Research Paper Mathematics

Important Topic of Graph Theory

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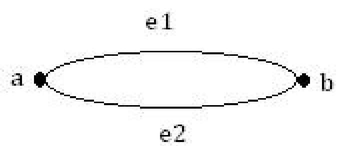
ABSTRACT

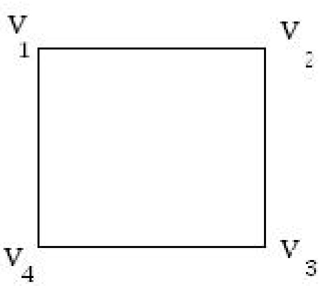
This paper defines of graph and sub-graph, isomorphic graph, complete graph, path and cycle, completed bipartite graph, matrices representation of graph. This paper in the last of induction method theorem different types of walk is related with matrices.

Keywords :

Definitions:
Let G be a graph. If two (or more) edges of G have the same end vertices then these edges are called Parallel.

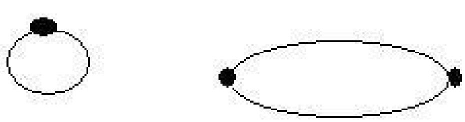


- A vertex of G which is not the end of any edge is called isolated.
- Two vertices which are joined by an edge are said to be adjacent or neighbours.
- The set of all neighbors of a fixed vertex of G is called. The neighbor – hood set of V and is denoted by N(V)



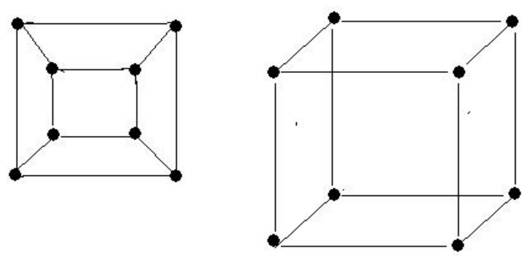
V_1 and V_2 are adjacent but V_1 and V_3 are not. The neighborhood Set N (V_2) of V_2 is {V_1,V_3}.

A graph is called simple IE it has no loops and no parallel edges.

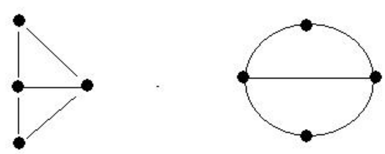


No simple graph(multi-graph)

Graph Isomorphism:
A graph G1=(V_1,E_1) is said to be isomorphic to graph G2=(V_2,E_2) if there is a one – to – one correspondence between the vertex set's V_1 and V_2 and a one – to – one correspondence between the edge sets E_1 and E_2 in such a way that if e_1 is an edge with end vertices u_1 and v_1 in G_1 then the corresponding edge e_2 in G_2 has it's end point's the vertices u_2 and v_2 in G_2 which correspond. to u_1 and v_1 respectively such a pair of correspondences is called a graph isomorphism.

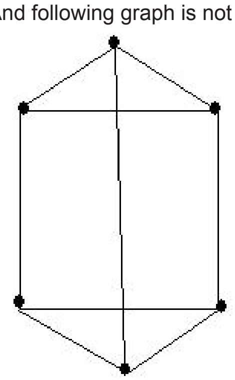


Pairs of isomorphic graphs

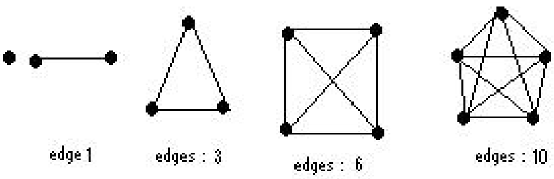


Pairs of isomorphic graphs

And following graph is not isomorphic to these graphs.



Complete graph:
Complete graph is a simple graph in which each pair of distinct vertices is joined by an edge.



(Complete graphs)

An empty (trivial) graph is a graph with no edges.

Paths and cycles.

