Volume : 2 | Issue : 12 | Dec 2013 ISSN - 2250-1991

Research Paper Mathematics

Partial Commutation on a Class of Indexed Siromoney Matrix Languages

* T. Kamaraj ** D. G. Thomas

* Department of Mathematics, Sathyabama University , Chennai-119

** Department of Mathematics, Madras Christian College, Chennai – 59

ABSTRACT

Trace theory on array languages is recently introduced formal method of analysis of concurrent systems in computer science by using partial commutation as a tool. The closure of Local array languages and Siromoney matrix languages (SML) under partial commutation are already discussed. In this paper we examine the closure of an extended form of SML under partial commutation and thereby we establish some interesting results.

Keywords : Indexed grammar, Arrays, Matrix grammar, Partial Commutation

1 Introduction

A two-dimensional language (picture language) is a set of rectangular array of patterns which appear in the studies concerning parallel computing and image analysis. A number of rectangular picture generating mechanisms such as two-dimensional grammars and automata have been introduced in the literature [1,4,5,6,7]. Siromoney matrix grammar(SMG)[6] is one of the earliest array models which used simple sequential and parallel rules for generating rectangular picture languages.

Based on the behavior of the elementary net theory, Mazurkiewicz introduced the concept of partial commutation and traces on strings [3] as a formal approach of analyzing the concurrent systems. An extension of partial commutation to two dimensional rectangular arrays has been recently attempted [2] and closure of local [1] and Siromoney matrix languages (SML) [6] under partial commutation have been discussed .

In this paper we consider an extended family of SML languages : Indexed Siromoney Matrix Languages(ISML)[7] which are unlike SML may contain pictures of given proportion about a horizontal line . We establish some interesting results on closure of a class of ISML under Partial commutation.

2 Preliminaries

In this section we recall the notions of indexed Siromoney matrix grammars [7] and partial commutation [2] on arrays.

Definition 1. A two-dimensional string (or a picture) over Σ is a two-dimensional rectangular array of elements of Σ. The set of all two-dimensional strings over Σ is denoted by Σ^{**}. A two-dimensional language over Σ is a subset of Σ^{**}.

Definition 2. A right linear indexed right-linear (RIR) grammar G is five-tuple < N, T, F, P, S > where N, T, F are finite, pairwise disjoint sets of nonterminals, terminals and indices respectively; P consists of two types of productions, namely

(i) right linear productions of the form

$A \to xBf,\ A \in N,\ x \in T \cup \{\varepsilon\},\ B = N \cup \{\varepsilon\}$ not all ε (empty element) and

(ii) indexed production of the form

$Af \to xB,\ A \in N,\ B \in N \cup \{\varepsilon\},\ x \in T \cup \{\varepsilon\},\ f \in F,$

and $S \in N$ is the start symbol.

Definition 3. A regular right linear indexed right linear Siromoney matrix grammar (R : RIR) SMG is a two tuple G = <G_1, G_2>, where G_1 = (N, I, P, S) is regular (R) horizontal grammar where N is a finite set of non terminals, I is a finite set of intermediate symbols $S_1, S_2, \ldots, S_k$ with $N \cap I = \varnothing$ and $S \in N$ is the start symbol of G_1.

G_2 = <$G_{21}, G_{22}, \ldots, G_{2k}$> where each G_{2i}, i = 1, 2, ..., k is called a vertical grammar and G_{2i} = <N_i, T, F_i, P_i, S_i> is an RIR grammar with N_i, F_i being distinct from N_j, F_j for $i \neq j$ $(1 \leq i, j \leq k)$. The intermediate symbol S_i of G_i is in N_i and is the start symbol of G_{2i}.

The derivations are as follows: G_1 generates finite strings of intermediates as in a Chomskian R grammar, i.e., $S \underset{G_1}{\Rightarrow} á \in I^+$.

The vertical derivation starts with a string of intermediates generated by G_1 and proceeds in parallel as in SMG [18]. Rules of the same type: $A \to a$ or $Af \to a$ with length of a being same (3, 2, 1 or 0), are applied in the vertical derivations. A derivation ends successfully on generating an array M over T by rewriting a row of non terminals with (without) indices by the rules of the form $Af \to a$ ($A \to a$). The array language generated by the grammar G is given by

. The language generated by (R : RIR) SMG can be referred as (R : RIR) SML

3 Partial Commutation on arrays

We now recall partial commutation rectangular arrays [3]

Definition 4. Let Σ be an alphabet and $p \in \Sigma^{**}$. The row partial commutation mapping $\varphi_R : \Sigma \times \Sigma \to \Sigma \times \Sigma$ is defined as $\varphi_R(a, b) = (b, a)$ and $\varphi_R(b, a) = (a, b)$ for $a, b \in \Sigma$. i.e., a and b commute with each other. It is a row partial commutation denoted by $(a\ \ b) \leftrightarrow (b\ \ a)$. It is applied on any row of p replacing ab by ba and vice versa. The set of all equivalence classes obtained by row partial commutation is a trace language. Let φ_R be the row partial commutation mapping such that $\varphi_R(p) = [p]$. If $L \subseteq \Sigma^{**}$, then $\varphi_R(L) = \{[p] \mid p \in L\}$.

