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A Research Paper on Static Analysis of Laminated Composite Plate

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ABSTRACT

This research work aims to analyze the static analysis of laminated composite plate with various boundary conditions and various types of load applications. In this paper, finite element analysis has been carried out for five laminated rectangular plate by considering the master element as a tetrahedron element. Numerical analysis has been carried out by developing programming in mathematical software MATLAB and the results obtained from MATLAB are giving good agreement with the results. Later, for the same structure, analysis has been carried out using finite element analysis software ANSYS 12.1. This job is helpful for obtaining the results not only at node points but also the entire surface of the rectangular plate. Finally, comparison has been done between the results obtained from FEA numerical analysis, and ANSYS results. Numerical results showed that, the results obtained by finite element analysis and ANSYS simulation results are in close agreement with the stress results. During this analysis, the optimal thickness of the plate has been obtained when the plate is subjected to different loading and boundary conditions.

Keywords : Composite plates, FEA analysis (ANSYS), Analytical solution (MATLAB).

INTRODUCTION

Composite materials have been used in the industry for many years because they perform better than the comparable homogenous isotropic materials. Composite Materials can be defined as a material with two or more distinct macroscopical phases. They consist of two or more materials combined in such a way that the individual materials are easily distinguishable.

Laminates are composite material where different layers of materials give them the specific character of a composite material having a specific function to perform. Fabrics have no matrix to fall back on, but in them, fibers of different compositions combine to give them a specific character. Reinforcing materials generally withstand maximum load and serve the desirable properties. Composites cannot be made from constituents with divergent linear expansion characteristics. The interface is the area of contact between the reinforcement and the matrix materials.

MATHEMATICL MODELLING

The use of mathematics is one of the many approaches to solving real-world problems. Others include experimentation either with scaled physical models or with the real world directly. Mathematical modelling is the process by which a problem as it appears in the real world is interpreted and represented in terms of abstract symbols. This makes mathematical modelling challenging and at the same time demanding since the use of mathematics and computers for solving real-world problems is very widespread and has an impact in all branches of learning.

The coordinate system for a laminated plate used in the present study is shown in figure 3.1. The xyz coordinate system is assumed to have its origin at the corner of the middle plane of the plate. The surfaces of the plate are at z=+h/2 and h is the thickness of the plate .ɵ is the angle of fibre orientation with the respect to the axis.1-,2- and 3-directions are principal axes in the longitudinal, transverse and normal directions ,respectively, it is assumed that the transverse deflection is small, so that the out-of-plane components of the in plane results , and are negligible[1] .

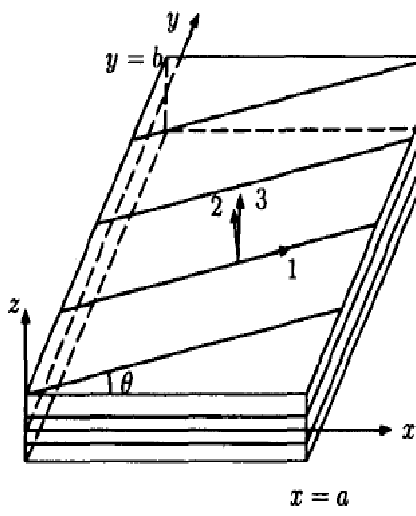

Figure 1: coordinate system for a laminated rectangular plate[1]

Plane stress for generally orthotropic plates, so that

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{bmatrix} = \begin{bmatrix} cos^2\theta & sin^2\theta & -2\sin\theta\cos\theta \\ sin^2\theta & cos^2\theta & 2\sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin\theta\cos\theta & (cos^2\theta - sin^2\theta)\end{bmatrix}\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix} \ldots (1)$$

Defining a new matrix called the laminate stiffness matrix as

$$[\bar{Q}] = [T]^{-1}[Q]\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 2\end{bmatrix}[T] \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

And letting:

$c = \cos\theta\,, s = \sin\theta,$

components are:

$$\bar{Q}_{11}=Q_{11}c^4 + 2(Q_{12}+2Q_{66})c^2s^2 + Q_{22}s^4$$

$$\bar{Q}_{12}=Q_{12}(c^4+s^4) + (Q_{11}+Q_{22}-4Q_{66})c^2s^2$$

$$\bar{Q}_{22}=Q_{11}s^4 + 2(Q_{12}+2Q_{66})\,c^2s^2+Q_{22}c^4$$

$$\bar{Q}_{16}=(Q_{11}-Q_{12}-2Q_{66})c^3s-(Q_{22}-Q_{12}-2Q_{66})cs^3$$

$$\bar{Q}_{26}=(Q_{11}-Q_{12}-2Q_{66})cs^3-(Q_{22}-Q_{12}-2Q_{66})c^3s$$

$$\bar{Q}_{66}=(Q_{11}+Q_{22}-2Q_{12}-2Q_{66})c^2s^2+Q_{66}(c^4+s^4)$$

If ɵ is any angle other than zero, there will be nonzero and terms. Putting this into equation,

