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Static and Dynamic Analysis of Functionally Graded Beam

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ABSTRACT

The improved technological development demands the use of highly efficient materials with improved mechanical properties like improved strength, stiffness etc. The requirement of improved properties demands the use of new hybrid materials like Functionally Graded Materials (FGM) for engineering applications. Understanding of FGM is required before having its implementation. The Finite element method provides a useful tool in modelling the dynamic behaviour of FGM to determine its natural frequency which would be used for the designing of mechanical component. The properties in the functionally graded material are assumed to vary according to power law. The natural frequencies were obtained for FG beams under various boundary conditions including CF, SF, CC, SS, and CS.

Keywords : Functionally Graded Beams; Free Vibration; Natural Frequency; Dynamic Analysis

INTRODUCTION

Functionally Graded Material (FGM) belongs to a class of advanced material characterized by variation in properties as the dimension varies. The overall properties of FGM are unique and different from any of the individual material that forms it. There is a wide range of applications for FGM and it is expected to increase as the cost of material processing and fabrication processes are reduced by improving these processes.

An application may require a material that is hard as well as ductile, there is no such material existing in nature. To solve this problem, combination (in molten state) of one metal with other metals or non-metals is used. This combination of materials in the molten state is termed alloying (recently referred to as conventional alloying) that gives a property that is different from the parent materials.

FGM can be considered second-generation composite materials. These materials are essentially two-phase particulate composites, e.g. ceramic and metallic alloy phases, microscopically engineered to have a smooth spatial variation of material properties in order to improve the overall composition performance.

FGMs are mainly used as thermal barriers where severe thermal environment exist (for example satellites thermal shield), however they have become popular in other applications such as heat exchanger tubes, biomedical implants, and high power electrical contacts which use in optoelectronics.

Composite materials will fail under extreme working conditions through a process called delimitation (separation of fibres from the matrix).This can happen for example, in high temperature application where two metals with different coefficient of expansion are used. To solve this problem, researchers in Japan in the mid 1980s, confronted with this challenge in an hypersonic space plane project requiring a thermal barrier (with outside temperature of 2000K and inside temperature of 1000K across less than 10 mm thickness), came up with a novel material called Functionally Graded Material (FGM). FGM occur in nature as bones, teeth etc.

FGM can be divided into two broad groups namely: thin and bulk FGM. Thin FGM are relatively thin sections or thin surface coating, while the bulk FGM are volume of materials which require more labor intensive processes. Thin section or surface coating FGM are produced by Physical or Chemical Vapour Deposition (PVD/CVD), Plasma Spraying, Self-propagating High-temperature Synthesis (SHS) etc. Bulk FGM is produced using powder metallurgy technique, centrifugal casting method, solid freeform technology etc.

FGM find their applications in aerospace, automobile, medicine, sport, energy, sensors, optoelectronic etc.

FGM are usually in the form of surface coatings, there are a wide range of surface deposition processes to choose from depending on the service requirement from the process.

A. Vapour Deposition Technique
B. Powder Metallurgy (PM)
C. Centrifugal Method
D. Solid Freeform (SFF) Fabrication Method

Process simulations may allow the prediction of suitable processing parameters for FGMs in the future and reduce the considerable amount of experimental effort which is still necessary to produce a graded material free of macro defects. Powder metallurgy, melt and polymer processing and modeling, the achievements of the priority program are the main type of manufacturing.

Introduction of Dynamic Analysis.

All real physical structures, when subjected to loads or displacements, behave dynamically. The additional inertia forces, from Newton's second law, are equal to the mass times the acceleration. If the loads or displacements are applied very slowly then the inertia forces can be neglected and a static load analysis can be justified. Hence, dynamic analysis is a simple extension of static analysis. In addition, all real structures potentially have an infinite number of displacements.

