



## SQC in Paper Machine – An Mr-Chart Approach

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### ABSTRACT

*This paper presents a statistical process control (SPC) chart for variables in a new dimension for the paper machine in the paper industry. The traditional control charts like X-bar and R charts may be inadequate sometimes when the process exhibits abnormal situations and those may be misleading and may lead to false decisions. To overcome such situations, a slightly revised control chart provides the correct picture to take right decisions without affecting the process unnecessarily.*

**Keywords : paper machine, statistical process control, MR-control chart**

No production process is good enough to produce all items of product exactly alike; a certain amount of inherent or natural variability will always exist. The variability may arise due to the presence of assignable causes or chance causes. Though the chance causes cannot be eliminated and out-of-control, the assignable causes can be discovered and eliminated. The use of statistical techniques helps us determine the presence of assignable cause and give a warning to make adjustments in the process and avoid the process going out-of-control in near future.

The statistical quality control techniques have been tried many times in the paper industry earlier, but there had always been a little bit of difficulty in exactly concluding the presence of assignable causes thus those techniques have been unsuccessful to some extent. There may be several technical reasons for this fact, but the main one is the non-acquaintance with certain control charts, referred to as special, and related to control continuous processes, as in the case of the paper machine.

These charts are exceptions with regard to the conventional control charts, so that it is important to understand for which reasons they should be adopted and how to work them out.

This paper tries to introduce the control chart called MR-chart, as it monitors simultaneously three characteristics of the process. In the end, a comparative example of this one and a traditional chart is presented.

### 2.Traditional Control Charts

The control chart, developed by Shewhart (1931), provides a basis for deciding whether the variation in the output is due to assignable causes. It admits that a given observation of a quality characteristic  $X(xt)$  obtained from a statistically stable process (with constant average  $m$  and standard deviation  $s$ ) can be adequately represented through the mathematical model:

$$xi = m + ei \quad \dots \quad (1)$$

where  $ei$  is normally and independently distributed, with mean 0 and standard deviation 1.

Shewhart made use of the area property of normal distribution and adopted the control limits for a random variable  $x$ , on

the control chart as follows:

$$\begin{aligned} \text{Lower Control Limit} &= \mu + 3\sigma_{\bar{x}} \\ \text{Central Line} &= \mu \quad \dots \quad (2) \\ \text{Upper Control Limit} &= \mu - 3\sigma_{\bar{x}} \end{aligned}$$

Due to the fact that the sample related means tend to have a normal distribution, by virtue of the Central Limit Theorem, as well as because the variance of the means is lower than the process variance (or individual values), it is usual to adopt mean ( $x$ -bar) and range (R) type charts. The latter are those most frequently used in every kind of industry and the paper making one is no exception.

The control limits for the x-bar chart are as follows:

$$\begin{aligned} \text{Lower Control Limit} &= \bar{X} + A_2 \bar{R} \\ \text{Central Line} &= \bar{X} \quad \dots \quad (3) \\ \text{Upper Control Limit} &= \bar{X} - A_2 \bar{R} \end{aligned}$$

where  $x$ -bar is the grand mean of the  $k$  samples obtained,  $A_2$  being defined as:

$$A_2 = \frac{3}{d_2 \sqrt{n}} \quad \dots \quad (4)$$

where  $d_2$  is a correction factor of the bias introduced by the replacement of  $s$  with  $R$ -bar in the formula and  $R$ -bar is the mean range defined as:

$$\bar{R} = \frac{1}{k} \sum R_i \quad (5)$$

The  $x$ -bar chart is used along with the Range chart (R), which has control limits as:

$$\begin{aligned} \text{Lower Control Limit} &= D_4 \bar{R} \\ \text{Central Line} &= \bar{R} \quad \dots \quad (6) \\ \text{Upper Control Limit} &= D_3 \bar{R} \end{aligned}$$

where  $D_3$  and  $D_4$  are also correction factors, a function of

the size of the sample (n), supplied in Annex A. Further details about these correction factors may be obtained at Montgomery2 (1996).