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix}=\begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & 2\bar{Q}_{16}\\ \bar{Q}_{12} & \bar{Q}_{22} & 2\bar{Q}_{26}\\ \bar{Q}_{16} & \bar{Q}_{26} & 2\bar{Q}_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_{xy}\end{bmatrix},$$

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix}=\begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16}\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26}\\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{bmatrix}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

It can be seen that a shear strain will produce normal stresses, and normal strains will contribute to a shear stress.

Constitutive Equations for a Laminate

Since the middle surface strains and curvatures are not a part of the summations, the laminate stiffness matrix and the h_k terms can be combined to form new matrices[1].

$$A_{ij}=\sum_{k=1}^{N}(\overline{Q_{ij}})_k(h_k-h_{k-1})\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

$$B_{ij}=\frac{1}{2}\sum_{k=1}^{N}(\overline{Q_{ij}})_k(h_k^2-h_{k-1}^2)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

$$D_{ij}=\frac{1}{3}\sum_{k=1}^{N}(\overline{Q_{ij}})_k(h_k^3-h_{k-1}^3)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

In matrix form, the constitutive equations can easily be written as:

$$\begin{bmatrix}N_x\\ N_y\\ N_{xy}\\ \cdots\\ M_x\\ M_y\\ M_{xy}\end{bmatrix}=\begin{bmatrix}A_{11} & A_{12} & A_{16} & \vdots & B_{11} & B_{12} & B_{16}\\ A_{12} & A_{22} & A_{26} & \vdots & B_{12} & B_{22} & B_{26}\\ A_{16} & A_{26} & A_{66} & & B_{16} & B_{26} & B_{66}\\ & \cdots & & \vdots & & \cdots & \\ C_{11} & C_{12} & C_{16} & & D_{11} & D_{12} & D_{16}\\ C_{12} & C_{22} & C_{26} & \vdots & D_{12} & D_{22} & D_{26}\\ C_{16} & C_{16} & C_{66} & \vdots & D_{16} & D_{26} & D_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\\ \cdots\\ K_x\\ K_y\\ K_{xy}\end{bmatrix}\ldots\ldots\ldots(7)$$

Written in contracted form,

$$\begin{bmatrix}N\\ -\\ M\end{bmatrix}=\begin{bmatrix}A & B\\ B & D\end{bmatrix}\begin{bmatrix}\varepsilon^0\\ -\\ K\end{bmatrix}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

This can be partially inverted to give:

$$\begin{bmatrix}\varepsilon^0\\ -\\ K\end{bmatrix}=\begin{bmatrix}A^* & B^*\\ C^* & D^*\end{bmatrix}\begin{bmatrix}N\\ -\\ K\end{bmatrix}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

Where,

$$[A^*]=[A]^{-1},$$

$$[B^*]=-[A]^{-1}[B],$$

$$[C^*]=[B][A]^{-1},$$

$$[D^*]=[D]-[B][A]^{-1}[B].$$

FINITE ELEMENT MODELING AND VALIDATION

Material Properties

The material used for the composite laminate is carbon/epoxy laminate. The unidirectional layer orthotropic properties for the material are given a

NO	PROPERTIES	VALUE	UNITS
1	Compressive strength-longitudinal	570	MPa
2	Compressive strength-transverse	570	MPa
3	Shear modulus-in plane	5	GPa
4	Young's Modulus-longitudinal	70	GPa
5	Young's Modulus -transverse	70	GPa

Geometry of Finite Element Model

The laminate composite plate model considered here includes symmetrical and unsymmetrical plate consisting of 5 layers, respectively, with ply thickness of 0.5'' each. We consider five laminates with orientations 0/90/0/90/0, respectively.