Therefore, the most critical phase of a structural analysis is to create a computer model, with a finite number of mass less

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members and a finite number of node (joint) displacements that will simulate the behavior of the real structure. The mass of a structural system, which can be accurately estimated, is lumped at the nodes. Also, for linear elastic structures the stiffness properties of the members, with the aid of experimental data, can be approximated with a high degree of confidence. However, the dynamic loading, energy dissipation properties and boundary (foundation) conditions for many structures are difficult to estimate. This is always true for the cases of seismic input or wind loads.

To reduce the errors that may be caused by the approximations summarized in the previous paragraph, it is necessary to conduct many different dynamic analyses using different computer models, loading and boundary conditions. It is not unrealistic to conduct 20 or more computer runs to design a new structure or to investigate retrofit options for an existing structure.

Because of the large number of computer runs required for a typical dynamic analysis, it is very important that accurate and numerically efficient methods be used within computer programs. Some of these methods have been developed by the author and are relatively new. Therefore, one of the purposes of this book is to summarize these numerical algorithms, their advantages and limitations.

2. MATHEMATICAL FORMULATION OF FG BEAM

The formulation of FG beam requires the complete knowledge about the distribution of material in the beam. It is also depended on the type of analysis we intend to perform. Here the mathematical formulation in two dimensional is given which could be extended easily for three dimensional as well. A functionally graded simply-supported beam of length (L), width(b), thickness (h), with coordinate system (xyz) having the origin O is shown in Fig. 1. In this study, it is assumed that the material properties of the beam such as, Young's modulus (E) and mass density (ρ) vary continuously along the beam axis according to a power law distribution as shown in the Fig.1, which is given by Eq. (5).

$$P(z) = (P_L - P_R)(1 - X/L)^k + P_R \qquad (5)$$

Where P_L and P_R are the corresponding material properties of the left and the right side of the beam, and k is the non-negative power-law exponent which represents the profile of material variation through the beam axis.

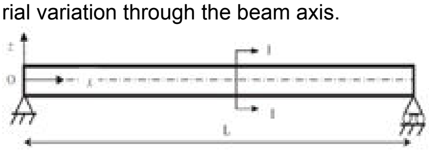

Figure 1: Functionally graded simply- supported beam. [14]

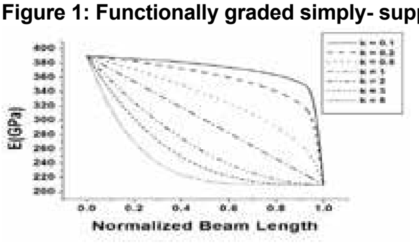

Figure 2: The variation of Young's modulus through the axial direction of the beam. [14]

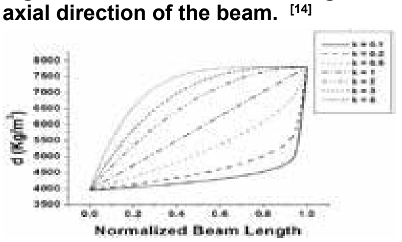

Figure 3: The variation of Mass density through the axial direction of the beam. [14]

The equation of motion for determining the natural frequencies is given by Eq. (6),

$$\mathrm{M}\ddot{d}_S + \mathrm{K}d_S = 0 \qquad (6)$$

Where, M is the global mass matrix of beam and K is the global stiffness matrix of beam, which are defined as follow,

The global mass-matrix is given by,

$$\mathrm{M} = \sum_{1}^{n} \mathrm{M}^{(e)} \qquad (7)$$

And the elemental mass matrix is given by,

$$\mathrm{M}^{(e)} = b\int_0^l \mathrm{N}^{\mathrm{T}} D_R N dx \qquad (8)$$

Where

$$\mathrm{N} = \begin{bmatrix} \mathrm{N}_{3i-2} & 0 & 0 & \mathrm{N}_{3j-2} & 0 & 0 \\ 0 & \mathrm{N}_{3i-1} & \mathrm{N}_{3i} & 0 & \frac{\delta \mathrm{N}_{3j-1}}{\delta x} & \frac{\delta \mathrm{N}_{3j}}{\delta x} \\ 0 & \frac{\delta \mathrm{N}_{3i-1}}{\delta x} & \frac{\delta \mathrm{N}_{3i}}{\delta x} & 0 & \frac{\delta^2 \mathrm{N}_{3j-1}}{\delta x^2} & \frac{\delta^2 \mathrm{N}_{3j}}{\delta x^2} \end{bmatrix} \qquad (9)$$

