



## Impact of the Past on E-Commerce Behaviour

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### ABSTRACT

*The main prerequisite for efficient management is an accurate overview of costs and assets. Information system develops in time, undergoes changes and is a significant part of a company's budget. The aim of this paper is to create a dynamical model for monitoring the value of information system's services in e-commerce. The model shall simulate the dynamics of an information system and express the value of important parameters. Structuring the model involved the use of analytic and synthetic methods, dynamical modelling and solution to a set of differential equations with delay. The necessary theory is explained briefly but sufficiently enough for expressing relations of the economic quantities examined. A concrete example and its solution carried out by means of Maple are shown at the end of the paper. A huge advantage of the model lies in its simplicity; it only deals with some specifically selected issues.*

**Keywords :** differential equations with delay, e-commerce, strategic management

### Introduction

The current state of world economy makes managers look for new methods and ways of cost-cutting, mainly in terms of support processes. Therefore many organizations have some specific internal and external processes including IS/IT architecture to support such methods and ways. One option is IS/IT development focusing on Business Intelligence, which is a way of achieving competitive advantage and which focuses on supporting decision-making processes, among others in e-commerce. The key application of the tool is to get precise information in a required form and in the right time, which increases the likelihood of making a relevant decision.

The aim of this paper is to create a model to monitor e-commerce as a dynamical system developing in time.

### Statement of the issue

The paper primarily deals with strategic management of e-commerce but it also touches on other areas.

The fundamental building block is development in strategic management, linked also to development in information and communications technology (ICT) and its use for evaluating system's efficiency, behaviour prediction, modelling boundary conditions or development of a company and the environment.

E-commerce is a rapidly developing field with prerequisites applying sophisticated solutions even in strategic management while using ICT.

Such solutions are then linked to modelling, in particular to creation of dynamical models which can depict a subject from perspectives required for making an informed decision. Quality information, gathered from real data using ICT, plays the key role in this respect.

### Strategic management in e-commerce

Key publications regarding strategic management of informatics in the Czech Republic are studies by Voříšek (2008), Molnár (2001) or Učeň (2008). The issue of harmony between information technologies and a company's business strategy has been dealt with by Luftman (2006), who sees it as one of

five main problems managers in big companies face.

At the same time, e-commerce is an activity tied to data level, thus enabling use of the database for acquiring information as well as for input of the intended model. Numerous studies have shown that strategic connection between IT and business strategy plays a significant role in a company's performance (Chan, Horner Reich, 2007). Strategy in terms of e-commerce has been studied by authors in (Desarbo, Di Benedetto, Song, Sinha, 2005), who have supported their findings by extensive international research.

Other studies dealing with the methodology of strategic planning in e-commerce are (Hackbarth & Kettinger, 2000), (Chang, Jackson, Grover, 2003) or (Collard, Licandro, Puch, 2008). Identification of particular elements of a system represents a key area determining the application of specific methods. In this respect, it is useful to respect development tendencies in strategic management. Strategic planning and development of a model which could contribute to successful risk management when planning the strategy of a start-up in e-commerce can be found for example in studies (Azumah, Koh, Maguire, 2007) and (Ferguson & Yen, 2007).

A significant prerequisite for purposeful strategic management of e-commerce is the ability of management to react to changing conditions both in the internal and external environment. The ability means, above all, constant decision-making in the right time.

### Methods

#### Modelling in economics

Economists have long been trying to apply models of dynamical systems in economy. State variables in economy can be represented by quantities of production, consumption, investment, etc. Original systems were based on the assumption that mathematical description of the monitored objects together with the description of their mutual relations is comprehensive in terms of understanding. Moreover, models include a factor of uncertainty hidden in the form of random elements. In some cases, though, quantities may be only seemingly random or their randomness is negligible and the quantities are dominated by dynamical relations. Mathematical description

can be provided by functional differential equations (FDE), or their specific kind – differential equations with delay argument.

Differential equations with delay can be used in such deterministic models in which we assume that all state quantities are determined not only by immediate values of quantities entering the solution but also by their previous states regardless of all random influences.

