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ALMOST CONTRA CONTINUITY ON GENERALIZED TOPOLOGICAL SPACES

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ABSTRACT

In this paper we introduce some almost contra continuous functions in generalized topological spaces such as almost contra (g, g')-continuous function, almost contra (α, g')-continuous function, almost contra (σ, g')-continuous function, almost contra (π, g')-continuous function, almost contra (β, g')-continuous function. We investigate their characterizations and relationships among such functions.

Keywords : almost contra (g, g')-continuous function, almost contra (α, g')-continuous function, almost contra (σ, g')-continuous function, almost contra (π, g')-continuous function, almost contra (β, g')-continuous function.

1. Introduction

Csaszar [1] has introduced the notions of generalized neighborhood systems and generalized topological spaces. He also introduced the notions of continuous functions and associated interior and closure operators on generalized neighborhood systems and generalized topological spaces. In particular, he investigated characterizations for generalized continuous functions in [1]. In [2] he introduced and studied the notions of g-α-open sets, g-semi-open sets, g-preopen sets and g-β-open sets in generalized topological spaces. Min [4, 5] has introduced and studied various types of continuous functions and almost continuous functions in generalized topological spaces. Contra continuous functions in generalized topological spaces are introduced by Jayanthi [3]. In this paper we introduce some almost contra continuous functions in generalized topological spaces such as almost contra (g, g')-continuous function, almost contra (α, g')-continuous function, almost contra (σ, g')-continuous function, almost contra (π, g')-continuous function, almost contra (β, g')-continuous function. We investigate their characterizations and relationships among such functions.

2. Preliminaries

Let X be a nonempty set and g be a collection of subsets of X. Then g is called a generalized topology (GT for short) on X iff $\phi \in g$ and $G_i \in g$ for $i \in I \neq \phi$ implies $G = \cup_{i\in I} G_i \in g$. The pair (X, g) is called as a generalized topological space (GTS for short) on X. The elements of g are called g-open sets and their complements are called g-closed sets. We denote the family of all g-closed sets in X by ${}_{g}c(X)$. The generalized closure of a subset S of X, denoted by $c_g(S)$, is the intersection of generalized closed sets including S. And the interior of S, denoted by $i_g(S)$, is the union of generalized open sets contained in S.

Note that $c_g(S) = X – i_g(X - S)$ and $i_g(S) = X – c_g(X - S)$ [1]

Definition 2.1: [2] Let (X, g_X) be a generalized topological space and $A \subseteq X$. Then A is said to be

(i) g-semi-open if $A \subseteq c_g(i_g(A))$,

(ii) g-preopen if $A \subseteq i_g(c_g(A))$,

(iii) g-α-open if $A \subseteq i_g(c_g(i_g(A)))$,

(iv) g-β-open if $A \subseteq c_g(i_g(c_g(A)))$.

The complement of g-semi-open (resp., g-preopen, g-α-open, g-β-open) is said to be g-semi-closed (resp., g-preclosed, g-α-closed, g-β-closed).

Let us denote the class of all g-semi-open sets, g-preopen sets, g-α-open sets and g-β-open sets on X by $\sigma(g_X)$ (σ for short), $\pi(g_X)$ (π for short), $\alpha(g_X)$ (α for short) and $\beta(g_X)$ (β for short) respectively.

Denote by $c_\sigma(X)$, $c_\pi(X)$, $c_\alpha(X)$ and $c_\beta(X)$, the closures of g-semi-closed sets, g-preclosed sets, g-α-closed sets and g-β-closed sets on X.

Definition 3.1: [3] Let (X, g_X) and (Y, g_Y) be GTS's. Then a function $f : X \to Y$ is said to be contra $(g_{X,} g_Y)$-continuous if for each g-open set U in Y, $f^{-1}(U)$ is g-closed in X.

3. Almost contra continuous functions on GTS's

In this section we have introduced the various types of almost contra continuous functions and investigated the relations among them.

Definition 3.1: Let (X, g_X) and (Y, g_Y) be GTS's. Then a function $f : X \to Y$ is said to be almost contra $(g, g_{'})$-continuous function at $x \in X$, if for each g-closed set V containing f(x), there exists a g-open set U containing x such that $f(U) \subseteq c_g(i_g(V))$.

Theorem 3.2: Let $f : X \to Y$ be a function on the GTSs (X, g_X) and (Y, g_Y). Then the following conditions are equivalent:

(i) f is an almost contra (g, g')-continuous function at $x \in X$.