Note: If every element of Σ commutes with each other then the partial commutation is called a total commutation.

Example 1: Consider the language L of mxn arrays(m>2,n>2) on $\Sigma = \{\bullet, a\}$ describing the H tokens. i.e. L contain pictures of the form

$$\begin{matrix} a & \bullet & \bullet & \bullet & a \\ a & \bullet & \bullet & \bullet & a \\ a & a & a & a & a \\ a & \bullet & \bullet & \bullet & a \\ a & \bullet & \bullet & \bullet & a \end{matrix}$$

If we apply row partial commutation $(a\ \ \bullet) \leftrightarrow (\bullet\ \ a)$ then $\varphi_R(L)$ =

$$\left\{ \begin{matrix} a & \bullet & a \\ a & a & a \\ a & \bullet & a \end{matrix}, \begin{matrix} a & a & \bullet \\ a & a & a \\ a & \bullet & a \end{matrix}, \begin{matrix} \bullet & a & a \\ a & a & a \\ a & \bullet & a \end{matrix}, \begin{matrix} a & \bullet & a \\ a & a & a \\ a & a & \bullet \end{matrix}, \ldots\ldots\ldots\ldots \right\}$$

Example 2: The language of squares over $\Sigma = \{0,1\}$ such that the principal diagonal elements are '1' and remaining elements are all '0'. i.e.,

$L = \{p \in \Sigma^{**} \mid \ell_1(p) = \ell_2(p)$, such that $a_{ii} = 1$, $a_{ij} = 0$ for $i = j, i = 1, 2, \ldots, n\}$

i.e. $L = \left\{ \begin{matrix} 10 \\ 01 \end{matrix}, \begin{matrix} 100 \\ 010 \\ 001 \end{matrix}, \ldots \right\}$

By applying the row commutativity $(0\ \ 1) \leftrightarrow (1\ \ 0)$ to each element of L, we get

$$\left\{ \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}, \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}, \ldots\ldots\ldots\ldots \right\}$$

$\varphi_R(L)$ where $\varphi_R(L)$ =

$$\left\{ \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}, \begin{matrix} 1 & 0 \\ 1 & 0 \end{matrix}, \begin{matrix} 0 & 1 \\ 0 & 1 \end{matrix}, \begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix}, \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}, \begin{matrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}, \ldots\ldots\ldots\ldots \right\}$$

Definition 5: Let $\Sigma = \{a,b\}$. The column partial commutation

φ_c is $\binom{\Sigma}{\Sigma} \to \binom{\Sigma}{\Sigma}$

is defined as $\varphi_c\binom{a}{b} \to \binom{b}{a}$ and $\varphi_c\binom{b}{a} \to \binom{a}{b}$

For $a,b \in \Sigma$. It is also denoted by $\binom{a}{b} \leftrightarrow \binom{b}{a}$.

It is applied on any column of $p \in \Sigma^{**}$. The equivalence class of p obtained by column partial commutation is a trace denoted by [p]. Then the trace of L is $\varphi_c(L) = \{[p] \ /\ p \in L\}$ where $L \subset \Sigma^{**}$.

Example 3: By applying column partial commutation $\binom{1}{0} \leftrightarrow \binom{0}{1}$ on the language in the example 2, $\varphi_c(L)$=

$$\left\{ \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}, \begin{matrix} 1 & 1 \\ 0 & 0 \end{matrix}, \begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix}, \begin{matrix} 0 & 0 \\ 1 & 1 \end{matrix}, \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}, \begin{matrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}, \ldots\ldots\ldots\ldots \right\}$$

Example 4: By applying column partial commutation $\binom{a}{\bullet} \leftrightarrow \binom{\bullet}{a}$ on the language in the example 1, $\varphi_C(L)$

$$\left\{ \begin{matrix} a & \bullet & a \\ a & a & a \\ a & \bullet & a \end{matrix}, \begin{matrix} a & a & a \\ a & \bullet & a \\ a & \bullet & a \end{matrix}, \begin{matrix} a & \bullet & a \\ a & \bullet & a \\ a & a & a \end{matrix}, \begin{matrix} a & \bullet & a \\ a & \bullet & a \\ a & a & a \\ a & \bullet & a \\ a & \bullet & a \end{matrix}, \begin{matrix} a & \bullet & a \\ a & a & a \\ a & \bullet & a \\ a & \bullet & a \\ a & \bullet & a \end{matrix}, \ldots\ldots\ldots \right\} =$$

4 Partial commutation on Indexed Siromoney Matrix languages

In this section, we examine the partial commutativity applied on languages generated by (R : RIR) SMG .