**Application of differential equations with delay in economics**

Many real systems mathematically modelled by dynamical models face the problem of delay in some of the quantities modelled. They are the following vector equations

$$x'(t) = f(x(t), x(t - \Delta_1), x(t - \Delta_2), \dots, x(t - \Delta_m)), \quad (1)$$

where the vector is represented by both the sought-for function  $x = x(t)$  and its derivation and by the right-hand side of the equation  $f$ . Delay  $\Delta_j, j = 1, 2, \dots, m$  can be either constant, in which case we discuss differential equations with constant delay, or the delays may be dependent on time  $t$ , i.e.  $\Delta_j = \Delta_j(t), j = 1, 2, \dots, m$  or on previous values of variables  $x_i, i = 1, 2, \dots, n$ , i.e.  $\Delta_j = \Delta_j(x_1, x_2, \dots, x_n), j = 1, 2, \dots, m$ , in which case we discuss differential equations with delay dependent on state variables, or even both on time and on state variables.

Delays of some quantities point to different effects of the problem modelled, such as transmissive, transport or inertial effects. When designing a model, the use of differential equations with delay variables enables formulation of "the memory impact" of particular quantities and their mutual relations.

Nowadays, there are already methods in place for solutions of dynamical models defined by differential equations with delay argument, which can be successfully applied.

The middle of the 20<sup>th</sup> century saw a boom not only in electro-technics, mechanics and chemical engineering, but also in biology, ecology, medicine, mathematical economics and other scientific and technological fields (relevant examples can be found in (Andreeva, Kolmanovskij, Sajchet, 1992) and (Titov & Uspenskij, 1969). All this led to the need of further development of the FDE theory.

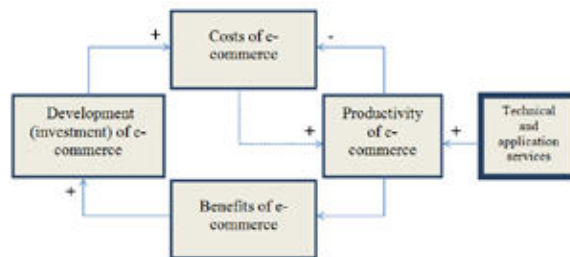
An extensive and comprehensive analysis of equations with delay argument was carried out by the participants of Perm seminar (Azbelev, Maximov, Rachmatullina, 1991) and (Azbelev, 2001); the findings were later systemized in monographs (Azbelev, 2003) and (Azbelev, Maximov, Rachmatullina, 2007).

In their monograph, Kobrinskij and Kuzmin (1981) pointed out the need of applying variable character of history in dynamical economic models, which impacts on systems' development and leads to fundamental changes in the whole process. In his publications, Simonov (2002), (2002b) and (2003) adjusted well-known micro- and macroeconomic models, e.g. Walras-Evans-Samuelson's model (WEC) with regard to the delay between supply and demand, Allen's model on one-commodity market, taking into account the delay in supplies and the dependence of supply and demand on price and the rate of price changes (Allen, 1971), Vidal-Wolf's model of one-product sale (Dychta, Camsonjuk, 2003), etc. Some authors are now rethinking Kalecky's model using differential equations with delay (e.g. (Asea, Zak, 1999), (Burn, Ash, 2005)).

**Analysis of an issue  
Model of a company's e-commerce**

As has already been said, the paper aims to design a model for monitoring e-commerce as a dynamical system developing in time. The model shall be designed from the perspective of an e-commerce keeper and the generalization the solution shall utilize should correspond to its strategic and conceptual management. Thus, it will not focus on the end user although

his interests enter the model within some input metrics.



**Fig. 1: Scheme of the model (Source: Authors)**

**E-commerce model using differential equations with delay**

In this chapter, we will attempt to design a model taking into account the impact of data from previous periods and we will create a dynamical model defined by a set of two differential equations with delay arguments.

Unlike linear ordinary differential equations with constant coefficients and their systems, direct methods of calculation cannot be used for differential equations and systems with delay. It is necessary to use numerical methods and, in this case, also so-called method of steps, which is one of basic methods for solving FDE with delay. The principle lies in dividing the interval for which the solution is being sought into subintervals, whose length is defined by the delay. We look for parts of solution on these subintervals, and the parts then make up the whole solution for the original interval.

Calculations were performed by Maple system. Maple, as mathematical software, is primarily used for symbolic solutions to calculations. It is similar to programs Mathematica and Maxima, which, however, provide much fewer functions. Maple can perform both analytical calculations with formulae and numerical calculation or graphic display of results. Maple has a pleasant user environment and, at the same time, offers a wide scale of options of quantitative methods for practical use, application problems, and scientific methods for many fields, etc.