(ii) $x \in i_g(f^{-1}(c_g(i_g(V))))$ for every g-closed subset V containing f(x).

(iii) $x \in i_g(f^{-1}(V))$ for every gr-closed subset V containing f(x).

(iv) For any gr-closed set V containing f(x), there exists a g-open set U containing x such that $f(U) \subseteq V$.

Proof: (i) $\Rightarrow$ (ii) Let V be any g-closed set containing f(x). By (i) there exists a g-open set U containing x such that $f(U) \subseteq c_g(i_g(V))$. Now $x \in U = i_g(U) \subseteq i_g(f^{-1}(f(U))) \subseteq i_g(f^{-1}(c_g(i_g(V))))$. Therefore $x \in i_g(f^{-1}(c_g(i_g(V))))$.

(ii) $\Rightarrow$ (iii) Let V be any gr-closed subset containing f(x). Then (ii) implies that $x \in i_g(f^{-1}(c_g(i_g(V))))$. Since V is a gr-closed set, $c_g(i_g(V)) = V$. Therefore $x \in i_g(f^{-1}(V))$.

(iii) $\Rightarrow$ (iv) Let V be any gr-closed subset containing f(x). By (iii) $x \in i_g(f^{-1}(V))$. Therefore there exists a g-open set say $U = i_g(f^{-1}(V))$ such that $U \subseteq f^{-1}(V)$. That is $x \in U$ and $f(U) \subseteq f(f^{-1}(V)) \subseteq V$. Hence $f(U) \subseteq V$.

(iv) $\Rightarrow$ (i) Let V be any g-closed subset of Y containing f(x). Then $f(x) \in V \subseteq c_g(i_g(V))$. But $c_g(i_g(V))$ is a gr-closed set. Therefore (iv) implies that there exists a g-open set U containing x such that $f(U) \subseteq c_g(i_g(V))$. Thus f is an almost contra (g, g')-continuous function.

Theorem 3.3: Let $f : X \to Y$ be a function on the GTSs (X, g_X) and (Y, g_Y). Then the following conditions are equivalent:

(i) f is an almost contra (g, g')-continuous function.

(ii) $f^{-1}(V) \subseteq i_g(f^{-1}(c_g(i_g(V))))$ for every g-closed subset V in Y.

(iii) $c_g(f^{-1}(i_g(c_g(F)))) \subseteq f^{-1}(F)$ for every g-open subset F in Y.

(iv) $c_g(f^{-1}(i_g(c_g(i_g(B))))) \subseteq f^{-1}(i_g(B))$ for every subset B in Y.

(v) $f^{-1}(c_g(B)) \subseteq i_g(f^{-1}(c_g(i_g(c_g(B)))))$ for every subset B in Y.

(vi) For every gr-open set V in Y, $f^{-1}(V)$ is a g-closed set in X.

(vii) For every gr-closed set V in Y, $f^{-1}(V)$ is a g-open set in X.

Proof: (i) $\Rightarrow$ (ii) Let V be a g-closed set in Y and $x \in f^{-1}(V)$. (i) implies that there exists a g-open set U in x such that $f(U) \subseteq c_g(i_g(V))$. Therefore $x \in i_g(f^{-1}(c_g(i_g(V))))$. Thus $f^{-1}(V) \subseteq i_g(f^{-1}(c_g(i_g(V))))$.

(ii) $\Rightarrow$ (iii) Let F be a g-open set in Y. Then Y – F is a g-closed set in Y. (ii) implies that $f^{-1}(Y – F) \subseteq i_g(f^{-1}(c_g(i_g(Y - F)))) = i_g(f^{-1}(Y - i_g(c_g(Y - F)))) = X - c_g(f^{-1}(i_g(c_g(F))))$. That is $X – f^{-1}(F) \subseteq X - c_g(f^{-1}(i_g(c_g(F))))$. This implies that $c_g(f^{-1}(i_g(c_g(F)))) \subseteq f^{-1}(F)$.

(iii) $\Rightarrow$ (iv) Let B be any set in Y. Then $i_g(B)$ is g-open in Y. By (iii), $c_g(f^{-1}(i_g(c_g(i_g(B))))) \subseteq f^{-1}(i_g(B))$.

(iv) $\Rightarrow$ (v) This can be easily proved by taking complement in (iv).