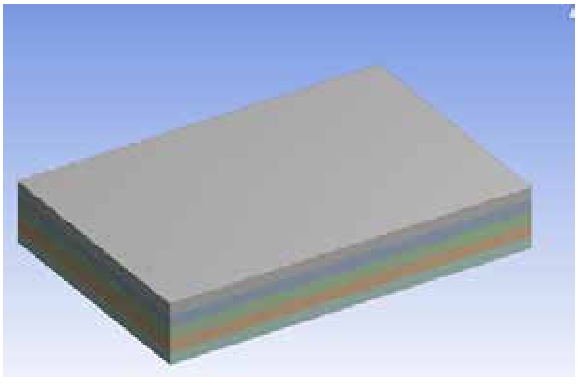

Figure 2: geometry of plate

The size of the element along the width of the area is maintained as per thickness of layer i.e. 0.5". The spacing ratio for the line divisions is negative to increase the density of the elements at the ends of the plate. Plate meshing is as shown in figure below,

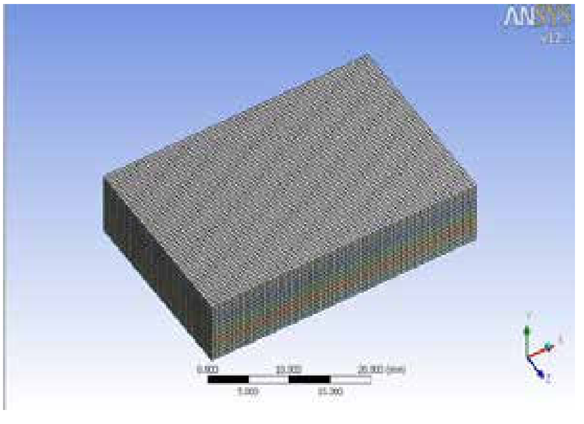

Figure 3: meshing geometry

Than after load apply on the plate and fixed support to the geometry ,Than after click to the equivalent von mises stress and stress generated on the plate as shown in figure below,

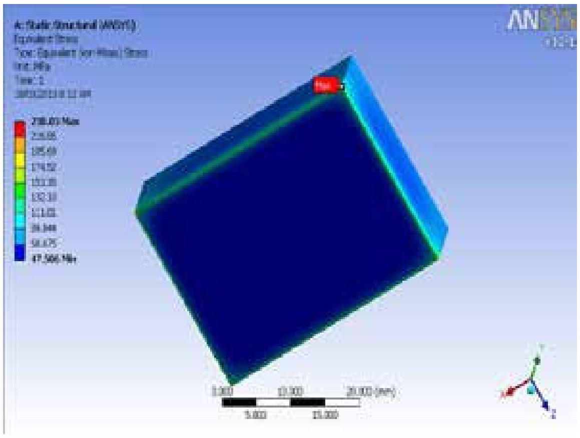

Figure 4: max stress result

ANALYTICAL SOLUTION

Matlab program

The program is coded using the MATLAB version 7.10.0 (R2010a). Since this is the first time of this programming language is used, many obstacles and hindrances have been met throughout the learning process due to the lack of understanding of Matlab language at the beginning of the study.

The most difficult part in the coding of the current program is to identify the error when the program could not function properly. The properties is same to used in ansys .

Finally Stress graph is generated from the stress result as shown in figure 5,

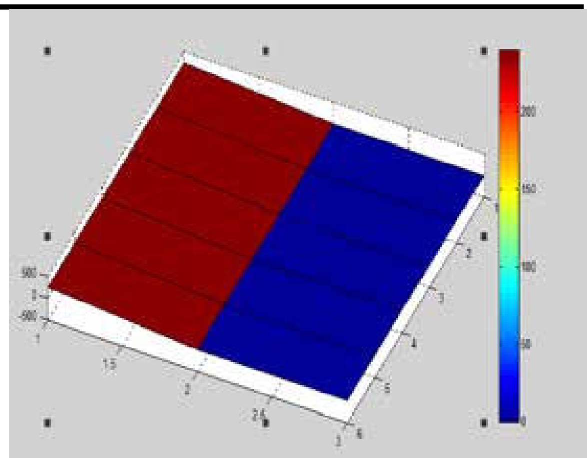

Figure 5: Stress graph

COMPARISION

GEOMETRY (PLATE)	ANSYS 12.1	ANALYTICAL (MATLAB)	DIFF
STRESS(MPa)	238.03	240	1.97%

CONCLUSION

Numerical analysis has been carried out by developing programming in mathematical software MATLAB and the results obtained from MATLAB are giving good agreement with the stress result is 240MPa. Than after for the same structure, analysis has been carried out using finite element analysis software ANSYS 12.1 and stress result is 238.03MPa.

Finally, comparison has been done between the results obtained from FEA numerical analysis and ANSYS result. Numerical results showed that, the results obtained by finite element analysis and ANSYS simulation results are in close agreement with the 1.97% difference result. During this analysis, the optimal thickness of the plate has been obtained when the plate is subjected to different loading and boundary conditions.

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