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Unsteady Convective Heat Transfer Through a Porous Medium in a Horizontal Wavy Channel with Oscillatory Flux

* Dr. M. Bharathi ** Prof. U. Rajeswara Rao

* Lecturer in Mathematics, Govt.Degree College for Men, Doctors Colony, B.Camp., Kurnool, 518002, Andhra Pradesh

** Department of Mathematics, Sri Krishnadevaraya University, S. V. Puram, Anantapur, Andhra Pradesh.

ABSTRACT

In this paper we analyze the unsteady convective heat transfer of a viscous fluid through a porous medium in a horizontal channel bounded by corrugated walls. The unsteadiness in the flow is due to the imposed oscillatory flow on the convective flow through the channel. The velocity and the temperature profiles were discussed in this paper.

Keywords: flux, corrugated channel, porous medium, unsteady convection.

1.Introduction: Unsteady convection flows play an important role in aerospace technology, turbo machinery and chemical Engineering. Such flows arise due to either in steady motion of a boundary or boundary temperature. Unsteadiness may also be due to oscillatory free stream velocity or temperature. The heat transfer through wavy channel has been a topic of interest in recent times owing to its applications in technological areas viz., transpiration cooling of re-entry vehicles and rocket boosters, cross-hatching of an ablative surface and film vaporization in combustion chambers.As the fluid flows through a tortuous path viz., the dilated – constricted geometry, there will be more intimate contact between them.

2.FORMULATION OF THE PROBLEM

We consider the unsteady motion of a viscous, incompressible fluid through a porous medium in a horizontal channel bounded by wavy walls in the presence of a constant heat source /sink.The Boussinesq approximation is used so that the density variation will be considered only in the buoyancy force . The viscous, Darcy and Ohmic dissipations are neglected in comparison to the flow by conduction and convection. We also take into account into the radiative heat transfer in the energy equation. Also the kinematic viscosity n, the thermal conducting k are treated as constants. We choose a rectangular Cartesian system 0 (x , y) with x-axis in the direction of motion and y-axis in the vertical direction and the walls are taken at y = ± Lf(dx/L), where 2L is the distance between the walls, f is a twice differentiable function and d is a small parameter proportional to the boundary slope. The magnetic Reynolds number R is chosen to be much less than unity so that the induced magnetic field can be neglected in compari1son to the applied field. The flow is maintained by an oscillatory volume flux rate for which a characteristic velocity is defined as

$$q(1+e^{i\omega t}) = (\frac{1}{L})\int_{-Lf}^{Lf} u\,dy \tag{2.1}$$

The equations governing the unsteady hydrodynamic flow and heat transfer in Cartesian coordinate system O (x, y, z), in the absence of any input electric field are Equation of continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.2}$$

Equation of linear momentum

$$\rho_e(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}) = -\frac{\partial p}{\partial x} + \mu(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}) - (\frac{\mu}{k})u \tag{2.3}$$

$$\rho_e(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}) = -\frac{\partial p}{\partial y} + \mu(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}) - (\frac{\mu}{k})v - \rho g \tag{2.4}$$

Equation of energy

$$\rho_e C_p(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}) = \lambda(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}) + Q - \frac{\partial(q_R)}{\partial y} \tag{2.5}$$

Equation of state

$$\rho - \rho_e = -\beta_e(T - T_e) \tag{2.6}$$

where ρ_e is the density of the fluid in the equilibrium state, Te is the temperature and in the equilibrium state, (u, v) are the velocity components along O(x, y) directions, p is the pressure, T is the temperature in the flow region,ris the density of the fluid,μ is the constant coefficient of viscosity ,Cp is the specific heat at constant pressure,λis the coefficient of thermal conductivity, k is the permeability of the porous medium ,β is the coefficient of thermal expansion,, Q is the strength of the constant internal heat source and qR is radiative heat flux.

Invoking Rosseland approximation(Brewester(*))for the radiative flux we get

$$q_R = -\frac{4\sigma^{\bullet}}{3\beta_R}\frac{\partial(T'^4)}{\partial y} \tag{2.7}$$

and expanding T'^4 about Te by using Taylor's series and neglecting higher order terms we get

$$T'^4 \cong 4T_e^3 T - 3T_e^4 \tag{2.8}$$

In the equilibrium state

$$0 = -\frac{\partial p_e}{\partial x} - \rho_e g \tag{2.9}$$

where $p = p_e + p_D, p_D$ being the hydrodynamic pressure and in this state the temperature gradient balances the heat flux generated by source Q.