$$D_R = \int_{-h/2}^{h/2} \begin{bmatrix} \rho & 0 & -\rho z \\ 0 & \rho & 0 \\ -\rho z & 0 & \rho z^2 \end{bmatrix} dz \qquad (10)$$

And n is the total number of discretized elements and l is elemental beam length.

Now the global stiffness matrix is given by

$$\mathrm{K} = \sum_{1}^{n} \mathrm{K}^{(e)} \qquad (11)$$

The elemental stiffness matrix is

$$\mathrm{K}^{(e)} = b\int_0^l \mathrm{B}^{\mathrm{T}} D_E \mathrm{B} dx \qquad (12)$$

Where

$$\mathrm{B} = \begin{bmatrix} \frac{\delta \mathrm{N}_{3i-2}}{\delta x} & 0 & 0 & \frac{\delta \mathrm{N}_{3j-2}}{\delta x} & 0 & 0 \\ 0 & \frac{\delta^2 \mathrm{N}_{3i-1}}{\delta x^2} & \frac{\delta^2 \mathrm{N}_{3i}}{\delta x^2} & 0 & \frac{\delta^2 \mathrm{N}_{3j-1}}{\delta x^2} & \frac{\delta^2 \mathrm{N}_{3j}}{\delta x^2} \end{bmatrix} \qquad (13)$$

$$D_R = \int_{-h/2}^{h/2} \begin{bmatrix} \rho & 0 & -\rho z \\ 0 & \rho & 0 \\ -\rho z & 0 & \rho z^2 \end{bmatrix} dz \qquad (14)$$

$$D_{\mathrm{E}} = \int_{-h/2}^{h/2} \begin{bmatrix} \mathrm{E} & -z\mathrm{E} \\ -z\mathrm{E} & z^2\mathrm{E} \end{bmatrix} dz \qquad (15)$$

and the shape functions are given by ,

$$\mathrm{N}_{3i-2} = \left(1 - \frac{x}{l}\right) \qquad (16)$$

$$\mathrm{N}_{3i-1} = \frac{1}{l^3}\left(l^3 - 3lx^2 + 2x^3\right) \qquad (17)$$

$$\mathrm{N}_{3i} = \frac{1}{l^2}\left(l^2 x - 2lx^2 + 2x^3\right) \qquad (18)$$

$$\mathrm{N}_{3j-2} = \left(\frac{x}{l}\right) \qquad (19)$$

$$\mathrm{N}_{3j-1} = \frac{1}{l^3}\left(3lx^2 - 2x^3\right) \qquad (20)$$

$$N_{3j} = \frac{1}{l^2}\left(x^3 - k^2\right) \quad (21)$$

Where x is the local coordinate of the beam element. The elemental stiffness matrix and mass matrix is calculated for each element and assembled to form the Global Stiffness and Mass matrix to solve for the natural frequencies and mode shapes. This section gives us the particulars about the mathematical background in simulating the FG beam to improve our understandinng in its dynamic behaviour.

3. RESULTS AND DISCUSSION

3.1 STRESS RESULTS

For the above mentioned end condition, the following boundary conditions are applied for each case. Here z is the displacement and θ is the slope of the deflection curve. ANSYS work bench is used for Dynamic Analysis of FG Beam. For the meshing in ANSYS, number of nodes 51900 and 9375 elements with tetrahedron elements. Force applied at free end is 1000N for C-F and S-F boundary condition and same force at midpoint for S-S, C-C and C-S boundary condition. Six natural frequencies results are taken for checking the Dynamic behaviour of beam from the work bench. The results of stress of different material and different natural frequency graphs are shown below in detail.