Change in costs of e-commerce ( $N$ ) is influenced by performance of the e-commerce ( $V$ ) and its development ( $I$ ):

$$\frac{dN}{dt} = f(I, V) \quad (2)$$

Change in costs decreases with rising performance  $V$  and rises with growing development  $I$ . The precise functional dependence is not known, and, as there is no suggestion that there is a non-linear relation between these variables, we can opt for linear dependence, i.e. direct proportion. However, we can suppose that the change will be influenced by behaviour of  $I$  in the past, hence the relation

$$f(I, V) = \alpha I(t) - \beta V(t - \Delta) \quad (3)$$

$\alpha, \beta$  are positive constants representing coefficients of particular areas which are:

$\alpha$  – coefficient of development and  $\beta$  – coefficient of e-commerce's performance, is (constant) length of time delay,  $t \in [0, T]$ ,  $T$  is length of the period examined.

This leads to the first equation of the model:

$$\frac{dN}{dt} = \alpha I(t) - \beta V(t - \Delta) \quad (4)$$

Initial condition is set as  $N(0) = N_0$ ; value  $N_0 > 0$  (initial costs cannot be negative).

Further

$$\frac{dV}{dt} = u + \frac{P(t) - N(t)}{P(t - \Delta) - N(t - \Delta)} \quad (5)$$

while realization parameter  $u$  (quality in use and service quality) shall not be negative – thus  $u > 0$  and, at the same time, it should reflect the performance, taking into account that the higher its value is, the higher performance of the whole e-commerce is.

Change in benefits ( $P$ ) is influenced by performance  $V$  and rises with a rise in performance and an increase  $I$  in period  $(t-\Delta) - t$ . Again, there is no suggestion of a non-linear relation; we can opt for a linear dependence:

$$\frac{dP}{dt} = \gamma V(t) - \nu h(I(t) - I(t - \Delta)) \tag{6}$$

$\gamma$  represents coefficient of proportionality of dependence between a change in investment  $I$  and performance  $V$ ,  $\nu h$  the power of the impact of the past.

Benefits  $P$  influence development  $I$ , while a rise in benefits means a rise in e-commerce

$$I = \delta P, \tag{7}$$

where  $\delta$  is coefficient of proportionality.

The whole model consists of two state variables  $N$ ,  $P$  and corresponding three linear differential equations with delay:

$$\frac{dN}{dt} = \alpha \delta P(t) - \beta V(t - \Delta) \tag{8}$$

$$\frac{dP}{dt} = \gamma V(t) - \nu h(\delta P(t) - \delta P(t - \Delta))$$

$$\frac{dV}{dt} = u * \frac{P(t) - N(t)}{P(t - \Delta) - N(t - \Delta)}$$

Initial conditions for these equations are:

$$N(0) = N_0$$

$$P(0) = P_0 \tag{9}$$

$$V(0) = V_0$$

Coefficients  $\alpha$ ,  $\beta$ ,  $\gamma$  and  $\delta$  are positive.