A few limitations of control charts are as follows:

The control charts are robust as to deviations from normality in the data, as it was demonstrated by Burr3 (1967) and Schilling, Nelson4 (1976), i.e. even when a process generates data with a distribution that cannot be admitted to be normal, even so the control charts will work satisfactorily.

When there is no statistical independence between the data (presence of serial correlation or autocorrelation), the model proposed by Shewhart is unsuitable and may lead to mistakes in the interpretation of the statistical stability of the process. In other words, there is an excessive generation of false alarms, i.e. several points will fall outside the control limits, indicating the presence of a special cause of variation, while in fact this one does not exist.

The control chart, which monitors the process centering, makes use of the variation inside the sample to establish the distance of its control limits with regard to the grand mean. Intrinsically it is admitted that the variation represented by R-bar in the formula (4) is suitable to define the amount of variation permitted for the sample related means. It is noted that the distance from the control limits to the mean line in the x-bar chart is a function of R-bar, as well as of factor A2, which on the other hand depends on the size of sample n.

**3. An Experiment with paper machine**

On a paper machine, 20 samples of five sub-sample sizes are taken in different time periods in a day of a run. These samples are tested for their weight. The results of 20 consecutive rolls are shown in Table 1.

With the conventional use of x-bar charts and Range (R), one sample in each roll was collected the mean (x-bar) and the range (R) per paper roll was calculated.

The values of grand mean (x-bar) and average range (R-bar) are obtained as follows:

$$\bar{x} = \frac{\sum_{i=1}^{20} \bar{x}_i}{20} = 75.1 \quad \bar{R} = \frac{\sum_{i=1}^{20} R_i}{20} = 1.60$$

Applying these results to the formulas (3) and (6) and remembering that in this case n = 5, the control charts of Figure 1, shown in the following, are obtained.

The control chart of the mean indicates the presence of a special cause of variation: stratification. In other words, the chart points to an apparently curious problem, which is the lack of variation in the process. When stratification appears, its cause is usually either in the way the samples have been collected or else, how they have been applied to the control limit calculation. In the particular case, the charts have been set up without analyzing which kind of variation is being pointed to on each of them.

| Roll | Values         | Mean | Range |
|------|----------------|------|-------|
| 1    | 76 76 75 76 75 | 75.6 | 1     |
| 2    | 74 76 75 75 75 | 75.0 | 2     |
| 3    | 78 76 76 76 76 | 76.4 | 2     |
| 4    | 76 75 74 76 75 | 75.2 | 2     |

2 Montgomery, D.C. Introduction to statistical quality control. 3ed. New York, Wiley, 1996  
 3 Burr, J.T. The effects of non-normality on constants for x-bar and R charts. Industrial Quality Control, Milwaukee, v.23, p.563-9, 1967.  
 4 Schilling, E.G.; Nelson, P.R. The effect of nonnormality on the control limits of x-bar charts. Journal of Quality Technology, Milwaukee, v.8, 1976.

| Roll | Values         | Mean | Range |
|------|----------------|------|-------|
| 5    | 76 75 74 75 74 | 74.8 | 2     |
| 6    | 73 74 74 75 73 | 73.8 | 2     |
| 7    | 73 74 72 72 72 | 72.6 | 2     |
| 8    | 73 73 73 72 72 | 72.6 | 1     |
| 9    | 72 73 73 73 73 | 72.8 | 1     |
| 10   | 73 71 73 73 73 | 72.6 | 2     |
| 11   | 75 75 76 75 75 | 75.2 | 1     |
| 12   | 76 75 74 75 74 | 74.8 | 2     |
| 13   | 77 76 74 75 75 | 75.4 | 3     |
| 14   | 76 76 76 76 76 | 76.0 | 0     |
| 15   | 76 75 77 77 76 | 76.2 | 2     |
| 16   | 77 76 77 77 77 | 76.8 | 1     |
| 17   | 77 76 77 77 77 | 76.8 | 1     |
| 18   | 75 77 78 78 76 | 76.8 | 3     |
| 19   | 76 76 76 76 77 | 76.2 | 1     |
| 20   | 77 76 76 76 77 | 76.4 | 1     |

