Research Paper Statistics

Optimization of Inventory Using Kanban Model in a Fixed Market for Two Brands - Need of an Hour

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Introduction:

Until now, inventory control was studied in a conventional fashion and therefore, perhaps the study was restricted to production and planning only. However with the globalization of the market some advanced technique was necessitated to bring about the equilibrium conditions in competitive situations(1). In this chapter a humble attempt is made to bring equilibrium condition in competitive situation using Kanban model with varying setup cost.

We have dealt with the determination of optimum inventory level when two brands of same product are competing in the fixed market. The objective of the model developed over here is to maximize the profit of the brand under consideration when the rival brand is also trying to maximize his profit. Here model is developed which is related with the determination of optimum inventory level for two brands of same product which are competing where market size is not fixed but it is growing. The more is spent on the inventory of different brands; more is the anticipated sales volume. We have utilized Kanban concept along which varying ordering cost which helps in maximization of profit of all brands in the market.

This model deals with the determination of optimal inventory level for two brands of a given product to maximize the profit contribution of the brand under consideration when the rival brand is also trying to maximize his profit, by optimizing the inventory level in the fixed market. The model has been considered under the JIT approach, where the costs are much reduced as compared to the conventional inventory approach.

Notations:

Q_i : Order quantity (units) of i^{th} brand, i = 1, 2.

A'_i : Cost of ordering (Rs/order) of i^{th} brand, i = 1, 2.

$A'_i = A_i + b_i Q_i$

P_i : Aggregate cost per shipment of i^{th} brand, i = 1, 2.

N_i : Number of shipments per order of i^{th} brand, i = 1, 2.

H_i: Cost of inventory holding of i^{th} brand, i = l, 2.

L_i: Labour cost for i^{th} brand, i=1,2.

S_i: Set up time for i^{th} brand, i=1,2.

M_i: Material cost for i^{th} brand, i=1,2.

b_i : Positive constant, i = 1, 2.

TFW: Time that a full container waits to be moved.

TFM: Time that a full container spends moving back to users work centre.

Assumptions:

1. Only two brands of a product are competing in the market.

2. The total anticipated sales volume (V) of the product is fixed.

3. The demand of the i^{th} brand (D_i) is unknown and it is assumed that it depends upon competitor's brand strategy.

4. The total number of runs (n_i) of the quantity produced be known for i^{th} brand i = 1,2 and Q_i denote the lot size in each production of i^{th} brand.

5. The set up cost for i^{th} brand is not fixed but it is $A_i + b_iQ_i$ per production. The logistic margin of the i^{th} brand is defined as the difference between unit price (p_i) and unit variable cost (C_i).

6. As soon as the inventory level reaches to zero, replenishment is made. The shortages are not allowed to occur.

7. Production of the commodity is uniform.

8. Lead time is zero.

9. Each competitor brand not only knows his number of production runs, inventory holding cost and logistic margin, but also the same for the opponent brand and tries to maximize his profit.

10. The buyer and the seller are operating in a JIT schedule.

Problem Formulation:

For the i^{th} brand, it is assumed that after each time t_i, the quantity Q_i is produced or supplied throughout the entire period. Thus

$D_i = n_iQ_i$; i = 1,2. (1)

The total annual cost is given by

TC_i=

$$\left[\frac{H_i}{2N_i}(TFW + TFM) + n_i b_i\right]Q_i + (A_i + L_iS_i + M_i)n_i$$

i=1,2 (2)

Here, we have considered a fixed market in which two brands are competing and total market potential represents the total anticipated sales of both the competitors under a given set of strategies.

The contribution of demand to market share of i^{th} brand is proportional to

$\frac{D_i}{D_1 + D_2}$; i = 1, 2 respectively.

Thus, share of the market M_i for i^{th} brand is given by

$$M_i = \frac{n_i Q_i}{n_1 Q_1 + n_2 Q_2}$$

The anticipated profit function for the i^{th} brand is given as:

Profit = (Total anticipated sales volume) (Margin of the profit) - (Inventory expenditure)-(Fixed expenditure)

That is,

$P_i = VM_ih_i - TC_i - F_i$; i =1,2

$$= V\left[\frac{n_i Q_i}{n_1 Q_1 + n_2 Q_2}\right] h_i - TC_i - F_i;$$

i = 1, 2. (3)

The problem here is to find out equilibrium points for both the competitor brand with the sense that if any brand deviates from the equilibrium values, his anticipated off goes down.