(v) $\Rightarrow$ (vi) Let V be any gr-open set in Y. Then Y- V is a gr-closed set in Y and hence it is a g-closed set. (v) implies that $f^{-1}(Y - V) = f^{-1}(c_g(Y - V)) \subseteq i_g(f^{-1}(c_g(i_g(c_g(Y - V))))) = i_g(f^{-1}(Y - V))$. That is $X – f^{-1}(V) \subseteq X – c_g(f^{-1}(V))$. Thus $c_g(f^{-1}(V)) \subseteq f^{-1}(V) \subseteq c_g(f^{-1}(V))$. Hence $c_g(f^{-1}(V)) = f^{-1}(V)$. This implies that $f^{-1}(V)$ is a g-closed set.

(vi) $\Rightarrow$ (vii) is obvious.

(vii) $\Rightarrow$ (i) Let V be a g-closed set in Y containing f(x). Therefore $f(x) \in V \subseteq c_g(i_g(V))$. Since $c_g(i_g(V))$ is a gr-closed set, (vii) implies that $f^{-1}(c_g(i_g(V)))$ is a g-open set. Therefore there exists a g-open set , say $U = f^{-1}(c_g(i_g(V)))$ such that $f(U) = f(f^{-1}(c_g(i_g(V)))) \subseteq c_g(i_g(V))$. Hence f is an almost contra (g, g')-continuous function.

Definition 3.4: Let (X, g_X) and (Y, g_Y) be GTS's. Then a function $f : X \to Y$ is said to be

(i) almost contra (α, g_Y)-continuous if for each gr-open set U in Y, $f^{-1}(U)$ is g-α-closed in X,

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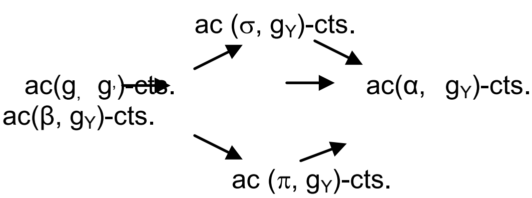
(ii) almost contra (σ, g_Y)-continuous if for each gr-open set U in Y, $f^{-1}(U)$ is g-semi-closed in X,

(iii) almost contra (π, g_Y)-continuous if for each gr-open set U in Y, $f^{-1}(U)$ is g-preclosed in X,

(iv) almost contra (β, g_Y)-continuous if for each gr-open set U in Y, $f^{-1}(U)$ is g-β-closed in X.

Remark 3.5: Let f: X $\rightarrow$ Y be a function between the GTS's X and Y. Then we have the following implications. Here ac means almost contra and cts. means continuous.



The reverse implications may not be true in general and this can be clearly known from the following examples.

Example 3.6: Let X = {a, b, c, d}. Consider two generalized topologies g_X = {ϕ, {a}, a, b, c}} on X. Define f : (X, g_X) $\rightarrow$ (X, g_X) as follows: f(a) = f(b) = d and f(c) = f(d) = a. Then f^{-1}({a}) = {c, d} and f^{-1}({a, b, c}) = {c, d}. We have f is almost contra (α, g_X)-continuous, almost contra (σ, g_X)-continuous, almost contra (π, g_X)-continuous and almost contra (β, g_X)-continuous but not almost contra (g_X, g_X)-continuous.

Example 3.7: Let X = {a, b, c}. Consider two generalized topologies g_1 = {ϕ, {b}, {c}, {b, c}} and g_2 = {ϕ, {c}} on X. Define f : (X, g_1) $\rightarrow$ (Y, g_2) as follows: f(a) = b, f(b) = c and f(c) = a. Then f^{-1}({c}) = {b}. We have f is almost contra (σ, g_2)-continuous and almost contra (β, g_2)-continuous but not almost contra (α, g_2)-continuous.

Example 3.8: Let X = {a, b, c}. Consider two generalized topologies g_1 = {ϕ, {a}, {b}, {a, b}} and g_2 = {ϕ, {a, b}} on X. Define f: (X, g_1) $\rightarrow$ (Y, g_2) as follows: f(a) = a, f(b) = c and f(c) = c. Then f^{-1}({a, b}) = {a}. We have f is almost contra (β, g_2)-continuous but not almost contra (π, g_2)-continuous.

Example 3.9: Let X = {a, b, c}. Consider two generalized topologies g_1 = {ϕ, {a, c}} and g_2 = {ϕ, {a}} on X. Define f : (X, g_1) $\rightarrow$ (Y, g_2) as follows: f(a) = f(b) = a and f(c) = b. Then f^{-1}({a}) = {a, b}. We have f is almost contra (β, g_2)-continuous but not almost contra (σ, g_2)- continuous. Also f is almost contra (π, g_2)-continuous but not almost contra (α, g_2)-continuous.