The boundary conditions for the velocity and temperature fields are u=0,

v=0,T=T1(h) on y=-Lf(dx/L) $u = 0,\ v = 0,\ T = T_2(\eta)$

on y=Lf(dx/L) (2.10)

In view of the continuity equation we define the stream function ψ as

u=yy, v=-yx (2.11)

Eliminating pressure p from equations (2.3) & (2.4)and using (2.11) the equations governing the flow in terms of y are

$$[(\nabla^2\psi)_t + \psi_x(\nabla^2\psi)_y - \psi_y(\nabla^2\psi)_x] = \nu\nabla^4\psi + \quad (2.12)$$

$$+\beta g(T-T_0)_x - (\frac{\nu}{k})\nabla^2\psi$$

$$\rho_e C_p(\frac{\partial\theta}{\partial t} + \frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}) = k_1\nabla^2\theta + Q - \frac{16T_e^3\sigma^*}{3\beta_R}\frac{\partial^2\theta}{\partial y^2} \quad (2.13)$$

Introducing the non-dimensional variables in (2 .12) & (2.13) as

$$x' = x/L,\ y' = y/L, t' = t\varpi, \Psi' = \Psi/qL, \theta = \frac{T-T_e}{T_2-T_e} \quad (2.14)$$

the governing equations in the non-dimensional form (after dropping the dashes) are

$$\gamma((\nabla^2\psi)_t + R\frac{\partial(\psi,\nabla^2\psi)}{\partial(x,y)}) = \nabla^4\psi + (\frac{G}{R})\theta_x - D^{-1}\nabla^2\psi \quad (2.15)$$

and the energy equation in the non-dimensional form is

$$P_1\gamma^2(\frac{\partial\psi}{\partial t}) + P_1R(\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}) = \nabla^2\theta + \alpha_1 \quad (2.16)$$

with the corresponding boundary conditions

$$\psi(+1) - \psi(-1) = (1 + ke^{i\varpi t})$$

$$\frac{\partial\psi}{\partial x} = 0,\ \frac{\partial\psi}{\partial y} = 0\ ,\theta = \frac{T_1-T_e}{T_2-T_e} = h, say \quad on\ y = -f(\delta x)$$

$$\frac{\partial\psi}{\partial x} = 0,\ \frac{\partial\psi}{\partial y} = 0\ ,\theta = 1 \quad on\ y = f(\delta x)$$

$$\frac{\partial\theta}{\partial y} = 0,\quad at\quad y = 0$$

(2.17)

where $R = \frac{qL}{\nu}$ (the Reynolds number) , $G = \frac{\beta g(T_2-T_e)L^3}{\nu^2}$ (the Grashof number)

$P = \frac{\mu c_p}{k_1}$ (the Prandtl number), $\gamma^2 = \frac{\varpi L^2}{\nu}$ (the Wormsley number)

$\alpha = \frac{QL^2}{k}$ (the heat source parameter), $N = \frac{4\sigma^* T_e^3}{\beta_R}$ (radiation parameter)

$P_1 = \frac{3NP}{3N+4}$, $\alpha_1 = \frac{3N\alpha}{3N+4}$

3. SOLUTION OF THE PROBLEM

Solving the equations (3.4) - (3.14) subject to the relevant boundary conditions we obtain

$$\theta_0 = -\frac{\alpha_1 f^2}{2}\eta^2 + a_1\eta + a_2$$

$$\psi_0 = b_1 + b_2\eta + b_3Ch(\beta_1\eta) + b_4Sh(\beta_1\eta) + b_6\eta^6 + a_7\eta^3 + a_8\eta^2 + a_9\eta$$

$$\bar{\theta}_0 = 0$$

$$\bar{\psi}_0 = c_4Sh(\beta_2\eta)$$

$$\theta_1 = a_{33}\eta^2 + a_{34}\eta^3 + a_{35}\eta^4 + a_{36}\eta^5 + a_{37}\eta^6 + a_{38}\eta^7 + (a_{45} + a_{46}\eta + a_{47}\eta^2)Ch(\beta_1\eta) + a_{41} + a_{49}\eta + a_{50}\eta^2)Sh(\beta_1\eta) + a_{51}\eta + a_{52}$$

$$\psi_1 = d_1 + d_2\eta + d_3Ch(\beta_1\eta) + d_4Sh(\beta_1\eta) + \phi_2(\eta)$$