The necessary and sufficient conditions are as follows:

Since both Q_1 and Q_2 are positive, the necessary and sufficient condition for maximum profit of i^{th} competitor (i = 1,2) are given by:

$$P_i = Vh_i\left[\frac{n_i Q_i}{n_1 Q_1 + n_2 Q_2}\right] - \left(A_i + L_i S_i + M_i\right) n_i - \left[\frac{H_i}{2N_i}(TFW + TFM) + n_i b_i\right] Q_i - F_i;$$

i =1,2. (4)

1. $\frac{\partial P_i}{\partial Q_i} = 0$ (5)

2. $\frac{\partial^2 P_i}{\partial Q_i^2} \leq 0$

fori=1

$$\frac{\partial P_1}{\partial Q_1} = \frac{Vn_1 n_2 Q_2 h_1}{\left(n_1 Q_1 + n_2 Q_2\right)^2} - \left[\frac{H_1 (TFW + TFM)}{2N_1} + n_1 b_1\right] \quad (6)$$

$$\frac{\partial^2 P_1}{\partial Q_1^2} = \frac{-2Vn_1^2 n_2 Q_2 h_1}{\left(n_1 Q_1 + n_2 Q_2\right)^3} \quad (7)$$

Similarly for i = 2,

$$\frac{\partial P_2}{\partial Q_2} = \frac{Vn_1 n_2 Q_1 h_2}{\left(n_1 Q_1 + n_2 Q_2\right)^2} - \left[\frac{H_2 (TFW + TFM)}{2N_2} + n_2 b_2\right] \quad (8)$$

$$\frac{\partial^2 P_2}{\partial Q_2^2} = \frac{-2Vn_2^2 n_2 Q_1 h_2}{\left(n_1 Q_1 + n_2 Q_2\right)^3} \quad (9)$$

From (7) and (9), it can be observed that the sufficient condition $\frac{\partial^2 P_i}{\partial Q_i^2}$ <0 for i = 1, 2 is satisfied for achieving maximum profit. Using necessary condition $\frac{\partial P_i}{\partial Q_i}$ = 0; i = 1, 2 result (6) and (8) can be written as

$$\frac{Vn_1n_2Q_2h_2}{\left(n_1Q_1+n_2Q_2\right)^2}-\left[\frac{H_1(TFW+TFM)}{2N_1}+n_1b_1\right]$$
= 0 (10)

And

$$\frac{Vn_1n_2Q_1h_2}{\left(n_1Q_1+n_2Q_2\right)^2}-\left[\frac{H_2(TFW+TFM)}{2N_2}+n_2b_2\right]$$
= 0 (11)

which yields

$$\frac{Vn_1n_2Q_2h_1}{\left(n_1Q_1+n_2Q_2\right)^2}=\left[\frac{H_1(TFW+TFM)}{2N_1}+n_1b_1\right] \quad (12)$$

And

$$\frac{Vn_1n_2Q_1h_2}{\left(n_1Q_1+n_2Q_2\right)^2}=\left[\frac{H_2(TFW+TFM)}{2N_2}+n_2b_2\right] \quad (13)$$

Simultaneous Optimization Conditions:

From (12) and (13)

$$\frac{Vn_1n_2Q_1h_2}{\left(\frac{H_1(TFW+TFM)}{2N_1}+n_1b_1\right)}=\left(n_1Q_1+n_2Q_2\right)^2 \quad (14)$$

$$\frac{Vn_1n_2Q_1h_2}{\left(\frac{H_2(TFW+TFM)}{2N_2}+n_2b_2\right)}=\left(n_1Q_1+n_2Q_2\right)^2 \quad (15)$$

Equating results (14) and (15) and dividing both sides of the equation by V, n_1, n_2 we obtain the equilibrium condition as:

$$\frac{Q_1}{Q_2}=\frac{h_1}{h_2}\frac{\left[\frac{H_2(TFW+TFM)}{2N_2}+n_2b_2\right]}{\left[\frac{H_1(TFW+TFM)}{2N_1}+n_1b_1\right]} \quad (16)$$