**Discussion**

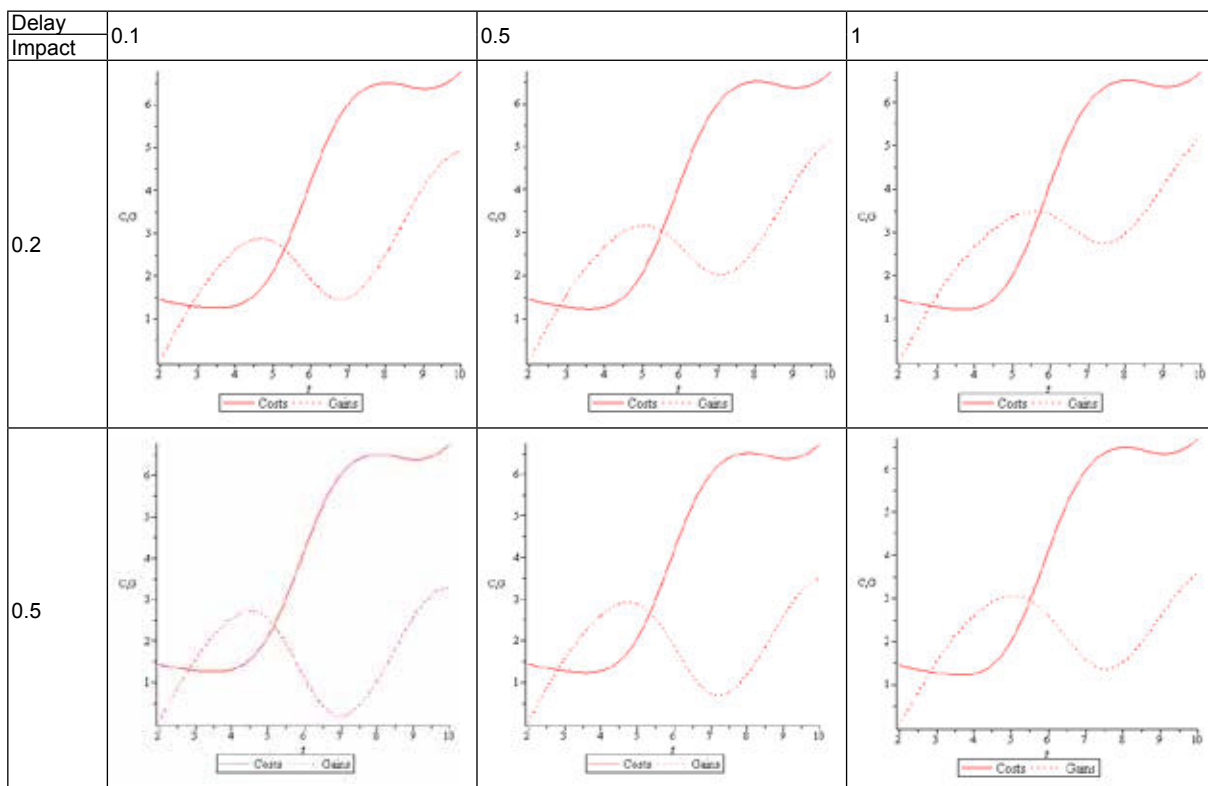
Simulation of model system's behaviour under various conditions

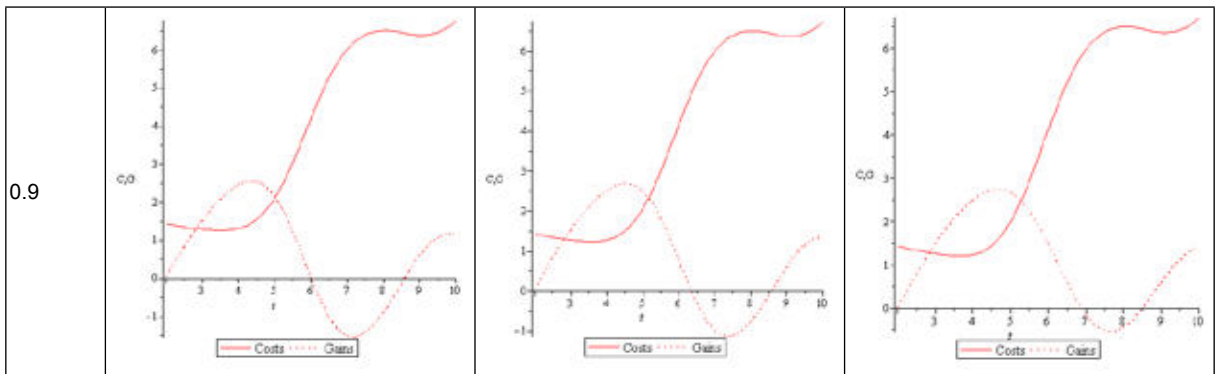
It must be emphasized that just like in case of differential equations without delay, in this case as well, the solution of the initial problem is defined unambiguously and, given the shape of the system and its attributes, the solution is continuously dependant on continuous changes in systems' coefficients, initial problem's parameters and potential non-homogeneity of equations' right-hand sides, as well as on changes in delay value and functions describing the "the past" of the functions examined in time preceding the analyzed interval of model's behaviour. All this enables changing all input data in the model and monitoring the impact of changes on final system's solutions – on the monitored model's characteristics.

Let's suppose that a model is designed for a start-up, a rapidly growing electronics shop with a relatively low performance. Given periodically repeated investment in equipment, in order to simulate costs of this company in the past the authors chose so-called historic function  $N(t)=0,1\cos(1,1t)+1,5$  and for benefits in the past function  $I(t)=|\cos(0,5 \sin(1,1t)+0,7t+2)+1-0,1t|$ . Function selected for the past performance is periodical  $V(t)=0,3\sin(t+0,1)+1$ . Time unit was 1 year.

