Importantly Topic of Graph Theory

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ABSTRACT

This paper defines of graph and sub-graph, isomorphic graph, complete graph, path and cycle, completed bipartite graph, matrices representation of graph. This paper in the last of induction method theorem different types of walk is related with matrices.

Keywords :

Definitions:
Let G be a graph. If two (or more) edges of G have the same end vertices then these edges are called Parallel.

A vertex of G which is not the end of any edge is called isolated.

Two vertices which are joined by an edge are said to be adjacent or neighbours.

The set of all neighbors of a fixed vertex of G is called. The neighbor – hood set of V and is denoted by N(V)

\[ V_1 \quad V_2 \]

\[ V_3 \quad V_4 \]

\[ e_1 \]

\[ e_2 \]

A graph is called simple IE it has no loops and no parallel edges.

Graph Isomorphism:
A graph \( G_1=(V_1,E_1) \) is said to be isomorphic to graph \( G_2=(V_2,E_2) \) if there is a one – to – one correspondence between the vertex set’s \( V_1 \) and \( V_2 \) and a one – to – one correspondence between the edge sets \( E_1 \) and \( E_2 \) in such a way that if \( e_i \) is an edge with end vertices \( u_i \) and \( v_i \) in \( G_1 \) then the corresponding edge \( e_j \) in \( G_2 \) has it’s end point’s the vertices \( u_j \) and \( v_j \) in \( G_2 \) which correspond. to \( u_i \) and \( v_i \) respectively such a pair of correspondences is called a graph isomorphism.

\[ \text{Pairs of isomorphic graphs} \]

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And following graph is not isomorphic to these graphs.

\[ \text{No simple graph(multi-graph)} \]
**Complete graph:**
Complete graph is a simple graph in which each pair of distinct vertices is joined by an edge.

![Complete Graph Diagram](image)

**Paths and cycles.**
A walk in a graph G is a finite sequence
\[ W = v_0, e_1, v_1, e_2, v_2, \ldots, e_k, v_k \]
whose terms are alternately vertices and edges such that, for \( 1 \leq i \leq k \)
the edge \( e_i \) has ends \( v_{i-1} \) and \( v_i \).

Thus each edge \( e_i \) is immediately preceded and succeeded by the two vertices with which it is incident.

We say that the above walk \( w \) is a walk (walk from \( v_0 \) to \( v_k \)) the vertex \( v_0 \) is called the origin of the walk \( w \), while \( v_k \) is called the terminus of \( w \). Note that \( v_0 \) and \( v_k \) need not be distinct.

In figure \( w_1 = v_1, e_1, v_2, e_4, v_4, e_6, v_6 \)
and \( w_2 = v_1, e_1, v_2, e_8, v_3 \)
Walk of length 4 and 2 respectively from \( v_1 \) to \( v_2 \) and from \( v_1 \) to \( v_3 \).

Cij = \( v_1, e_1, v_2, \ldots, v_j - 1, v_i, v_j + 1, v_j + 2, \ldots, v_n, v_1 \)

Now deleting edges \( v_1, v_2 \) and \( v_9, v_2 \) as show in figure and
\[ w(v_1) + w(v_1) + w(v_1) + w(v_2) < w(v_1) + w(v_2). \]

The \( cij \) is of smaller length then \( c \) in this case we replace \( c \) by \( cij \) and perform a similar comparison on \( cij \). We repeat the procedure until we reach a cycle which can’t be improved upon by using the same technique.

**Complete bipartite graph:**
**Figure:** - Complete graph

Now, figure complete bipartite graph

G be a graph and set of vertices \( V \) of G and \( V = X \cup Y \) and \( X \cap Y = \emptyset \) and \( X \neq \emptyset \), \( Y \neq \emptyset \) and in a way each edges of G has one end in X and second end in Y the G is called bipartite, the partition \( v = x \cup y \) is called a bipartition of G.

A complete bipartite graph is a complete graph and bipartite graph It is denote by \( K_{mn} \) where \( m \) has \( n \) vertices.

**The Matrix representation of graphs:**
Let \( G \) be a graph with \( n \) vertices, \( v_1, \ldots, v_n \).
The adjacency matrix of \( G \) with respect to \( n \) vertices of \( G \) is the \( n \times n \) matrix
\[ A(G) = (a_{ij}) \]
where simple.

**The travelling salesman problem:**
Let \( c = v_1, v_2, \ldots, v_n \) be initial Hamiltonian cycle in our complete graph \( G \), where i,j. Sneh that \( 1 < i + 1 < j \leq n \), we can form a new Hamiltonian cycle \( cij \) from \( c \) given by

\[ A(\overrightarrow{i}) = Aij \]

Where simply.
As further example $b_{23} = b_{32} = 2$ and are two walks of length 2
From vertex 2 to vertex 3.

There two walks of length 2

**Theorem:**
Let $G$ be a graph with $n$ vertices $v_1 \ldots v_n$ and let $A$ denote the adjacency matrix of $G$ with respect to this listing of the vertices. Let $k$ be any positive integer and let $A^k$ denote the matrix multiplication of $k$ copies of $A$. Then the $(i|j)$ entry of $A^k$ is the number of different $v_i \rightarrow v_j$ walks in $G$ of length $k$.

**Proof.**
We prove this theorem by mathematical induction on $k$.

For $k = 1$ then $(i, j)$ entry of $A$ is the number of different $v_i \rightarrow v_j$ walks in graph $G$ of length 1

Which is clearly this result for $k = 1$ .

Now suppose this result is true for $A^k$ when $k \leq N - 1$.

Now we prove this result for $k + 1$ as $A^{k+1} = A_k \times A$, from the definition of matrix multiplication we get.

Let $G_j = \sum_{t=1}^n t$ element of $A^k \times t$ element of $A$.

Now every $v_i \rightarrow v_j$ walks of length $k + 1$ contains of a $v_i \rightarrow v_j$ walks of length $k$ and $v_t$ is adjacent to $v_j$, followed by an edge $v_t \rightarrow v_j$ since there are $t$ such walks of length $k$ and $a + j$ such edges for each vertex $v_t$, the total number of all $v_i \rightarrow v_j$ walks is

$$\sum_{t=1}^n a_t + j$$

then above induction $k$ is true $= 1$ $k + 1$ is true then by indented method the proof is complete.

**Conclusion:**
This paper in various topic of graph theory are discussed various example and diagram.

And also connected path and cycle represent various real problems to convert easily. And last any graph converted to matrices.

![Diagram showing graph theory concepts](https://via.placeholder.com/150)