(a) C-F

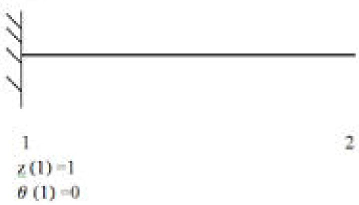

Figure 4: Fix-Free End condition.

For Fix-Free or Clamped-Free boundary condition, we fixed the beam at one end and another end is free as shown is Fig 4. At free end we applied 1000N load at free end. The stress results for this boundary condition for different material and material distribution are shown in Fig 5-7. In Fig 5 the Max stress generated is 33.156 MPa for horizontal material variation FG Beam. In Fig 6 the Max stress generated is 39.975 MPa for Steel Beam. In Fig 7 the Max stress generated is 0.59029 MPa for vertical material variation FG Beam.

For the boundary condition stress results are shown in the next page.

(i) For horizontal material variation FG Beam

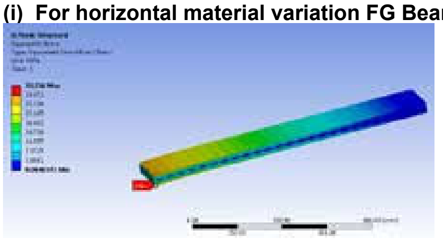

Figure 5: Stress result for horizontal material variation FG Beam for Fix-Free.

(ii) For Steel Beam

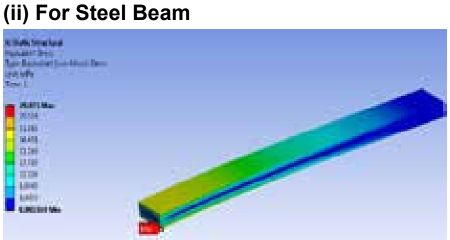

Figure 6: Stress result for steel beam for Fix-Free

(iii) For vertical material variation FG Beam

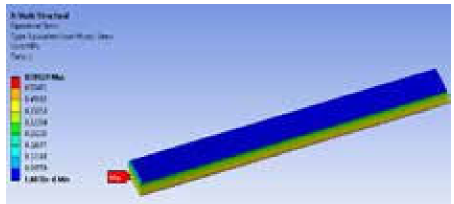

Figure 7: Stress result for vertical material variation FG Beam for Fix-Free

(b) S-F

Figure 8: SS-Free End condition

For Simply Supported-Free boundary condition, we simply supported the beam at one end and another end is free as shown in Fig 8. At free end we applied 1000N load at free end. The stress results for this boundary condition for different material and material distribution are shown in Fig 9-11. In Fig 9 the Max stress generated is 31.972 MPa for horizontal material variation FG Beam. In Fig 10 the Max stress generated is 86.482 MPa for Steel Beam. In Fig 11 the Max stress generated is 17.23 MPa for vertical material variation FG Beam