Table 1 – Basis weight Values of 20 Paper Rolls

The range (R) chart always presents a variation called within sample, i.e. in the present situation the basis weight variation in machine cross direction, since all five specimens are thus obtained. On the other hand, the x-bar chart shows another type of variation, called between samples, i.e. the basis weight variation in machine direction.

It is known by the practice in controlling this process that the machine cross-direction variations are of a totally distinct nature from that of the machine direction variations. Consequently, it can be said that using the machine cross direction variation (represented here by R-bar) to establish how much the process can vary in machine direction (x-bar) is a totally mistaken practice.

**Results**

|                     | Xbar  | Range |
|---------------------|-------|-------|
| x-bar value         | 75.10 |       |
| R bar               | 1.60  |       |
| Upper control limit | 76.02 | 3.38  |
| Center line         | 75.10 | 1.60  |
| Lower control limit | 74.18 | 0     |

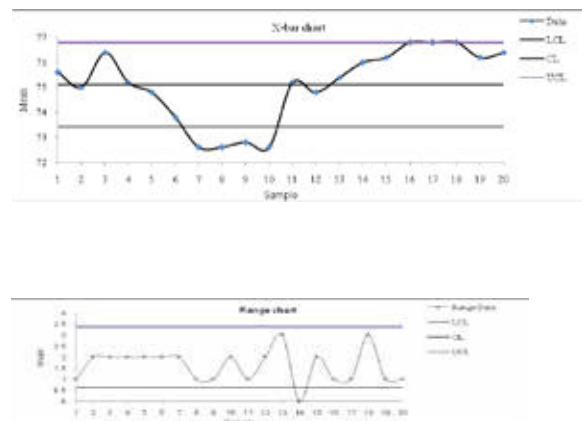


Figure 1 – Control Charts for Basis Weight Means and Ranges

The x-bar control chart shows that several points are falling outside the control limits, but the R-chart has no evidence for presence of variation in the process. However, while checking the process parameters, everything is in perfect order. Hence it is decided to go for further analysis through MR-chart using the moving averages.

**4. MR Control Chart**

On a conventional chart for variables, the R-bar value determines the distance at which the control limits are from the mean line on the x-bar chart. In other words, the variation within sample determines the difference that can exist in the variation between samples, before the latter is considered as statistically significant.

However, there are situations in which the variation inside the sample does not serve as a good basis for establishing the control limits of x-bar. Cases where this occurs are:

- in batch manufacturing of lots, where the differences between lots are pronounced due to the inherent raw material variation and there is no possibility of reducing it;
- in continuous product manufacturing (paper machine, for instance), where the machine cross direction variation is no suitable basis to establish the machine direction variation range due to their completely opposite natures.

The MR control chart is a combination of the mean and range charts (x-bar and R) with the charts for individual values and moving range (x-MR), according to Ramos (2000), so that they make it possible to simultaneously control more than two types of variation.

The R chart will monitor the variation within sample. Consequently:

$$\begin{aligned} \text{Lower Control Limit} &= D_4 \bar{R} \\ \text{Central Line} &= \bar{R} \quad \dots \quad (6) \end{aligned}$$

$$\text{Upper Control Limit} = D_3 \bar{R}$$

The MR chart, for its part, will serve as a basis to establish the distance of the control limits to the mean line on the x-bar chart. Therefore:

$$\begin{aligned} \text{Lower Control Limit} &= D_4 \overline{MR} \\ \text{Central Line} &= \bar{R} \quad \dots \quad (7) \end{aligned}$$

$$\text{Upper Control Limit} = D_3 \overline{MR}$$

Finally, the x-bar chart will be calculated by means of the formulas:

$$\begin{aligned} \text{Lower Control Limit} &= \bar{X} + J_2 \overline{MR} \\ \text{Central Line} &= \bar{X} \quad \dots \quad (8) \end{aligned}$$

$$\text{Upper Control Limit} = \bar{X} - J_2 \overline{MR}$$

where J2 is given in Annexure-A.