$$\phi_2(\eta) = c_{24}\eta + c_{25}\eta^2 + c_{26}\eta^3 + c_{27}\eta^4 + c_{28}\eta^5 + c_{29}\eta^6 + c_{30}\eta^7 + c_{31}\eta^8 + c_{32}\eta^9 + (c_{33} + c_{34}\eta + c_{35}\eta^2 + c_{36}\eta^3 + c_{37}\eta^4)Sh(\beta_1\eta) + (c_{38} + c_{39}\eta) + (c_{45} + c_{46}\eta)Sh(2\beta_1\eta)$$

$$\bar{\theta}_1 = d_{27}(1 - \frac{Sh(\beta_2\eta)}{Sh(\beta_2)}) + d_{29}(\eta^2 - \frac{Ch(\beta_2\eta)}{Ch(\beta_2)}) + d_{32}(\eta^2 - 1)Ch(\beta_2\eta) + d_{34}(\eta Sh(\beta_2\eta) + \frac{Ch(\beta_2\eta)}{Ch(\beta_2)}) + d_{28}(\eta - \frac{Sh(\beta_2\eta)}{Sh(\beta_2)}) + d_{31}(\eta Ch(\beta_2\eta) + \frac{Sh(\beta_2\eta)}{Sh(\beta_2)}Ch(\beta_2))$$

$$\bar{\psi}_1 = e_{28} + e_{29}\eta + e_{30}Ch(\beta_2\eta) + e_{31}Sh(\beta_2\eta) + \phi_3(\eta)$$

$$\phi_3(\eta) = e_6\eta^2 + e_7\eta^3 + e_8\eta^4 + e_9Sh(\beta_1\eta) + e_{10}Ch(\beta_1\eta) + (e_{11} + e_{12}\eta + e_{13}\eta^2 + e_{14}\eta^3)Ch(\beta_2\eta) + (e_{15} + e_{16}\eta + e_{17}\eta^2 + e_{18}\eta^3 + e_{19}\eta^4)Sh(\beta_2\eta) + e_{20}Sh(\beta_4\eta) + e_{21}Sh(\beta_5\eta) + e_{22}Ch(\beta_4\eta) + e_{23}Ch(\beta_5\eta) + e_{24}Sh(\beta_4\eta) + e_{25}\eta Sh(\beta_5\eta) + e_{26}\eta Ch(\beta_4\eta) + e_{27}\eta Ch(\beta_5\eta)$$

4. SHEAR STRESS AND NUSSELT NUMBER

The shear stress on the channel walls is given by

$$\tau = \frac{\sigma_{xy}(1 - f'^2) + (\sigma_{yy} - \sigma_{xx})f'^2}{(1 + f'^2)}$$

where

$$\sigma_{ij} = -p\delta_{ij} + 2\mu e_{ij}$$

$$\sigma_{xx} = \frac{\partial u}{\partial x}, \sigma_{yy} = \frac{\partial v}{\partial y}, \sigma_{zz} = \frac{\partial w}{\partial z}, \sigma_{xy} = 0.5(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x})$$

and the corresponding expressions are

$$(\tau)_{\eta=+1} = ((1 - f'^2)f^2(e_{48} + ke^{it}e_{50}) + \delta((1 - f'^2)f^2(g_{21} + ke^{it}g_{19}) - (\frac{2f'}{f})(g_{25} + ke^{it}g_{26}) + O(\delta^2))/(1 + f'^2)$$

$$(\tau)_{\eta=-1} = ((1 - f'^2)f^2(e_{49} + ke^{it}e_{50}) + \delta((1 - f'^2)f^2(g_{22} + ke^{it}g_{20}) - (\frac{2f'}{f})(g_{29} + ke^{it}g_{31}) + O(\delta^2))/(1 + f'^2)$$

The local rate of heat transfer coefficient(Nusselt number Nu) on the walls has been calculated using the formula

$$Nu = \frac{1}{\theta_m - \theta_w}(\frac{\partial\theta}{\partial y})_{\eta=\pm1}$$

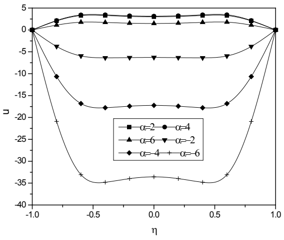
where $\theta_m = 0.5\int_{-1}^{1}\theta\, dy$