From (16), we have

$$Q_2=\frac{h_2}{h_1}\left[\frac{\frac{H_1(TFW+TFM)}{2N_1}+n_1b_1}{\frac{H_2(TFW+TFM)}{2N_2}+n_2b_2}\right]Q_1 \quad (17)$$

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and from (14) we have

$$\frac{V\left[n_1 n_2 \frac{h_2}{h_1} \frac{\frac{H_1(TFW+TFM)}{2N_1}+n_1 b_1}{\frac{H_2(TFW+TFM)}{2N_2}+n_2 b_2} Q_1\right]}{\left(n_1 Q_1 + n_2 b_2\right)^2}$$

$$h_1 = \frac{1}{2}\left[\frac{H_1(TFW+TFM)}{N_1} + 2n_1 b_1\right] \quad (18)$$

This means that

$$Q_1 = \frac{\left(\frac{H_2(TFW+TFM)}{2N_2} + n_2 b_2\right)\left(n_1 Q_1 + n_2 Q_2\right)^2}{Vn_1 n_2 h_2} \quad (19)$$

Using the result in equation (17) and on further simplification, we obtain

$$Q_1^o = h_1\left[\frac{H_1(TFW+TFM)}{2N_1} + n_2 b_2\right] G_1 \quad (20)$$

where

$$G_1 = \frac{Vn_1 n_2 h_1}{\left[n_1 h_1\left(\frac{H_2(TFW+TFM)}{2N_2} + n_2 b_2\right) + n_2 h_2\left(\frac{H_1(TFW+TFM)}{2N_1} + n_1 b_1\right)\right]} h_1 \quad (21)$$

Similarly optimum inventory level for brand 2 can be obtained as

$$Q_2^o = h_2\left(\frac{H_1(TFW+TFM)}{2N_1} + n_1 b_1\right) G_2 \quad (22)$$

where

$$G_2 = \frac{Vn_1 n_2 h_2}{\left[n_2 h_2\left(\frac{H_1(TFW+TFM)}{2N_1} + n_1 b_1\right) + n_1 h_1\left(\frac{H_2(TFW+TFM)}{2N_2} + n_2 b_2\right)\right]^2} \quad (23)$$

Sensitivity Analysis:

Let brand-I be the brand under consideration and brand II be the opponent brand. We measure the sensitivity of net profit contribution for brand-I with respect to its inventory quantity as well as that of his opponent.

1. Change In The Inventory Level Of Brand Under Consideration:

Let us assume that new inventory level of given brands is $Q_1' = Q_1 + \delta$, where δ is a small non-zero constant. Hence, from (4) new profit contribution function is given by

$$P_1' + F_1 = Vh_1 \frac{n_1 Q_1 + n_1 \delta}{n_1 Q_1 + n_2 \delta + n_2 Q_2} - \left[\left(\frac{H_1(TFW+TFM)}{2N_1} + n_1 b_1\right).(Q_1 + \delta)\right]$$

Considering D = $n_1 Q_1 + n_2 Q_2$, we have

$$P_1^{'} + F_1 = Vh_1 \frac{n_1Q_1 + n_1\delta}{D}\left(1+\frac{n\delta}{D}\right)^{-1}$$

$$-\left(\frac{1}{2}\frac{H_1(TFW+TFM)}{N_1}+n_1b_1\right)^{\delta} - \left[\left(\frac{H_1(TFW+TFM)}{2N_1}+n_1b_1\right)(Q_1)\right]$$

$$P_1^{'} + F_1 = Vh_1 \frac{n_1Q_1 + n_1\delta}{D}\left(1-\frac{n_1\delta}{D}+\frac{(n_1\delta)^2}{D^2}\right)$$

$$-\frac{1}{2}\left(\frac{H_1(TFW+TFM)}{N_1}+n_1b_1\right) - C_{A1}$$

That is,

$$P_1^{'} + F_1 = (P_1 + F_1) + \frac{V_1h_1n_1\delta}{D}\left[1+\frac{n_1Q_1+n_1\delta}{D}\left(-1+\frac{n_1\delta}{D}\right)-\left(\frac{H_1(TFW+TFM)}{2N_1}+n_1b_1\right)\right]\delta$$

