Volume : 2 | Issue : 7 | July 2013 ISSN - 2250-1991

Research Paper Engineering

Investigation and Analysis of Quarter Car Automotive Suspension System Using Mathematical Model

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ABSTRACT

Quarter car automotive suspension model is one of the bests and simplest way for the analysis of vehicle ride or ride comfort with two degrees of freedom system. It represents the automotive system at each wheel i.e. the motion of the axle and of the vehicle body at any one of the four wheels of the vehicle. The information obtained from the mathematical model could be used for investigation and analysis of the system. State space modeling is one of the best ways of presenting differential equations describing a dynamic system. This paper studies the mathematical model of quarter car automotive two degrees of freedom suspension system. From the state space modeling, matrix formation is carried out, and from the formation of matrices, stability, controllability, observability and many other useful attributes of the system can be calculated.

Keywords : Passive suspension, quarter car, full car suspension, ride comfort, vehicle body accelerationt

Mathematical Models:
The analysis, design and synthesis of complex physical systems can be simplified by making certain type of models like physical models or mathematical models.

The mathematical description of the dynamic characteristics of a physical model is called a mathematical model. The solution of the governing equations gives the response of the dynamic system to an input function.

A given physical system would have different mathematical models depending upon the variables considered and coordinate system chosen. The choice of a particular model is based on a compromise between the simplicity of a solution and accuracy desired.

Full Car Suspension:
A "Full Car" Suspension System is a 7 degrees of freedom system. The 7 degrees of freedom of the full car model are the heave, pitch, and roll of the vehicle body and the vertical motions of each of the four unsprung masses.

Half Car Suspension:
A "half car" suspension system is a 4 degrees of freedom system. In the half car model, the pitch and heave motions of the vehicle body and the vertical translations of the front and rear axles are represented.

Quarter Car Passive Suspension Model:
The quarter car suspension model is nothing but representation of suspension system of quarter of the vehicle i.e. one wheel of the vehicle. The vehicle is connected to the road via a linear spring and damper in parallel, which represents the primary suspension, and a spring in series with suspension, which represents the vertical stiffness of the tire of the vehicle. It also consists of two masses, one sprung mass and another unsprung mass of the vehicle. The vehicle body is represented by the 'sprung' mass while the mass due to axles and tires are represented by the 'unsprung' mass.

Figure 1 shows the mathematical model of quarter car passive suspension system.

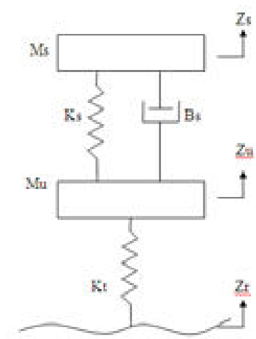

Figure:1

Where,

Ms : Sprung mass of the vehicle
Mu : Unsprung mass of the vehicle.(i.e. mass which is not supported by suspension system.)
Ks : Spring stiffness (suspension)
Kt : Tire stiffness
Bs : Damping coefficient for suspension
Zs, Zu, Zr : Vertical displacements from static equilibrium of sprung mass, unsprung mass, and road respectively.

Assumptions:
Following assumptions are made while making the mathematical model of quarter car suspension system.

1. Tire is modeled as a linear spring without damping.
2. The behavior of spring and damper are linear.
3. The tire is always in contact with the road surface.
4. There is no rotational motion in wheel and body.
5. The effect of friction is neglected.

Body Diagram:
For the unsprung mass, the force will act as shown in free body diagram below in fig 2.

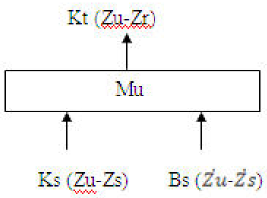

Figure:2: FBD of unsprung mass

For the sprung mass, the force will act as shown in free body diagram below in fig 3.

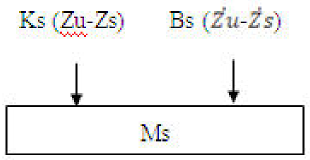

Figure:3: FBD of sprung mass

For the whole system, the free body diagram will consist of sprung mass and unsprung mass as shown below in fig 4.

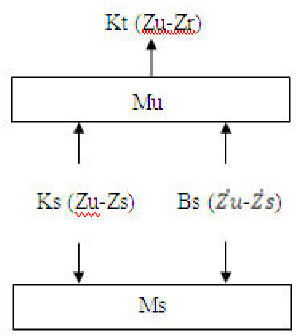

Figure 4: FBD of total system

Equations of Motion:
As this is the two degrees of freedom system, number of equations of motion will be two, i.e one for unsprung mass and one for sprung mass.