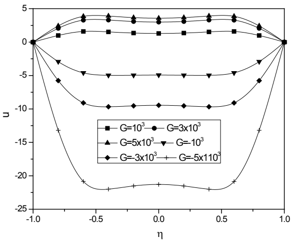
and the corresponding expressions are

$$(Nu)_{\eta=+1} = \frac{2(e_{40} + \delta(e_{41} + ke^{it}e_{42})}{f(e_3 + \delta(e_{30} + ke^{it}e_{39}) - 1)}$$

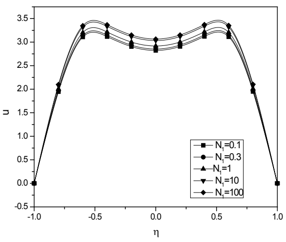
$$(Nu)_{\eta=-1} = \frac{2(e_{43} + \delta(e_{44} + ke^{it}e_{45})}{f(e_{37} + \delta(e_{38} + ke^{it}e_{39}) - h)}$$

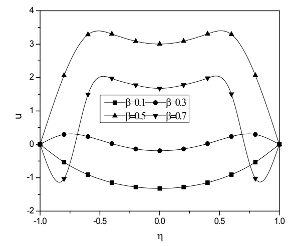
5. CONCLUSIONS : The primary aim of our analysis is to investigate the behaviour of the velocity and temperature with respect to the governing parameters G, R, β,α N1,D^{-1}. Velocity u exhibits a reversal flow in the entire fluid region in the cooling case and the region of reversal flow enhances with increase in |G|. It is found that for α>0 we find reversal flow in entire fluid region and enlarges with increase in | α |. For an increase in |N1| we notice that there is no reversal flow in the dilated channel. An increase in |N1| results in an enhancement for β > 0. Lesser the permeability of the porous medium smaller |u| for β > 0 in the vicinity of the reversal flow appears in the mid region for R >140. The maximum |u| occurs at h= 0.4for R=35 and drifts towards the upper boundary for higher values of R=70 and again for R=140 it occurs in the mid plane. Temperature increase in G > O enhances in the lower region and reduces in the upper region, while a reversed effect is observed with increase in |G| (<O).Higher the strength of the heat source a < 4 larger the actual temperature and for higher a > 6 smaller the actual temperature in the flow region .Lesser the permeability of the porous medium larger the actual temperature in the lower half and smaller the temperature in the upper half.Temperature experiences on enhancement with radiation parameter N1 <10 while it reduces with higher N1 >100



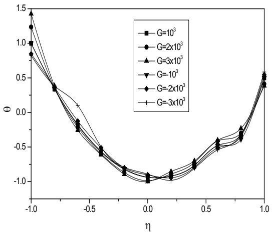


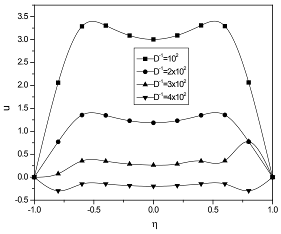
Fig(1) Variation of u with G Fig(2) Variation of u with α
N=1 α=2 N_1=4 N=1 N1=4 β=0.5



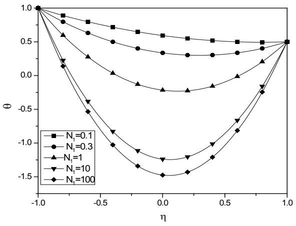


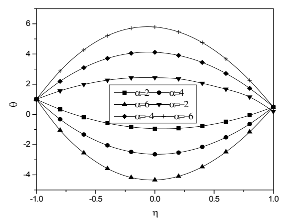
Fig(3) Variation of u with β Fig(4) Variation of u with N1G=10_3
N=1 N_1=4 α=2 G=10^3 N=1 D^{-1}=10^2 α=2 β=0.5 R=35



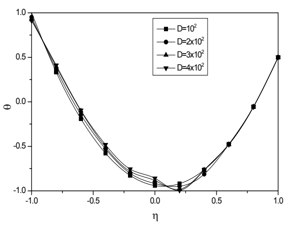


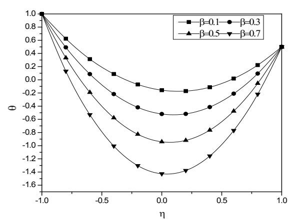
Fig(5) Variation of u with D^{-1} Fig(6) Variation of θ with G G=10^3 N=1 N_1=4 α=2 β=0.5 R=35 G=10^3 N=1 D^{-1}=10^2 α=2 β=0.5 R=35





