Volume : 2 | Issue : 4 | April 2013 ISSN - 2250-1991

Research Paper Management

# Probabilistic Periodic Review Model with a Mixture of Backorder and Lost Sales Under Constraint and Varying Holding cost

* D. S. Parihar

* Assistant Professor, Department of Management, Ansal University, Gurgaon, Sector 55, Golf Course Road, Gurgaon, Haryana - 122003 (India)

ABSTRACT

*In this study a probabilistic periodic review model under constraint and varying holding cost is developed with a mixture of backorders and lost sales. The demand in the protection interval is assumed to be normally distributed. The effect of model parameters on inventory circle, maximum inventory level and expected total cost are discussed. The models with total backorders or lost sales are special cases of the present study. The numerical example shows that with an increase in backorders the expected total cost decreases. The effect of the variability in demand in protection time is also discussed. It is found that as the variability in demand in protection time increase the maximum inventory level increases and expected total cost decreases.*

Keywords : Inventory, Protection interval, Lost sales.

## Introduction

The cost and operation of inventory depends a great deal on what happens when the system is out of stock. Especially, for products with high direct profitability and/or high sale value, the cost of lost demand is high. A manager's intension is always to explore the possibility of improving the current system behaviors so as to minimize (maximize) the total cost (profit). It can be observed from the real markets, for the well known products or fashionable goods, such as certain brand products, hi-fi equipments and cloths, customers prefer asking their unsatisfied demands to be backordered. Beside the products themselves, there is a potential factor that may motivate the customers, desire for backorders. This factor is the image of the selling shop. In general, the products that are sold by shops with positive image (customers friendly), customers may be more willing to wait for their desired items. In order to establish good image and to enhance customer's loyality, many efforts such as up grading the servicing facilities, maintaining the high quality of products and increasing expenditure on advertisement could be made by a selling shop. In addition other endeavors such as mailing greeting cards and providing free gifts can also be done to establish a good relationship with customers.

There are several research papers that discuss the partial backorder situation

(e.g.; Owyang and Wu (1998); Moon and Choi (1998); Hariga and Ben-daya (1999); Lan et al (1999). Montgomery et al (1973) is amoung the first to analyze and solve the stochastic demand, inventory problems with partial backorders, where the continuous review and periodic review cases are presented. Most of the inventory models discuss two extreme situations regarding the demand process when items are stockout. They are: (1) all of the demand within shotage period is backordered and (2) all of the demand within shortage period is lost sale. Although the second model is atleast as important as the first, it has received for less attention from researches, then the first. This is because the lost sales inventory models are much less analytically tractable then backorder models. Following are some research papers which deals stockout situations with all of the demand within shortage period is lost sale or partially backordered. Nick T. thmopoulos (2004) discussed how to estimate the lost sale demand, how to control the lost sale and how to measure an efficient service level.

Bore-Ren chaung etal (2004) investigated stochastic periodic review inventory model with optional lost sales caused by investment strategy. A.G.De Kok (1985) proposed the approximations for a lost sales productions inventory control model with service level constraints. Martin I. Reiman (2004) considered the single items, single location, and continuous review inventory modelwith lost sales. Chander Sekhar Das (1983) discussed (Q,r) inventory model with time weighted backorders.

In recent papers researchers discussed probabilistic Economic Order Quantity model with various constraints. Hala A-Fergang (2005) developed a periodic review probabilistic multi-items inventory system with zero lead time under constraints and varying order cost. He investigated the probable safety stock multi-item, single source inventory model with zero lead time and varing order cost under two constraints, one of them of the expected molding cost and the other on the expected cost of safety stock. Kotab Abd-EL-Hamid Mahmood Kota and Huda Mohamed Hamid Al- Sanbari (2011) studies a constraints probabilistic Economic Order Quantity model under varying order cost and zero lead time using geometric programming. Hala Aly Fergangxy and Naglaa Hassan El-Sodang (2011) Investigated a probabilistic periodic review (Qm, N) Backorders and lost sales inventory models under constraints and varying holding cost and normally distributed protection interval demand. He discussed two situations: (1) All of the demand during shortage period is backordered and (2) All of the demand within shortage period is lost sale.

The aim of this study is to study a mixture probabilistic periodic review inventory model with partial backorders (lost sales) under constraints and vaying holding cost. The lead time demand is assumed to be normally distributed. Further it is intended to see the effect of demand variation in lead time and the maximum inventory level and the minimum expected annual total cost.

## Model

The model with a mixture of back orders and lost sales is developed with the following assumptions.