Mu $\ddot{Z}u$ + Kt (Zu-Zr) + Ks (Zu-Zs) + Bs ($\dot{Z}u$-$\dot{Z}s$) = 0 ---------------- (i)

Ms $\ddot{Z}s$ - Ks (Zu-Zs) - Bs ($\dot{Z}u$-$\dot{Z}s$) = 0 ----------------- (ii)

Equation (i) & (ii) represents the equations of motion for the system considered.

Second order Matrix Form:
The second order matrix form of quarter car passive suspension system contains mass matrix, damping matrix and stiffness matrix.

Mass Matrix :

$$\begin{bmatrix} Ms & 0 \\ 0 & Mu \end{bmatrix}$$

Damping Matrix :

$$\begin{bmatrix} Bs & -Bs \\ -Bs & Bs \end{bmatrix}$$

Stiffness Matrix :

$$\begin{bmatrix} Ks & -Ks \\ -Ks & Ks+Kt \end{bmatrix}$$

Now, all these three matrices together will form the standard second order matrix form for the quarter car passive suspension system as shown.

$$\begin{bmatrix} Ms & 0 \\ 0 & Mu \end{bmatrix}\begin{pmatrix} \ddot{Z}s \\ \ddot{Z}u \end{pmatrix}+\begin{bmatrix} Bs & -Bs \\ -Bs & Bs \end{bmatrix}\begin{pmatrix} \dot{Z}s \\ \dot{Z}u \end{pmatrix}+$$
$$\begin{bmatrix} Ks & -Ks \\ -Ks & Ks+Kt \end{bmatrix}\begin{pmatrix} Zs \\ Zu \end{pmatrix}-\begin{pmatrix} 0 \\ Kt \end{pmatrix} Zr = 0$$

State Space Model:
A state space representation is a mathematical model of a physical system as a set of input, output and state variables related to differential equations. To abstract from the number of inputs, outputs and states, the variables are expressed as vectors and the differential and algebraic equations are written in matrix form. The state space representation provides a convenient and compact way to model and analyze systems with multiple inputs and outputs. The use of the state space representation is not limited to systems with linear components and zero initial conditions. “State space” refers to the space whose axes are the state variables. The state of the system can be represented as a vector within that space.

State Variables:
The internal state variables are the smallest possible subset of the system variables that can represent the entire state of the system at any given time. State variables must be linearly independent; a state variable cannot be a linear combination of other state variables.

The minimum number of state variables required to represent a given system, is usually equal to the order of the system’s defining differential equation.

So here two state variables are selected, and then derivation of each has been taken.

x_1 = Zs – Zu

x_2 = $\dot{Z}s$

x_3 = Zu - Zr

x_4 = $\dot{Z}u$

Now taking derivatives of each state variables,

$\dot{x}_1 = \dot{Z}s = x_2$

$\dot{x}_2 = \ddot{Z}s =$

$\frac{1}{Ms}[Ks(Zu - Zs) + Bs(\dot{Z}u - \dot{Z}s)]$

$\dot{x}_3 = \dot{Z}u = x_4$

$\dot{x}_4 = \ddot{Z}u =$

$-\frac{1}{Mu}[Kt(Zu - Zr) + Ks(Zu - Zs) + Bs(\dot{Z}u - \dot{Z}s)]$

Now, these variables can be written in full matrix form.

Transfer Function:
The transfer function of a linear time invariant system or a section of the system represents the ratio of the Laplace transform of the output to the Laplace transform of the input. The Laplace transforms are obtained directly from the differential equations describing the system. The transfer function thus contains basic information concerning the essential characteristics of a system without any regard to initial conditions or excitation.

For the simplicity in the making equations, let’s assume,
Zu = x_1
Zs = x_2
Zr = x_3

As we have two equations of motion (equation i & ii) with us ,

Mu $\ddot{Z}u$ + Kt (Zu-Zr) + Ks (Zu-Zs) + Bs ($\dot{Z}u$-$\dot{Z}s$) = 0

$Ms\,\ddot{Z}s$ - Ks (Zu-Zs) - Bs ($\dot{Z}u$-$\dot{Z}s$) = 0
Let's use the notations as mentioned above.
$Mu\,\ddot{x}_1 + Kt(x_1-x_3) + Ks\,(x_1-x_2) + Bs(\dot{x}_1-\dot{x}_2) = 0$ -------- (iii)
$Ms\,\ddot{x}_2 - Ks\,(x_1-x_2) - Bs\,(\dot{x}_1-\dot{x}_2) = 0$ --- (iv)

Laplace Transform:
The Laplace transform technique provides a useful and simple method to evaluate the performance of a control system. This integral transform is used to simplify the solution of linear differential equations by converting the differential equation into an algebraic equation.