For the boundary condition stress results are shown below

(i) For horizontal material variation FG Beam

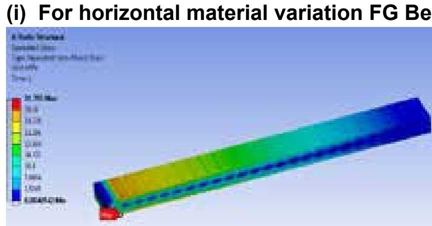

Figure 9: Stress result for horizontal material variation FG Beam for SS-Free

(ii) For Steel Beam

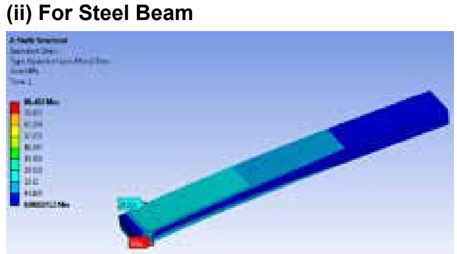

Figure 10: Stress result for Steel beam for SS-Free

(iii) For vertical material variation FG Beam

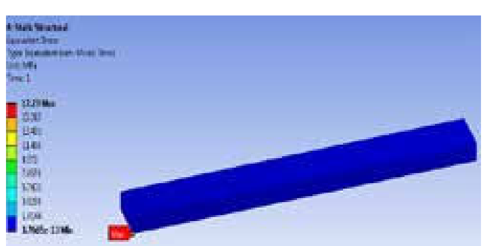

Figure 11: Stress result for vertical material variation FG Beam for SS-Free

Figure 12: SS-SS End condition

For Simply supported-Simply supported boundary condition, we simply supported the beam at both end as shown in fig 12. At mid of beam we applied 1000N load at midpoint of beam. The stress results for this boundary condition for different material and material distribution are shown in Fig 13-15. In Fig 13 the Max stress generated is 15.475 MPa for horizontal material variation FG Beam. In Fig 14 the Max stress generated is 5.1081 MPa for Steel Beam. In Fig 15 the Max stress generated is 0.065764 MPa for vertical material variation FG Beam.

For the boundary condition stress results are shown below

(i) For horizontal material variation FG Beam

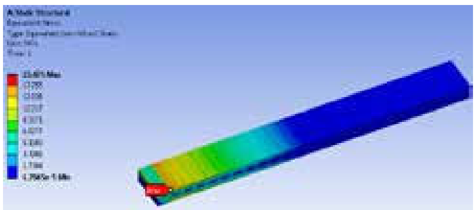

Figure 13: Stress result for horizontal material variation FG Beam for SS-SS

(ii) For Steel Beam

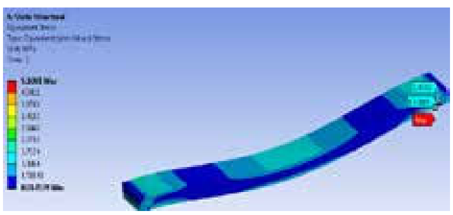

Figure 14: Stress result for Steel Beam for SS-SS

(iii) For vertical material variation FG Beam

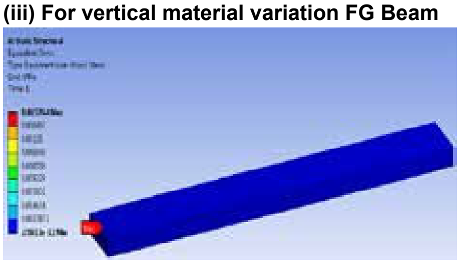

Figure 15: Stress result for vertical material variation FG Beam for SS-SS

Figure 16: Fix-Fix End condition

For Fix-Fix boundary condition, we Fixed the beam at both end. At mid of beam we applied 1000N load at midpoint of beam. The stress results for this boundary condition for different material and material distribution are shown in Fig 17-19. In Fig 17 the Max stress generated is 16.793 MPa for horizontal material variation FG Beam. In Fig 18 the Max stress generated is 2.8804 MPa for Steel Beam. In Fig 19 the Max stress generated is 0.0071084 MPa for vertical material variation FG Beam.

