y), q) ≥ α, ν_V(f(x)f(y), q) ≤ β. Hence f($A_{(\alpha,\beta)}$) is a Q-level subnearring of a Q-intuitionistic L-fuzzy subnearring V of R^I.

2.12 Theorem: The anti-homomorphic pre-image of a Q-level subnearring of a Q-intuitionistic L-fuzzy subnearring of a nearring R^I is a Q-level subnearring of a Q-intuitionistic L-fuzzy subnearring of a nearring R.

Proof: Let (R, +, .) and (R^I , +, .) be any two nearrings and f : R → R^{I} be an anti-homomorphism. That is, f(x + y) = f(y) + f(x) and f(xy) = f(y)f(x), for all x and y in R. Let V be a Q-intuitionistic L-fuzzy subnearring of a nearring R^{I} and A be the anti-homomorphic pre-image of V under f. Clearly A is a Q-intuitionistic L-fuzzy subnearring of a nearring R. Let f(x) and f(y) in R^{I}, implies x and y in R and q in Q. Let f($A_{(\alpha, \beta)}$) be a Q-level subnearring of V. That is, μ_V(f(x), q) ≥ α and ν_V(f(x), q) ≤ β; μ_V(f(y), q) ≥ α and ν_V(f(y), q) ≤ β; μ_V((-f(y)) + f(x), q) ≥ α, μ_V(f(y)f(x), q) ≥ α and ν_V((-f(y)) + f(x), q) ≤ β, ν_V(f(y)f(x), q) ≤ β. We have to prove that $A_{(\alpha, \beta)}$ is a Q-level subnearring of A. Now, μ_A(x, q) = μ_V(f(x), q) ≥ α, which implies that μ_A(x, q) ≥ α; μ_A(y, q) = μ_V(f(y), q) ≥ α, which implies that μ_A(y, q) ≥ α. Now, μ_A(x–y, q) = μ_V(f(x–y), q) = μ_V(f(–y) + f(x), q) = μ_V((–f(y)) + f(x), q) ≥ α, which implies that μ_A(x–y, q) ≥ α. Also, μ_A(xy, q) = μ_V(f(xy), q) = μ_V(f(y)f(x), q) ≥ α, which implies that μ_A(xy, q) ≥ α. And, ν_A(x, q) = ν_V(f(x), q) ≤ β, which implies that ν_A(x, q) ≤ β and ν_A(y, q) = ν_V(f(y), q) ≤ β, which implies that ν_A(y, q) ≤ β and ν_A(x–y, q) = ν_V(f(x– y), q) = ν_V(f(–y)+ f(x), q) = ν_V((–f(y)) + f(x), q) ≤ β, which implies that ν_A(x– y, q) ≤ β. And, ν_A(xy, q) = ν_V(f(xy), q) = ν_V(f(y)f(x), q) ≤ β, which implies that ν_A(xy, q) ≤ β. Therefore, μ_V(f(x) –f(y), q) ≥ α, ν_V(f(x)– f(y), q) ≤ β and μ_V(f(x)f(y), q) ≥ α, ν_V(f(x)f(y), q) ≤ β. Hence $A_{(\alpha, \beta)}$ is a Q-level subnearring of a Q-intuitionistic L-fuzzy subnearring A of R.

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