Theorem 1: If a picture language L is a (R : RIR) SML then $\varphi_C(L)$ is also a (R : RIR) SML

Proof: First we note that the number of occurrences of each terminal in every column of a picture under Column partial commutation does not change. For a (R:RIR)SML L defined on two symbols, we claim that there exists a (R:RIR) SMG $G' = < G_1', G_2' >$ generating $\varphi_C(L)$. To show this, let L be a (R:RIR)SML on alphabet {a, b}. If a column of $p \in \varphi_C(L)$ contains only a's and no b's, then the vertical production rules of G_2' are of the form $I \to aA_1g$, $A_1 \to A_2f$, $A_1 \to aA_1f$, $A_2f \to aA_3$, $A_3f \to aA_3$, $A_3g \to a$. If a column of $p \in \varphi_C(L)$ contains only one b and remaining a's, then the vertical production rules of G_2' are of the form $I \to bA_1g/aB_1g/aC_1g$, $A_1 \to A_2f$, $A_1 \to aA_1f$, $A_2f \to aA_3$, $A_3f \to aA_3$, $A_3g \to a$, $B_1 \to B_2f$, $B_1 \to aB_1f$, $B_2f \to aB_3$, $B_3f \to aB_3$, $B_3g \to b$, $D_1 \to D_2f$, $C_1 \to aC_1f$, $C_1 \to bD_1f$, $D_1 \to aD_1f$ $D_2f \to aD_3$, $D_3f \to aD_3$, $D_3g \to a$. If a column of $p \in \varphi_C(L)$ contains two or more b's and remaining a's, then the vertical production rules of G_2' are of the form $bA_1g/aB_1g/aC_1g/aE_1g/bF_1g/bH_1g$ and the rules involving A_1,B_1,C_1 are same as that of the case where p contains only one b remaining a's and the rules involving E_1,F_1,H_1 are similar to that of A_1,B_1,C_1 respectively except the difference that the symbol a is replaced by a and vice versa.

By induction on the size of the alphabet (greater than or equal to 2) we can show that, given a (CF:RIR)SML *L*, there exists a RMG (CFMG, CSMG) G' generating $\varphi_C(L)$. Hence first part of the theorem follows.

Theorem 2 : If a picture language L is a (R: RIR) SML then $\varphi_R(L)$ need not be a (R : RIR) SML.

Proof : Consider the language on $\{\cdot, a\}$describing the H tokens, given in example 1. This language is already shown to be an (R: RIR) SML [7, Example 1]

In $\varphi_R[L(G)]$, we observe that each row (except last) consists of two a's (at any positions) and remaining ·'s. The middle row contains all a's. This means every column has a chance of having more than one a (entire column containing a's also possible) .

So if we develop any (R:RIR)SMG $G' = \langle G_1', G_2' \rangle$ for $\varphi_R[L(G)]$ all the vertical production rules in G_2' should produce the column strings as discussed above.

This indicates that $L(G')$ must contain a picture of the form

$$\begin{matrix} a & a & a \\ a & a & a \\ a & a & a \end{matrix}$$, which shows that $L(G') \neq \varphi_R[L(G)]$

5 Conclusion

In this work we have considered row partial commutation φ_R, column partial commutation φ_c on a class of Indexed Siromoney matrix Languages and shown the closure under φ_c but not under φ_R. The question of whether similar results hold good for other classes of Indexed Siromoney matrix grammar and also other general forms of array Grammars like Kolam Array grammars should be investigated. Application of Partial commutation on 2D languages for analyzing concurrent system is explored in future.

REFERENCES

1. Giammarresi, D., Restivo, A. (1997): Two-dimensional languages. In: Handbook of Formal Languages, vol. 3, pp. 215–226. Springer, Berlin | 2. Kamaraj, T et al.,(2012) : Partial commutation on array languages In R.P. Barneva et al. (Eds.): IWCIA 2012, LNCS 7655, pp. 196–208, 2012.Springer-Verlag Berlin ,Heidelberg. | | 3. Mazurkiewicz, A. (1995): Introduction to Trace Theory. In: Diekert, V., Rozenberg, G. (eds.) The Book of Traces, pp. 3–41. World Scientific Publishers, Singapore | | 4. Rosenfeld, A. (1979): Picture languages: Formal Models for Picture Recognition. Academic Press | | 5. Rosenfeld, A., Siromoney, R. (1993): Picture languages - A survey. Languages of Design 1, 229–245 | | 6. Siromoney, G., Siromoney, R., Krithivasan, K. (1972): Abstract families of matrices and picture languages. Computer Graphics and Image Processing 2, 284–307 | 7. Subramanian, K.G., Revathi, L., Siromoney,R., (1989): Siromoney array grammars and applications, Int. J. of Pattern Recognition and Artificial Intelligence, 3, 333□351. |