For the boundary condition stress results are shown below

(i) For horizontal material variation FG Beam

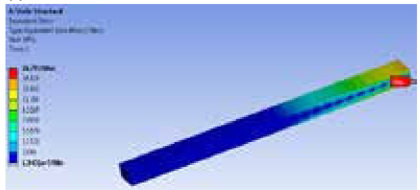

Figure 17: Stress result for horizontal material variation FG Beam for Fix-Fix

(ii) For Steel Beam

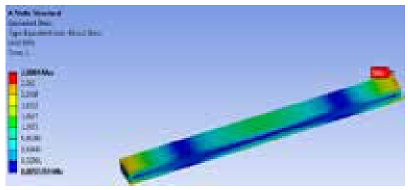

Figure 18: Stress result for Steel Beam for Fix-Fix

(iii) For vertical material variation FG Beam

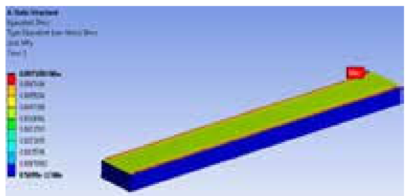

Figure 19: Stress result for vertical material variation FG Beam for Fix-Fix

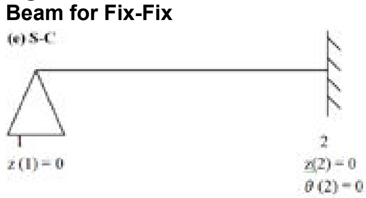

Figure 20: SS-Fix End condition

For Fix-Simply supported boundary condition, we fixed at one end and at another end simply supported the beam as shown in Fig 20. At mid of beam we applied 1000N load at midpoint of beam. The stress results for this boundary condition for different material and material distribution are shown in Fig 21-23. In Fig 21 the Max stress generated is 15.475 MPa for horizontal material variation FG Beam. In Fig 22 the Max stress generated is 5.0901 MPa for Steel Beam. In Fig 23 the Max stress generated is 0.085346 MPa for vertical material variation FG Beam.

For the boundary condition stress results are shown below

(i) For horizontal material variation FG Beam

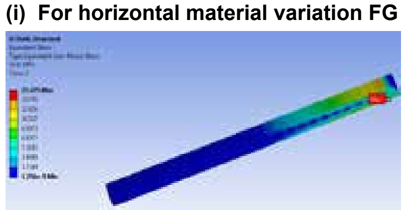

Figure 21: Stress result for horizontal material variation FG Beam for Fix-SS

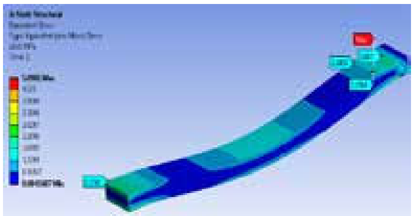

Figure 22: Stress result for Steel beam for Fix-SS

(iii) For vertical material variation FG Beam

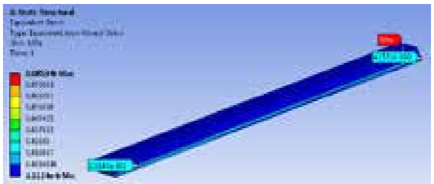

Figure 23: Stress result for vertical material variation FG Beam for Fix-SS

3.2 Natural Frequency Results
3.2.1 For Fix-Free
The graph between natural frequency and ordinal frequency of FGB is shown is in the Fig.24 for Fix-Free condition. The ordinal frequency is taken in X- axis and natural frequency is taken in Y-axis. The first six natural frequencies are taken for the plot the graph. The first natural frequencies are 64.185 Hz, 64.305 Hz, 67.828 Hz, 68.138 Hz, 74.557 Hz and 77.254 Hz. The minimum and maximum natural frequencies for the graph are 64.185 Hz and 77.254 Hz.

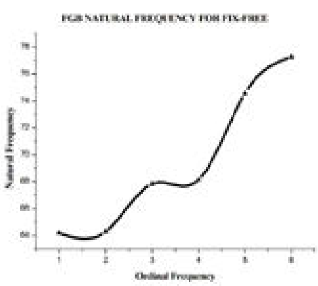

Figure 24: Graph for Fix-Free

3.2.2 For SS-Free
The graph between natural frequency and ordinal frequency of FGB is shown is in the fig.25 for SS-Free condition. The ordinal frequency is taken in X- axis and natural frequency is taken in Y-axis. The first six natural frequencies are taken for the plot the graph. The first natural frequencies are 64.185 Hz, 64.307 Hz, 67.828 Hz, 68.138 Hz, 74.557 Hz and 77.254 Hz. The minimum and maximum natural frequencies for the graph are 64.185 Hz and 77.254 Hz.