1. The system is periodic review, inventory is replenished at equal time intervals.
2. The inventory cycle N is defined as the time between the placements of two successive orders.
3. The average number of cycles per year can be written as 1/N.
4. The lead time L is constant.
5. The purchase cost is constant independent of quantity ordered.
6. The cost Cr of making a review is independent of variable Qm. where Qm is the maximum inventory level.
7. Holding cost per unit is a varying function of review time N, give as $C_h$ (N) = $C_h N^{\beta}$, $.0.0 \le \beta \le 0.1$
8. The cost $C_b$ of a backorder is independent of time at which the backorders exist.
9. The cost $C_l$ of a lost sale is constant.
10. During stock out period only a fraction $\gamma (0 \le \gamma \le 1)$ of the demand is backordered and the remaining fraction is lost sale.

The relevant annual expected cost is given by
E(TC) = E(PC)+E(RC)+E(OC)+E(HC)+E(BC)+E(LC) (1)

Where,
E (PC) = the expected annual purchase cost
E (OC) = the expected annual ordering cost
E (HC) = the expected annual holding cost
E (BC) = the expected backorder cost
E (LC) = the expected lost sale cost
E (TC) = the expected annual total cost

Since the annual purchase cost is constant, equation (1) takes the following form.

E(TC) = E(RC)+E(OC)+E(HC)+E(BC)+E(LC) (2)

The different terms in equation (2) are given as

$$\mathrm{E(HC)} = C_{h}N^{\beta}\left(Q_m - DL - \frac{DN}{2}\right) + (1-\gamma)\ C_h N^{\beta} \int_{Q_m}^{\infty} (X - Q_m) f(X; L+N)dx \quad (3)$$

$$\mathrm{E(BC)} = \gamma C_b \int_{Q_m}^{\infty} (X - Q_m) f(X; L+N)dx \quad (4)$$

$$\mathrm{E(LC)} = C_l\,(1-\gamma) \int_{Q_m}^{\infty} (X - Q_m) f(X; L+N)dx \quad (5)$$

$$\mathrm{E(OC)} = \frac{C_o}{N} \quad (6)$$

$$\mathrm{E(RC)} = \frac{C_r}{N} \quad (7)$$

The expected annual cost is given by

$$\mathrm{E(TC)} = C_h N^{\beta}\left(Q_m - \mu - \frac{DN}{2}\right) + C_h(1-\gamma) \int_{Q_m}^{\infty} (X - Q_m) f(X; L+N)dx + \gamma \frac{C_b}{N} \int_{Q_m}^{\infty} (X - Q_m) f(X; L+N)dx + C_l\left(\frac{1-\gamma}{N}\right) \int_{Q_m}^{\infty} (X - Q_m) f(X; L+N)dx \quad (8)$$

Where, D is the annual demand rate and $\mu$ the expected value of protection interval demand.

The optimization of E (TC) subject to constraints $C_r \le K_r$ is carried out with Lagrange multiplies method, where $K_r$ is the expected annual review cost. The Lagrange function is given by

$$L(Q_m, N) = \frac{C_r}{N} + \frac{C_o}{N} + C_h N^{\beta}\left(Q_m - \mu - \frac{DN}{2}\right) + C_h(1-\gamma)N^{\beta} \int_{Q_m}^{\infty} (X - Q_m) f(X; L+N)dx + \frac{\gamma C_b}{N} \int_{Q_m}^{\infty} (X - Q_m) f(X; L+N)dx + C_l \frac{(1-\gamma)}{N} \int_{Q_m}^{\infty} (X - Q_m) f(X; L+N)dx + \lambda\left(\frac{C_r}{N} - K_r\right) + \frac{C_o}{N} \quad (9)$$

Where, $\lambda$ is the Lagrange multiplier.

The optimum value $Q_m^*$ and $N^*$ are found by setting the corresponding first partial derivatives of L ($Q_m$, N) equal to zero as follows:

$$\frac{\partial L(Q_m, N)}{\partial Q_m}|N = 0 \quad (10)$$

$$\frac{\partial L(Q_m, N)}{\partial Q_m}|Q_m = 0 \quad (11)$$

The equation (10) and (11) give the maximum values N and Qm as shown in appendix A. The solution procedure is described in appendix A.

The following equations are found with equation (10) and (11) respectively.

$$\int_{Q_m}^{\infty} f(X; N^*)dx = \frac{C_h N^{(\beta+1)}}{\gamma C_b + C_h(1-\gamma)N^{(\beta+1)} + (1-\gamma)C_l} \quad (12)$$

$$\left[\gamma C_b + (1-\gamma)C_l - C_h\beta(1-\gamma)N^{(\beta+1)}\right]\int_{Q_m}^{\infty} f(X; L+N^*)dx + C_o + (1+\lambda)C_r + C_h\beta(Q_m^* - \mu)N^{*(\beta+1)} - \frac{C_h D}{2}(1+\beta)N^{*(\beta+1)} \quad (13)$$

Model with the normally distributed protection demand.