**5. The Paper Machine Experiment modified**

Since the control of variation in both machine and machine cross directions are important, the MR graph is a sensible option. The same data of Table 1 are re-presented in the following (Table 2). However, a new column was added, that of the moving range of values taken in pairs (the modulus of the highest minus the lowest value).

Following is obtained with the results of this last table:

$$MR = \frac{\sum_{i=1}^{20} MR_i}{19} = 0.63$$

The remaining grand mean and mean range values remain unchanged. The control limits for the MR chart are:

$$\begin{aligned} \text{Lower Control Limit} &= \\ \bar{X} + J_2 \overline{MR} &= 75.1 + 2.66(0.63) = 76.78 \\ \text{Central Line} &= \bar{X} = 75.1 \quad \dots \quad \dots \quad (8) \end{aligned}$$

$$\text{Upper Control Limit} =$$

The MR-chart is presented in Figure 2. By analyzing the control graphs, it can be remarked that the process is not stable (there is some points fall outside the control limits of x-bar chart which indicates the presence of special variation causes having an influence on weights).

| Roll | Values |    |    |    |    | Mean | Range | Moving Range |
|------|--------|----|----|----|----|------|-------|--------------|
| 1    | 76     | 76 | 75 | 76 | 75 | 75.6 | 1     | -            |
| 2    | 74     | 76 | 75 | 75 | 75 | 75.0 | 2     | 0.6          |
| 3    | 78     | 76 | 76 | 76 | 76 | 76.4 | 2     | 1.4          |
| 4    | 76     | 75 | 74 | 76 | 75 | 75.2 | 2     | 1.2          |
| 5    | 76     | 75 | 74 | 75 | 74 | 74.8 | 2     | 0.4          |
| 6    | 73     | 74 | 74 | 75 | 73 | 73.8 | 2     | 1.0          |
| 7    | 73     | 74 | 72 | 72 | 72 | 72.6 | 2     | 1.2          |
| 8    | 73     | 73 | 73 | 72 | 72 | 72.6 | 1     | 0            |
| 9    | 72     | 73 | 73 | 73 | 73 | 72.8 | 1     | 0.2          |
| 10   | 73     | 71 | 73 | 73 | 73 | 72.6 | 2     | 0.2          |
| 11   | 75     | 75 | 76 | 75 | 75 | 75.2 | 1     | 2.6          |
| 12   | 76     | 75 | 74 | 75 | 74 | 74.8 | 2     | 0.4          |
| 13   | 77     | 76 | 74 | 75 | 75 | 75.4 | 3     | 0.6          |
| 14   | 76     | 76 | 76 | 76 | 76 | 76.0 | 0     | 0.6          |
| 15   | 76     | 75 | 77 | 77 | 76 | 76.2 | 2     | 0.2          |
| 16   | 77     | 76 | 77 | 77 | 77 | 76.8 | 1     | 0.6          |
| 17   | 77     | 76 | 77 | 77 | 77 | 76.8 | 1     | 0            |
| 18   | 75     | 77 | 78 | 78 | 76 | 76.8 | 3     | 0            |
| 19   | 76     | 76 | 76 | 76 | 77 | 76.2 | 1     | 0.6          |
| 20   | 77     | 76 | 76 | 76 | 77 | 76.4 | 1     | 0.2          |

Table 2 – Basis Weight Values of 20 Paper Rolls

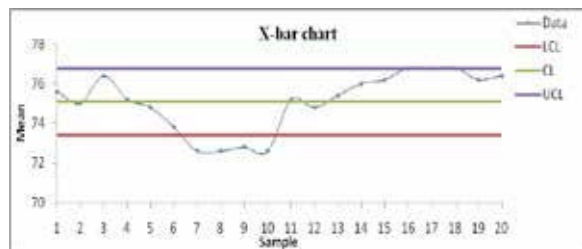


Figure 2 – MR Control Chart.