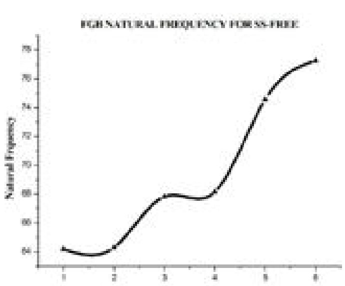

Figure 25: Graph for SS-Free

3.2.3 For SS-SS
The graph between natural frequency and ordinal frequency of FGB is shown is in the fig.26 for SS-SS condition. The ordinal frequency is taken in X- axis and natural frequency is taken in Y-axis. The first six natural frequencies are taken for the plot the graph. The first natural frequencies are 61.151 Hz, 63.953 Hz, 66.608 Hz, 68.099 Hz, 74.552 Hz and 77.254 Hz. The minimum and maximum natural frequencies for the graph are 61.151 Hz and 77.254 Hz.

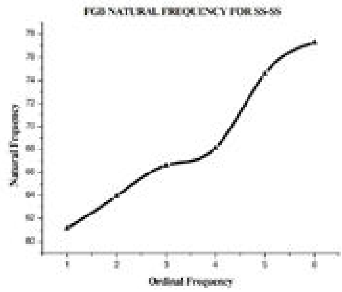

Figure 26: Graph for SS-SS

3.2.4 For Fix-Fix
The graph between natural frequency and ordinal frequency of FGB is shown is in the fig.27 for Fix-Fix condition. The ordinal frequency is taken in X- axis and natural frequency is taken in Y-axis. The first six natural frequencies are taken for the plot the graph. The first natural frequencies are 60.321 Hz, 63.953 Hz, 66.584 Hz, 68.085 Hz, 73.780 Hz and 74.552 Hz. The minimum and maximum natural frequencies for the graph are 60.321 Hz and 74.552 Hz.

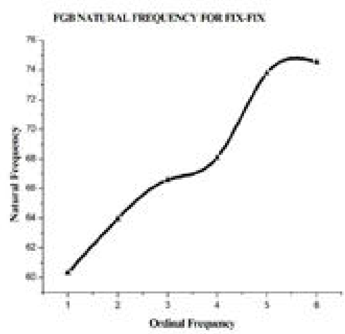

Figure 27: Graph for Fix-Fix

3.2.5 For Fix-SS
The graph between natural frequency and ordinal frequency of FGB is shown is in the fig.28 for Fix-SS condition. The ordinal frequency is taken in X- axis and natural frequency is taken in Y-axis. The first six natural frequencies are taken for the plot the graph. The first natural frequencies are 61.149 Hz, 63.953 Hz, 66.608 Hz, 68.809 Hz, 74.552 Hz and 77.254 Hz. The minimum and maximum natural frequencies for the graph are 61.149 Hz and 74.254 Hz.

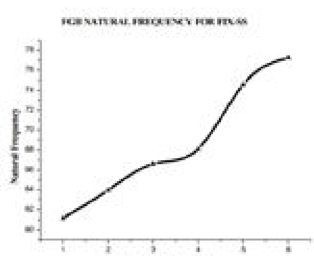

Figure 28: Graph for Fix-SS

CONCLUSIONS

The natural frequency of FG beam is found to be increased with foundation modulus. It is been concluded that the free vibration characteristics and dynamic behaviour of a Functionally Graded Beam (FGBs) for different material distribution are analyzed numerically by finite element method in the present work. It is assumed that the material properties of the beam vary continuously through the thickness or axial direction according to power-law form. The natural frequency of FG Beam (horizontal material variation) is gradually increased. The natural frequency of the FGM helps in designing a better structure which avoids resonance during its functioning. By comparing both materials the result which we got is FG beam is lighter in weight than steel beams.

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