With the assumption that the demand in lead time follows the normal distribution. The probability density function is given by

$$f(X; L+N) = \frac{1}{\sigma\sqrt{L+N}.\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)}, 0 < x < \infty \quad (14)$$

Where,
$\mu = D(L+N)$ and

$\sigma\sqrt{L+N}$ is standard deviation

With the help of equation (14) and (12) the following equation is obtained.

$$1 - \phi\left(\frac{Q_m^* - \mu}{\sigma\sqrt{L+N}}\right) = \frac{C_h N^{*(\beta+1)}}{\left[C_h(1-\gamma)N^{*(\beta+1)} + \gamma C_b + (1-\gamma)C_l\right]} F \quad (15)$$

From equation (13) the optional inventory cycle is the solution of the following equation.

$$\frac{C_h D}{2}(1+\beta)N^{*(\beta+2)} - C_h\beta\left[Q_m^* - \mu + (1-\gamma)\left\{\sigma\sqrt{L+N^*}\,\theta\left(\frac{Q_m^* - \mu}{\sigma\sqrt{L+N^*}}\right) + (\mu - Q_m^*)\left(1 - \phi\left(\frac{Q_m^* - \mu}{\sigma\sqrt{L+N^*}}\right)\right)\right\}\right] N^{*(\beta+1)} + C_o + (1+\lambda)C_r + [\gamma C_b + (1-\gamma)C_l]\left[\sigma\sqrt{L+N^*}\,\theta\left(\frac{Q_m^* - \mu}{\sigma\sqrt{L+N^*}}\right) + (\mu - Q_m^*)\left\{1 - \phi\left(\frac{Q_m^* - \mu}{\sigma\sqrt{L+N^*}}\right)\right\}\right] = 0$$

(16)

Where $\theta$ and $\phi$ denote the standard normal p.d.f and cumulative distribution function (c.d.f) respectively.

Equation (15) and (16) are solved numerically using iteration process. The solution procedure is given in appendix A.

Numerical example
In order to illustrates the model the inventory cycle with following data in considered:

D= Rs.600per year, L=6 months, σ =25.981, μ =450, $C_h$ =Rs.3, Co =Rs.13, $C_r$ =Rs.12, $C_b$ =Rs.25, Ce =Rs .25 and Kr =44.3

The results obtained by the above example are shown in Tables (1-2)

**Results and discussion**
In this study a probabilistic periodic review ($Q_m$, and N) model under constraint and varying holding cost is developed with a mixture of back order and lost sales. Hala Aly Fergany and Naglaa Hasan El-Sudan (2011) models with back order or lost sales situations are particular cases of the present study. The demand in the protection interval is assumed to be normally distributed the results of the numerical example are illustrated through Tables (1-2)

**Table1. Effect of changes in model parameter in the inventory cycle, maximum inventory level and total cost.**

| | $\beta = 0.02$ | | | $\beta = 0.06$ | | | $\beta = 0.1$ | | |
|---|---|---|---|---|---|---|---|---|---|
| $\gamma$ | N* | $Q_m^*$ | Min. E (TC) | N* | $Q_m^*$ | Min. E (TC) | N* | $Q_m^*$ | Min. E (Tc) |
| 0.0 | 0.2709263 | 501.783 | 873.726 | 0.2708938 | 502.238 | 887.542 | 0.2715043 | 502.487 | 900.147 |
| 0.2 | 0.2709192 | 501.554 | 872.388 | 0.2709143 | 502.238 | 886.301 | 0.2710134 | 502.698 | 899.422 |
| 0.4 | 0.2709960 | 501.557 | 870.968 | 0.2709256 | 502.239 | 884.782 | 0.2718053 | 502.497 | 897.675 |
| 0.6 | 0.2709289 | 501.555 | 869.648 | 0.2709084 | 502.010 | 883.830 | 0.2711545 | 502.475 | 897.061 |
| 0.8 | 0.2709919 | 501.326 | 868.278 | 0.2713727 | 502.026 | 882.186 | 0.2709432 | 502.468 | 896.090 |
| 1.0 | 0.2709600 | 501.326 | 866.874 | 0.2708902 | 501.781 | 881.324 | 0.2710292 | 502.470 | 894.861 |

Table1 shows the effects of changes in model parameters on the inventory cycle, maximum inventory level and the expected annual total cost. It is found that as the parameters $\beta$ in its given range there is small variation in inventory cycle and maximum inventory level. The expected annual total cost creases with an increase ab $\beta$.The numerical result of the example shows that as the back orders of the demand increases the expected total cost decreases.