A walk in a graph G is a finite sequence

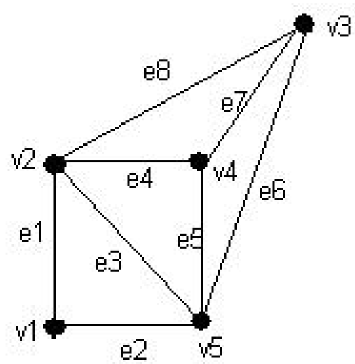
$W = v_0\, e_1\, v_1\, e_2\, v_2$ ----- $v_{k-1}\, e_k\, v_k$

Whose terms are alternately vertices and edges such that, for $1 \le i \le k$

The edge ei has ends vi-1 and VI

Thus each edge ei is immediately preceded and succeded by the two vertices with which it is incident.

We say that the above walk w is a v_0---v_k walk (walk from v_0 to v_5) the vertex v_0 is called the origin of the walk w, while v_k is called the terminus of w. Note that v_0 and v_k need not be distinct.



In figure $w_1 = v_1\, e_1\, v_2\, e_4\, v_4\, e_5\, v_5\, e_6\, v_3$ and $w_2 = v_1\, e_1\, v_2\, e_8\, v_3$ Walk of length 4 and 2 respectively from v_1 to v_3 and from v_1 to v_3 . in w_1 , $v_1 \neq v_3$ then walk w_1 is called open walk.

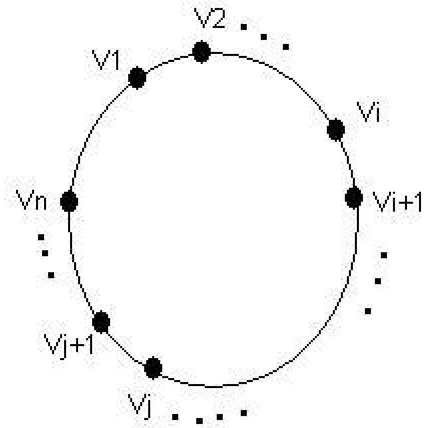
$W_3 = v_1\, e_1\, v_2\, e_4\, v_5\, e_2\, v_1$ is closed walk if the edges e_1 , e_2 --- e_r of the walk $w = v_0\, e_1\, v_1\, e_2\, v_2$ --- $e_r v_r$ are destined then wise called a trail.

$w4 = v_1\, e_1\, v_2\, e_8\, v_3\, e_6\, v_5$ is called trail because e_1 , e_8 , e_6 are all destined.

If the vertices of the walk all are distinct then w is called a path.

Result : - Given any two vertices v_1 and v_2 of a graph G , every $v_1\, v_2$ walk (outen'n) a $v_1 - v_2$ path.

The travelling salesman problem:
Let $c = v_1\, v_2$ --- $v_n\, v_1$ be initial Hamiltonian cycle in our complete graph G, where i.j. Sneh that $1 < i + 1 < j \le n$. we can form a new Humiltonian cycle cij from c given by

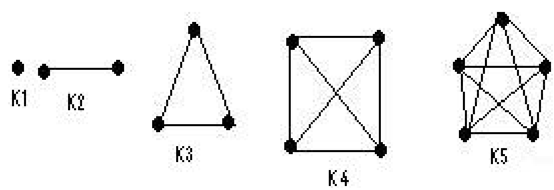


$Cij = v_1\, v_2$ --- $v_i\, v_j\, v_j - 1 - vi + 1\, v_j + 1\, v_j + 2$ -- $v_n v_1$

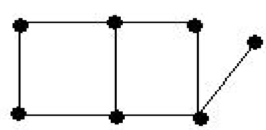
Now deleting edges $v_i v_i + 1$ and $v_j\, v_j + 1$ as show in figure and $w(v_i v_j) + w(v_i + 1\ v_j + 1) < w\ (v_i\ v_i{+}1) + w\ (v_j\ w_j + 1)$.

The cij is of smaller length then c in this case we replace c by cij and perform a similar comparison on cij. We repeat the procedure until we reach a cycle which can't be improved upon by using the same technique.

Complete bipartite graph:
Figure: - Complete graph



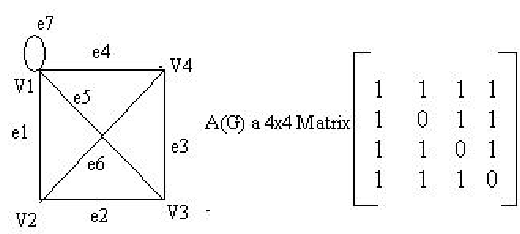
Now, figure complete bipartite graph



G be a graph and set of vestex V of G and $V = X \cup Y$ and $X \cap Y = \emptyset$ and $X \neq \emptyset$, $Y \neq \emptyset$ and in a way each edges of G has one end in X and second end in Y the G is called bipartite, the partitin $v = x \cup y$ is called a bipartition of G.