Remark 3.10: Every contra continuous function is almost contra continuous in GTSs, but the converse is not true in general as seen from the following example.

Example 3.11: Let X = {a, b, c}. Consider two generalized topologies g = {ϕ, {a}, {c},{a, c}} and g'= {ϕ, {a},{a, b}} on X. Define f : (X, g) $\rightarrow$ (X, g') as an identity mapping. Then f is almost contra continuous but not contra continuous, since {a}is g-open in (X, g) but it is not g-closed in (X, g') as $c_g(f^{-1}(\{a\})) = c_g(\{a\}) = \{a, b\} \neq \{a\}$

Theorem 3.12: A mapping $f : X \rightarrow Y$ is almost contra (α, g_Y) continuous if and only if it is both almost contra (π, g_Y)-continuous and almost contra (σ, g_Y)-continuous.

Proof: Necessity: It is clear from the above diagram.

Sufficiency: Let B be a gr-open set in Y. Then by hypothesis, $f^{-1}(B)$ is both g-preclosed and g-semi-closed in X. Therefore $c_g(i_g(f^{-1}(B))) \subseteq f^{-1}(B)$ and $i_g(c_g(f^{-1}(B))) \subseteq f^{-1}(B)$. We have $i_g(i_g(c_g(f^{-1}(B)))) \subseteq i_g(f^{-1}(B))$. That is $i_g(c_g(f^{-1}(B))) \subseteq i_g(f^{-1}(B))$. Now $c_g(i_g(c_g(f^{-1}(B)))) \subseteq c_g(i_g(f^{-1}(B))) \subseteq f^{-1}(B)$. Hence $f^{-1}(B)$ is a g-α-closed set in X. Thus f is almost contra (α, g_Y)-continuous.

Theorem 3.13: Let $f: X \rightarrow Y$ be a function from two GTS's. Then the following conditions are equivalent:

(i) f is almost contra (π, g_Y)-continuous,

(ii) $f^{-1}(A)$ is g-preopen in X for every gr-closed set A in Y,

(iii) for each $x \in X$ and each gr-closed set F in Y containing f(x), there exists a g-preopen set U in X containing x such that $f(U) \subseteq F$,

(iv) for each $x \in X$ and each gr-open set V in Y non-containing f(x), there exists a g-preclosed set K in X non-containing x such that $f^{-1}(V) \subseteq K$,

(v) $f^{-1}(i_g(c_g(G)))$ belongs to the family of all g-preclosed sets in X for every g-open subset G of Y,

(vi) $f^{-1}(c_g(i_g(F)))$ belongs to the family of all g-preopen sets in X for every g-closed subset F of Y

Proof: (i) ⇔ (ii) Let A be gr-closed in Y. Then Y – A is gr-open in Y. By (i) $f^{-1}(Y - A)$ is g-preclosed in X. That is $X - f^{-1}(A) = f^{-1}(Y - A)$ is g-preclosed in X. Hence $f^{-1}(A)$ is g-preopen in X. Reverse can be obtained similarly.

(ii) ⇒ (iii) Let F be a gr-closed set in Y containing f(x). (ii) implies that $f^{-1}(F)$ is g-preopen in X and $x \in f^{-1}(F)$. Take $U = f^{-1}(F)$. Then $f(U) \subseteq F$.

(iii) ⇒ (ii) Let F be gr-closed in Y and $x \in f^{-1}(F)$. From (iii) there exists a g-preopen set U_x in X containing x such that $U \subseteq f^{-1}(F)$. We have $f^{-1}(F) = \cup_{x \in f^{-1}(F)} U_x$. Thus $f^{-1}(F)$ is g-preopen in X.

(iii) ⇔ (iv) Let V be a gr-open set in Y non-containing f(x). Then Y – V is a gr-closed set containing f(x). By (iii) there exists a g-preopen set U in X containing x such that $f(U) \subseteq Y - V$. Hence $U \subseteq f^{-1}(Y - V) \subseteq X - f^{-1}(V)$ and then $f^{-1}(V) \subseteq X - U$. Take H = X – U. We obtain that H is a g-preclosed set in X non-containing x.

Reverse can be obtained similarly.

(i) ⇔ (v) Let G be a g-open subset of Y. Since $i_g(c_g(G))$ is gr-open, then by(i), it follows that $f^{-1}(i_g(c_g(G)))$ is a g-preclosed set in X.