Table2. Effect of customers demand variability in protection interval on inventory cycle, maximum inventory level

and total cost. $\gamma$ = 0.2

| | σ=10 | | | σ=10 | | |
|---|---|---|---|---|---|---|
| $\beta$ | N* | $Q_m^*$ | Exp. Min. (TC) | N* | $Q_m^*$ | Exp. Min. (TC) |
| 0.01 | 0.2708889 | 469.755 | 868.381 | 0.2709323 | 529.0224 | 868.286 |
| 0.02 | 0.2709302 | 469.8433 | 872.941 | 0.2710003 | 529.3770 | 871.812 |
| 0.03 | 0.2709229 | 469.9310 | 876.477 | 0.2709094 | 529.7236 | 875.475 |
| 0.04 | 0.2709358 | 469.9312 | 879.9611 | 0.2709772 | 529.7272 | 878.910 |
| 0.05 | 0.2709734 | 470.0195 | 883.384 | 0.2709224 | 530.0756 | 881.596 |
| 0.06 | 0.2709287 | 470.1067 | 886.803 | 0.2709719 | 530.4290 | 885.791 |
| 0.07 | 0.2709768 | 470.1950 | 890.129 | 0.2709228 | 530.7780 | 889.221 |
| 0.08 | 0.2708918 | 470.1940 | 893.474 | 0.2709359 | 530.7780 | 892.541 |
| 0.09 | 0.2708886 | 470.1940 | 896.745 | 0.2709890 | 530.7810 | 895.000 |

Table 2 shows the effect of customers demand variability in protection interval on inventory cycle, maximum inventory level and expected total cost. As the variability in customer demand in protection interval increases, the variation in inventory cycle is small. The values of maximum inventory level are higher in the case of large variability $(\sigma = 4)$ in compared to lower variability$(\sigma = 0)$. At higher customers order variability the expected total cost is less in compared to less variability situation.

**Conclusion**

A probabilistic periodic review model under constraint and varying holding cost is developed with a mixture of backorders and lost sales. The models with complete backorders or lost sales are obtained as special cases of the present study. The effect of changes in model parameters on inventory cycle, maximum inventory level and total expected cost is illustrated with a numerical example. The numerical results show that as the backorders of the demand in protection interval increases, the expected total cost decreases. It is found that as the variability in demand in the protection interval increases, the total expected cost decreases and the maximum inventory level increases.

**REFERENCES**

Bor-Ren chuang, Liang-Yuh Ouyang and Yu-Jen Lin, 2004. “Investigating in lost sales reduction on periodic review inventory model with a service level constraint”. Tamui Oxford Journal of Mathematical Science 20(2), 187-204. Aletheia University. | Chandrashekhar Das, 1983, “Q, r inventory models with time-weighted backorders”, J. opl.Soc. 34 (5) 401-412. | De Kok, A.G. 1985, “Approximations for a lost sales production/Inventory control model with service level constraints”. Management Science, 31 (6), 729-737. | Hala Aly Fergany and Naglaa Hassan El-Sadany(2011),’Probabilistic Periodic review (Qm, N) backorder and lost sales inventory models under constraints and varying holding cost normally distributed protection interval demand. | Hala A. Fergany, 2005, ‘periodic review probabilistic multi-item inventory system with zero lead time under constraints and varying order cost’. American Journal of Applied Science, 2 (8), 1213-1217. | Hariga, M. and Ben-Daya, M., 1999. “Some stochastic inventory models with deterministic variable lead time”. European Journal of Operations Research 1113, 42-51. | Kotab Abd –El- Hamid Mahmood Kotb and Huda Mohamed Hamid Al-Shanbari, 2011, ‘Constrained Probabilistic Economic Order Quantity model under varying cost and zero lead time via geometric programming. Journal of Mathematics and Statistics 7 (4), 343-347. | Lan, S.P., Chu, P., Chung, K.J.Wan, W.J. and L , R.1999, “A simple method to locate the optimal solution of the inventory model with variable lead time”. Computers and Operation Research, 26, 599-605. | Martin I, Reiman, 2004, “A new and simple policy for the continuous review lost sales inventory model”. Bell Labs, Lucent Technologies, Murray Hill, N.J. | Montgomery, D.C., Bazaraa, M.S. and Keswani, A.K., 1973. “Inventory models with a mixture of backorders and lost sales”. Naval research Logistics Quarterly, 20, 255-263. | Moon, I and Chai, S., 1998. “A note on lead time and distributional assumptions in continuous review inventory models”. Computers and operations research, 25, 1007-1012. | Nick T. Thomopoulos, 2004, “Demands, backorders, service level, lost sales and effective service level”. International Applied Business Research, Annual Conference Proceedings Pureto Rice, March 2004. | Ouyang, L.Y. and Wu, K.S., 1998. “A minimize distribution free procedure for mixed inventory model with variable lead time”. International journal of model production economics, 56-57, 511-516.