The revised X-bar chart shows the process is not stable and indicates the presence of assignable causes as is evident in the MR-chart. Hence it is suggested to the quality control department to thoroughly check all the process parameters before taking the next run.

**6. Conclusions**

It is extremely important in the paper industry to use MR chart to explore the presence of assignable causes, as sometimes the usual charts may not indicate the presence of the same. A separate template in MS-Excel is created by the author for this chart.

If an appropriate chart is not selected, it may cause several problems in the quality control and improvement. Thus, using conventional mean and range (x-bar and R) control charts will lead to wrong conclusions. The MR control chart is a suitable solution.

**7. Literature**

Burr, J.T. The effects of non-normality on constants for x-bar and R charts. *Industrial Quality Control*, Milwaukee, v.23, p.563-9, 1967.

Montgomery, D.C. *Introduction to statistical quality control*. 3 ed. New York, Wiley, 1996

Schilling, E.G.; Nelson, P.R. The effect of nonnormality on the control limits of x-bar charts. *Journal of Quality Technology*, Milwaukee, v.8, 1976.

Shewhart, W.A. *Economic control of quality of manufactured product*. Milwaukee, ASQC Quality Press, 1989.

Wheeler, D.J. *Advanced topics in statistical process control*. Knoxville, SPC, 1996.

[http://paper.ijcsns.org/07\\_book/201001/20100119.pdf](http://paper.ijcsns.org/07_book/201001/20100119.pdf)

**Annex A - FACTORS FOR CONTROL LIMIT CALCULATION**

| n  | A <sub>2</sub> | A <sub>3</sub> | J <sub>2</sub> | B <sub>3</sub> | B <sub>4</sub> |
|----|----------------|----------------|----------------|----------------|----------------|
| 2  | 1.880          | 2.695          | 2.660          | -              | 3.267          |
| 3  | 1.023          | 1.954          | 1.772          | -              | 2.568          |
| 4  | 0.729          | 1.628          | 1.457          | -              | 2.266          |
| 5  | 0.577          | 1.427          | 1.290          | -              | 2.089          |
| 6  | 0.483          | 1.287          | 1.184          | 0.030          | 1.970          |
| 7  | 0.419          | 1.182          | 1.109          | 0.118          | 1.882          |
| 8  | 0.373          | 1.099          | 1.054          | 0.185          | 1.815          |
| 9  | 0.337          | 1.032          | 1.010          | 0.239          | 1.761          |
| 10 | 0.308          | 0.975          | 0.975          | 0.284          | 1.716          |

| n  | D <sub>3</sub> | D <sub>4</sub> | D     | c <sub>4</sub> | d <sub>2</sub> |
|----|----------------|----------------|-------|----------------|----------------|
| 2  | -              | 3.267          | 0.709 | 0.798          | 1.128          |
| 3  | -              | 2.574          | 0.524 | 0.886          | 1.693          |
| 4  | -              | 2.282          | 0.446 | 0.921          | 2.059          |
| 5  | -              | 2.114          | 0.403 | 0.940          | 2.326          |
| 6  | -              | 2.004          | 0.375 | 0.952          | 2.534          |
| 7  | 0.076          | 1.924          | 0.353 | 0.959          | 2.704          |
| 8  | 0.136          | 1.864          | 0.338 | 0.965          | 2.847          |
| 9  | 0.184          | 1.816          | 0.325 | 0.969          | 2.970          |
| 10 | 0.223          | 1.777          | 0.314 | 0.973          | 3.078          |

Source: Montgomery D. C. *Introduction to statistical quality control*. 3rd ed. New York, John Wiley, 1996.