The converse can be proved easily.

(ii) ⇔ (vi) Proof is similar as (i) ⇔ (v).

Theorem 3.14: Let $f : X \to Y$ be a function from two GTS's. Then the following conditions are equivalent:

(i) f is almost contra (π, g_Y)-continuous.

(ii) $f^{-1}(c_g(V))$ is g-preopen in X for every g-β-open set V in Y.

(iii) $f^{-1}(c_g(V))$ is g-preopen in X for every g-semiopen set V in Y.

(iv) $f^{-1}(i_g(c_g(V)))$ is g-preclosed in X for every g-preopen set V in Y.

Proof: (i) $\Rightarrow$ (ii) Let V be any g-β-open set in Y. Then $V \subseteq c_g(i_g(c_g(V)))$ and $c_g(V) = c_g(i_g(c_g(V)))$. Therefore $c_g(V)$ is gr-closed in Y. By (i) $f^{-1}(c_g(V))$ is g-preopen in X.

(ii) $\Rightarrow$ (iii) is obvious.

(iii) $\Rightarrow$ (iv) Let V be a g-preopen set in Y. Then $Y - i_g(c_g(V))$ is gr-closed and hence it is g-semiopen. By (iii), $f^{-1}(c_g(Y - i_g(c_g(V))))$ is g-preopen. That is $f^{-1}(c_g(Y - i_g(c_g(V)))) = X - f^{-1}(i_g(c_g(V)))$ is g-preopen and hence $f^{-1}(i_g(c_g(V)))$ is g-preclosed in X.

(iv) $\Rightarrow$ (i) Let V be any gr-open set in Y. Then V is g-preopen in Y. Therefore $i_g(c_g(V)) = V$. By (iv), $f^{-1}(i_g(c_g(V))) = f^{-1}(V)$ is g-preclosed in X. Hence f is almost contra (π, g_Y)-continuous.

(v) $\Rightarrow$ (ii) Let A be g-closed in Y. Then $f^{-1}(A) \subseteq X$. (v) implies that $f^{-1}(A) \subseteq c_g(i_g(f^{-1}(c_g(f(f^{-1}(A)))))) \subseteq c_g(i_g(f^{-1}(c_g(A)))) = c_g(i_g(f^{-1}(A)))$. Hence $f^{-1}(A)$ is g-semi-open in X.

Theorem 3.15: Let $f : X \to Y$ be a function from two GTS's. Suppose that one of the following conditions hold:

(i) $f^{-1}(c_g(B)) \subseteq i_g(c_\beta(f^{-1}(B)))$ for each subset B in Y,

(ii) $c_g(i_\beta(f^{-1}(B))) \subseteq f^{-1}(i_g(B))$ for each subset B in Y,

(iii) $f(c_g(i_\beta(A))) \subseteq i_g(f(A))$ for each subset A in X,

(iv) $f(c_g(A)) \subseteq i_g(f(A))$ for each g-β-open set A in X.

Then f is almost contra (β, g_Y)-continuous.

Proof: (i) $\Rightarrow$ (ii) is obvious by taking complement in (i).

(ii) $\Rightarrow$ (iii) Let $A \subseteq X$, then $f(A) \subseteq Y$. Now (ii) implies $c_g(i_\beta(f^{-1}(f(A)))) \subseteq f^{-1}(i_g(f(A)))$. That is $c_g(i_\beta(A)) \subseteq c_g(i_\beta(f^{-1}(f(A)))) \subseteq f^{-1}(i_g(f(A)))$. Hence $f(c_g(i_\beta(A))) \subseteq f(f^{-1}(i_g(f(A)))) \subseteq i_g(f(A)$.

(iii) $\Rightarrow$ (iv) Let $A \subseteq X$ be g-β-open. Then $f(c_g(i_\beta(A))) \subseteq i_g(f(A)$. That is $f(c_g(A)) = f(c_g(i_\beta(A))) \subseteq i_g(f(A)$, since $i_\beta(A) = A$. Hence $f(c_g(A)) \subseteq i_g(f(A))$.