Now, taking Laplace transform of equations of motion (iii) & (iv),

$Mu\,S^2 X_1(s) + Kt\,X_1(s) – Kt\,X_3(s) + Ks\,X_1(s) – Ks\,X_2(s) + Bs\,S\,X_1(s) - Bs\,S\,X_2(s) = 0.$

$Ms\,S^2 X_2(s) – Ks\,X_1(s) + Ks\,X_2(s) - Bs\,S\,X_1(s) + Bs\,S\,X_2(s) = 0.$

Now simplifying the above equations, we have,

$[Mu\,S^2 + Kt + Ks + Bs\,S]\,X_1(s) - [Ks + Bs\,S]\,X_2(s) - Kt\,X_3(s) = 0.$ ------- (v)

$-[Ks + Bs\,S]\,X_1(s) + [Ms\,S^2 + Ks + Bs\,S]\,X_2(s) = 0.$ ------- (vi)

In Matrix form,

$$\begin{bmatrix} (Mu\,S^2 + Kt + Ks + Bs\,S) & -(Ks + Bs\,S) \\ -(Ks + Bs\,S) & (Ms\,S^2 + Ks + Bs\,S) \end{bmatrix} \begin{bmatrix} X1(S) \\ X2(S) \end{bmatrix} + \begin{bmatrix} -Kt \\ 0 \end{bmatrix} X_3 = 0.$$

Now, as the value of unsprung mass is very less compared to sprung mass, for the simplicity, we can eliminate the value of X_1.

Applying Cramer's rule,

$$\frac{X_2(s)}{X_3(s)} = \frac{\begin{vmatrix} (Mu\,S^2 + Kt + Ks + Bs\,S) & (-Kt\,X_{3(s)}) \\ -(Ks + Bs\,S) & 0 \end{vmatrix}}{\Delta}$$

Where,

$\Delta = (Mu\,S^2 + Kt + Ks + Bs\,S)(Ms\,S^2 + Ks + Bs\,S) - (Ks + Bs\,S)(Ks + Bs\,S)$

$= Mu\,Ms\,S^4 + Mu\,Ks\,S^2 + Mu\,Bs\,S^3 + Kt\,Ms\,S^2 + Kt\,Ks + Kt\,Bs\,S + Ks\,Ms\,S^2 + (Ks)^2 + Ks\,Bs\,S + Bs\,Ms\,S^3 + Bs\,Ks\,S + (Bs\,S)^2 – (Ks)^2 – 2\,Ks\,Bs\,S – (Bs\,S)^2$

$= Mu\,Ms\,S^4 + (Mu\,Bs + Bs\,Ms)\,S^3 + (Mu\,Ks + Kt\,Ms + Ks\,Ms)\,S^2 + Kt\,Bs\,S + Kt\,Ks$

$= Mu\,Ms\,S^4 + [Mu + Ms]\,Bs\,S^3 + [Kt\,Ms + (Mu+Ms)Ks]\,S^2 + Kt\,Bs\,S + Kt\,Ks$

Thus,

$$\frac{\text{Laplace transform o/p}}{\text{Laplace transform i/p}} = \frac{Kt\,(Ks+Bs\,S)}{Mu\,Ms\,S_4 + [Mu + Ms]\,Bs\,S_3 + [Kt\,Ms + (Mu+Ms)Ks]S_2 + Kt\,Bs\,S + Kt\,Ks}$$

This is nothing but the transfer function of the quarter car passive suspension system.

Conclusion:
In this paper the mathematical model of quarter car passive suspension system has been studied. From the state space modeling, matrix formation was carried out, and from the formation of matrices, by taking Laplace transform, the transfer function of the system was carried out. Quarter car automotive suspension model is one of the bests and simplest way for the analysis of vehicle ride or ride comfort with two degrees of freedom system. The same fundamentals again can be applied to 'half car' & 'full car' suspension systems for the analysis. As in case of quarter car suspension system, sprung mass acceleration is used to quantify ride quality; sprung mass acceleration can be obtained by substituting values of sprung mass, unsprung mass, damping coefficient and stiffness values of tire & suspension in transfer function.

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