Fig(7) Variation of θ with α Fig(8) Variation of θ with N1 G=10^3 N=1 D^{-1}=102 α=2 β=0.5 R=35 G=10^3 N=1 D^{-1}=10^2 β=0.5 R=35





Fig(9) Variation of θ with β Fig(10) Variation of θ with D^{-1} G=10^3 N=1 R=35 α=2 G=10^3 N=1 D^{-1}=10^2 R=35 α=2

Table.1
Shear Stress(t) at y=1
P=0.71 , b=0.5,N1=0.5

G/t	I	II	III	IV	V	VI	VII	VII	VIII	IX
5x10^2	-0.0691	0.9575	1.4715	2.6146	4.2348	-1.2145	-2.3877	3.5511	6.7326	11.2994
10^3	-1.3945	0.2919	1.1380	2.0655	3.9762	-1.5131	-0.2314	5.8245	14.314	26.986
2x10^3	-1.8476	0.0505	1.0136	1.0759	3.4788	6.6708	23.916	12.4912	37.946	77.3255
-5x10^2	4.7683	3.3765	2.6810	3.8226	4.7724	8.0727	12.836	1.1257	0.0811	-0.9168
-10^3	8.2754	5.1285	3.5567	4.4813	5.0513	17.025	30.103	0.9762	1.0373	2.6455
-2x10^3	17.4614	9.7186	5.8510	5.9095	5.6300	43.483	83.7008	2.8103	11.5826	29.3548
R	35	70	140	35	35	35	35	35	35	35
D^{-1}	102	102	102	2x102	3x102	102	102	102	102	102
a	2	2	2	2	2	4	6	-2	-4	-6

Table.2
Shear Stress(t) at y=1
P=0.71,a=2

G/t	I	II	III	IV	V
5x10^2	0.552723	-7869.999	-0.069158	18.02892	62.79616
10^3	-1.141633	-12943.14	-1.394509	17.68003	-100.3893
2x10^3	-3.901508	-14643.58	-1.847693	-33.65599	-4444.742
-5x10^2	4.569334	10674.08	4.768339	-31.33224	-281.8507
-10^3	6.891011	24125.58	8.275419	-80.80804	-784.0807
-2x10^3	12.16043	59341.32	17.46143	-228.7236	-2434.26
g	5	10	5	5	5
b	0.3	0.5	0.5	0.7	0.9

Table.3
Nusselt Number(Nu) at y=1
P=0.71, b=0.5,g=5

G/t	I	II	III	IV	V	VI	VII	VII	VIII	IX
$5x10^2$	0.1215	0.1192	0.1181	0.1188	0.1182	0.3378	0.4809	9.5239	1.6318	1.2546
10^3	0.1261	0.1215	0.1192	0.1201	0.1188	0.3742	0.5925	9.5554	1.7262	1.4412
$2x10^3$	0.1354	0.1261	0.1215	0.1229	0.1201	0.478	1.1018	11.3001	2.1991	2.5582
$-5x10^2$	0.1123	0.1146	0.1158	0.1161	0.1169	0.2793	0.3318	11.1514	1.6255	1.1263
-10^3	0.1076	0.1123	0.1146	0.1148	0.1163	0.2537	0.2694	13.5816	1.7316	1.1798
$-2x10^3$	0.0978	0.1076	0.1123	0.1123	0.115	0.2028	0.1238	42.7715	2.5551	3.176
R	35	70	140	35	35	35	35	35	35	35
D^{-1}	102	102	102	2x102	3x102	102	102	102	102	102
a	2	2	2	2	2	4	6	-2	-4	-6

Table.4
Nusselt Number(Nu) at y=1
P=0.71, b=0.5,a=2

G/Nu	I	II	III	IV	V
$5x10^2$	0.0407	0.2499	0.1215	0.1744	0.2004
10^3	0.428	0.3676	0.1261	0.1794	0.1657
$2x10^3$	0.047	0.5603	0.1354	0.1799	1.6013
$-5x10^2$	0.0367	-0.0009	0.1123	0.1559	0.1677
-10^3	0.0348	-0.0825	0.1076	0.1414	0.0975
$-2x10^3$	0.031	-0.0704	0.0978	0.933	-0.9294
	5	10	5	5	5 γ
β	0.3	0.5	0.5	0.7	0.9

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