Suppose (iv) holds: Let $A \subseteq Y$ be gr-open. Then A is g-open in Y. We have $f^{-1}(A) \subseteq X$ and $i_\beta(f^{-1}(A))$ is g-β-open in X. (iv) implies that $f(c_g(i_\beta(f^{-1}(A)))) \subseteq i_g(f(i_\beta(f^{-1}(A)))) \subseteq i_g(f(f^{-1}(A))) \subseteq i_g(A) = A$. Now $c_g(i_\beta(f^{-1}(A))) \subseteq f^{-1}(f(c_g(i_\beta(f^{-1}(A))))) \subseteq f^{-1}(A)$. We have $c_g(i_g(f^{-1}(A))) \subseteq c_g(i_\beta(f^{-1}(A))) \subseteq f^{-1}(A)$. Therefore $f^{-1}(A)$ is a g-preclosed set and hence a g-β-closed set. Thus f is almost contra (β, g_Y)-continuous.

Theorem 3.16: Let $f : X \to Y$ be a function from two GTS's. Then the following conditions are equivalent:

(i) f is almost contra (β, g_Y)-continuous,

(ii) for each $x \in X$ and each gr-closed set B containing f(x), there exists a g-β-open set A in X and $x \in A$ such that $A \subseteq f^{-1}(B)$,

(iii) for each $x \in X$ and each gr-closed set B containing f(x), there exists a g-β-open set A in X and $x \in A$ such that $f(A) \subseteq B$.

Proof: (i) ⇒ (ii) Let $B \subseteq Y$ be gr-closed and $f(x) \in B$. By hypothesis, $f^{-1}(B)$ is g-β-open in X. Therefore $i_\beta(f^{-1}(B)) = f^{-1}(B)$. Put $A = i_\beta(f^{-1}(B))$. Then A is a g-β-open set in X and $A \subseteq f^{-1}(B)$.

(ii) ⇒ (iii) Let $B \subseteq Y$ be gr-closed and $f(x) \in B$. By hypothesis, there exists a g-β-open set A in X and $x \in A$ such that $A \subseteq f^{-1}(B)$. Thus $f(A) \subseteq f(f^{-1}(B)) \subseteq B$. Thus $f(A) \subseteq B$.

(iii) ⇒ (i) Let B be gr-closed in Y. Let $x \in X$ and $f(x) \in B$. By hypothesis, there exists a g-β-open set A in X and $x \in A$ such that $f(A) \subseteq B$. This implies $x \in A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(B)$. That is $x \in f^{-1}(B)$. Since A is g-β-open, $A = i_\beta(A) \subseteq i_\beta(f^{-1}(B))$. Hence $x \in i_\beta(f^{-1}(B))$. Therefore $f^{-1}(B) = \cup_{x\in f^{-1}(B)} x \subseteq i_\beta(f^{-1}(B)) \subseteq f^{-1}(B)$. Thus $i_\beta(f^{-1}(B)) = f^{-1}(B)$ and $f^{-1}(B)$ is g-β-open. Hence f is almost contra (β, g_Y)-continuous.

Theorem 3.17: Let $f : X \to Y$ be a function from two GTS's. Suppose one of the following conditions hold:

(i) $f(c_\beta(A)) \subseteq i_g(f(A))$ for each subset A in X.

(ii) $c_\beta(f^{-1}(B))) \subseteq f^{-1}(i_g(B))$ for each subset B in Y.

(iii) $f^{-1}(c_g(B)) \subseteq i_\beta(f^{-1}(B))$ for each subset B in Y.

Then f is almost contra (β, g_Y)-continuous.

Proof: (i) ⇒ (ii) Let $B \subseteq Y$. Then $f^{-1}(B) \subseteq X$. (i) implies that $f(c_\beta(f^{-1}(B))) \subseteq i_g(f(f^{-1}(B))) \subseteq i_g(B)$. Now $f^{-1}(f(c_\beta(f^{-1}(B)))) \subseteq f^{-1}(i_g(B))$. Therefore $c_\beta(f^{-1}(B)) \subseteq f^{-1}(f(c_\beta(f^{-1}(B)))) \subseteq f^{-1}(i_g(B))$. Hence $c_\beta(f^{-1}(B))) \subseteq f^{-1}(i_g(B))$.

(ii) ⇒ (iii) is obvious by taking complement in (ii).

Suppose (iii) holds: Let $B \subseteq Y$ be gr-closed. Then it is g-closed. By hypothesis, $f^{-1}(c_g(B)) \subseteq i_\beta(f^{-1}(B))$. That is $f^{-1}(B) = f^{-1}(c_g(B)) \subseteq i_\beta(f^{-1}(B)) \subseteq f^{-1}(B)$. Therefore $f^{-1}(B)$ is g-β-open in X. Hence f is almost contra (β, g_Y)-continuous.

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