A complete bipartite graph is a complete graph and bipartite graph It is denote by 4min where x has on vertices and y has n vertices.

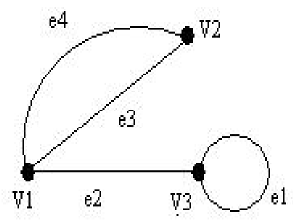
The Matrix representation of graphs:
Let G bea graph with n vertices , v_1 , ---- v_n. The adjacency matrix of G with respect to n vertices of G is the nxn matrix A(G) = (4ij) where the (i1j) th entry aij is the number of edges joining the vertex v_i to the vertex v_j.



Whare simply.

$$\begin{pmatrix} (1,1) & (2,1) & (3,1) & (4,1) \\ (1,2) & (2,2) & (3,2) & (4,2) \\ (1,3) & (2,3) & (3,3) & (4,3) \\ (1,4) & (2,4) & (3,4) & (4,4) \end{pmatrix}$$

Nor. Second example



Give matrix.

$$\begin{pmatrix} 0 & 1 & 1 \\ 2 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

Now suppose A = $\begin{pmatrix} 0 & 2 & 1 \\ 2 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$

Then $A^2 = \begin{pmatrix} 0 & 2 & 1 \\ 2 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 2 & 1 \\ 2 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} 5 & 0 & 1 \\ & & \\ & & \end{pmatrix}$

0 4 2 = (bij) (say)
1 2 2

and in fact bij gives the number of walks of length 2 from vertex I to vertex j for example b_{11} = 5 and the 5 walks of length 2 from vertex 1 to 1.

$v_1\ e_2\ v_3\ e_2\ v_1$
$v_1\ e_3\ v_2\ e_3\ v_1$
$v_1\ e_3\ v_2\ e_4\ v_1$
$v_1\ e_4\ v_2\ e_4\ v_1$
$v_1\ e_4\ v_2\ e_3\ v_1$

As farther example b23 = b_{32} = 2 and are 2 walks of length 2

From vertex 2 to vertex 3.
$v_2\ e_3\ v_1\ e_2\ v_3$
$v_3\ e_2\ v_1\ e_3\ v_2$

There two walks of length 2

Theorem:-
Let G be a graph with n vertices v_{1-4} v_n and let a denote the adjacency matrix of G with respect to this listing of the vertices. Let k be any positive integer and let A^k denote the matrix multiplication of k copies of A, Then the (i1j) the entry of A^k is the number of different $v_i - v_j$ walks in G of length k.

Proof.
We proof this theorem by mathematical induction on k.

For , k = 1 then (i , j)th entry of A is the number of different $v_i - v_j$ walks in graph G of length 1

Which is clearly this result for k = 1

Now suppose this result is true for A^k when where k 1 – N – {1}

Now we prove this result for k + 1 has A^k = (bij) we are assuming that bij is the number of different walks of length k from v_i to v_j and we want to prove that if A^{k+1} = (cij) is the number of different walk of length k+1 from v_i to v_j set A = (aij), Since A^{k+1} = Ak × A , from the definition of matrix multiplication we get.

Gj = $\sum_{t=1}^{n}$ (i , t) tn element of A^k) × (+ , j) the element of A

$$= \sum_{t-1}^{n} bit\ a + j$$

Now every $v_i - v_j$ walks of length k + 1 contains of a $v_i - v_j$ walks of length k. and v_t is adjacent to v_j , followed by an edge $v_t\ v_j$ since there are bit such walks of length k and a + j such edges for each vertex v_{tn} the total number of all $v_i - v_j$ walks is

$$\sum_{t=1} bit\ atj\ ,$$

then above induction k is true = 1 k + 1 is true then by indented method the proof is complete.

Conclusion:
This paper in various topic of graph theory are discussed various example and diagram.

And also connected path and cycle represent various real problems to convert easily. And last any graph converted to matrices.

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