Under optimization condition, we have

$$\frac{Vh_1n_1n_2Q_2}{D^2\left(\frac{H_1(TFW+TFM)}{2N_1}+n_1b_1\right)} = 1$$

and hence the above expression can be rewritten as

$$P_1^{'} + F_1 = (P_1 + F_1)\frac{Vh_1n_1\delta}{D}\left[1+\frac{n_1Q_1+n_1\delta}{D}\left(-1+\frac{n_1\delta}{D}\right)\right]$$

$$-\left(\frac{H_1(TFW+TFM)}{2N_1}+n_1b_1\right)\delta\left(\frac{Vh_1n_1n_2Q_2}{D^2\left(\frac{H_1(TFW+TFM)}{2N_1}+n_1b_1\right)}\right)$$

$$P_1^{'} + F_1 = (P_1 + F_1) + \frac{Vh_1n_1\delta}{D}\left[\frac{-n_1\delta}{D}+\frac{n_1\delta(n_1Q_1+n_1\delta)}{D^2}\right]$$

Which means that

$$P_1^{'} + F_1 - (P_1 + F_1)\frac{Vh_1(n_1\delta)^2}{D^3}\left[n_2Q_2 - n_1\delta\right] \quad (24)$$

It can be observed that above quantity is negative only if $n_2Q_2 > n_1\delta$. This suggests that if Brand-I deviates from its optimal policy, its profit goes down. (25)

2. Change In The Competitor's Inventory Level: (26)

Let us suppose that the new inventory level of the opponent is $Q_2' = Q_2 + \delta$ (27)

$$P_1^{'} + F_1 = Vh_1\frac{n_1Q_1}{n_1Q_1+n_2Q_2+n_2\delta} - TC_1 = Vh_1\left[\frac{n_1Q_1}{D}\left(1-\frac{n_2\delta}{D}\right)^{-1}\right] - TC_1$$

$$P_1^{'} + F_1 = Vh_1\left[\frac{n_1Q_1}{D}\left(1-\frac{n_2\delta}{D}\right)+\left(\frac{n_2\delta}{D}\right)^2\right] - TC_1$$

(Ignoring higher powers of δ) that is,

$$P_1^{'} + F_1 - (P_1 + F_1) = \frac{-(n_1Q_1)(n_2\delta)}{D^3}Vh_1\left[n_1Q_1 + n_2(Q_2+\delta)\right] \quad (28)$$

Here, the difference only depends upon the value δ, since V, Q_1, Q_2, n_1 and n_2 are all positive and $Q_2 > \delta$. This means that if a competitor is increasing his inventory level ($\delta>0$), the profit of brand under consideration will go down and if he is decreasing his level ($\delta<0$), the profit of the brand under consideration will increase.

Hypothetical Problem:

There are two brands X_1 and X_2 competing in the market then the total anticipated sales volume is 5000 units. Given: (TFW+TFM) =1.2, δ = 0.01

X_i	h_i	H_i	n_i	A_i	L_i	S_i	b_i	M_i	N_i
X_1	4	0.6	5	0.1	6	10	0.20	0.64	2
X_2	8	0.4	6	0.2	5	12	0.30	0.36	2

The optimum inventory levels are:

Brand	Optimum inventory level (Rs.)	Optimum profit contribution
X_1	1335.98	11064.88
X_2	614.86	3792.89

$$\frac{Q_1^o}{Q_2^o} = \frac{1335.98}{614.86} = 2.1728$$

According if brand X_2 deviates from its optimal strategy, and if its new inventory level is Rs.600, then brand X_1 also has to change his strategy to maintain his equilibrium and now its optimal inventory level will be

Remarks:

When two brands of same product are competing in market of varying size, using Kanban model it is observed that the determination of inventory level of brand under consideration depends upon the products fixed total anticipated sales volume, market expansion parameters, number of production runs logistic margin, set up cost and inventory holding cost for both the brands. If any brand deviates from its optimum inventory level or if any brand tries to increase the inventory level from optimal inventory level, than its profit goes down.

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