The input parameters of the system shall be values  $\alpha=1.5$   $\beta=0,2$   $\gamma=1.4$ ,  $\delta=1.2$ ,  $u=1.3$ . Further, the authors shall assume a varying influence of the past for the model, therefore we consecutively choose parameter  $\nu h$  (0.2/ 0.5/0.9), and length of delay  $\Delta$  is chosen consecutively 1, 0.5 a 0.1.

Graphical solution of the system corresponding to simulation of combined values of parameters  $\nu h$  and  $\Delta$ , was achieved by following the above described method of steps.



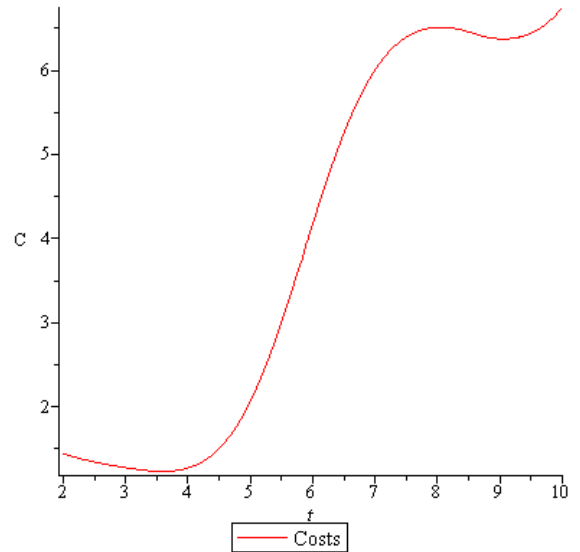


**Fig. 2: Graphical solution of the system simulation**

Fig. 2 shows that the impact of delay or the intensity of the delay's impact is reflected primarily in the amount of benefits, while costs do not seem to be influenced. The reason is that the amount of costs is captured as a real number in given time. Thus, a possible change is shown as immediate in the model and the delay is negligible. The graph also shows a usual period of system's stabilization with no need for higher service costs, except for periodical influences. However, costs can never fall to zero.

The curve of benefits supports the assumption that an average lifespan of an information system is five to eight years, necessarily followed by a substantial investment. The stronger the impact of the past is, the shorter the lifecycle period is and, at the same time, the lower the maximum achievable benefit is. If the delay is longer, the amplitude for benefit curve is lower. It is apparent that longer delay stabilizes benefits of the model for a company.

If we try to graphically analyze the impact of change in delay on costs, we can see from Fig 3 that the impact is probably small. One graph contains costs curves for delay  $\Delta=0.5$  and for changing values of parameter  $vh$  (0.2, 0.5 and 0.9 successively). Similarly, Fig 4 shows changes in costs function, this time for parameter  $vh=0.5$  and changing length of delay  $\Delta$  (1, 0.5 a 0.1 successively). It follows unambiguously from Fig 4 that the intensity of the delay impact on costs is totally unimportant in this case.



**Fig. 4: Costs 2**

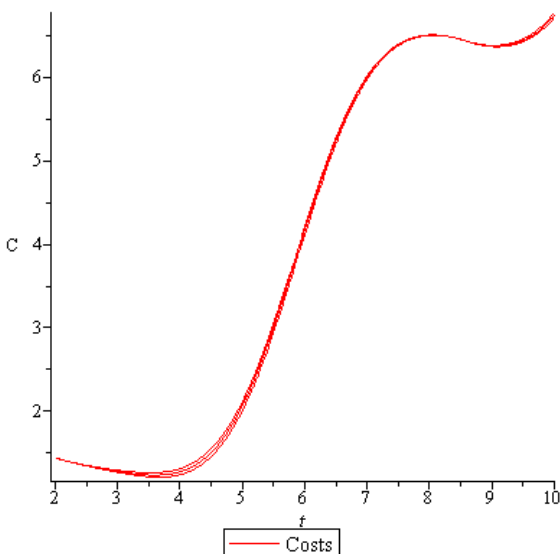
**Conclusion**

Monitoring costs and benefits of e-commerce is a fundamental prerequisite for efficient corporate management. Costs and benefits develop in time, undergo changes and represent a significant factor for the company's budget.

The new dynamical model displays behaviour of a company's e-commerce as a set of two linear differential equations with delay argument. Its analysis required the use of modern solution methods of differential equations with delay arguments. A major advantage of the model is its relative simplicity (besides linearity, it is constant coefficients and delays) and the fact that it only deals with several selected components of economic system. It is a basic model which can be made more accurate for a specific subject if need be. Such approach can then be used not only for processing input factors of the system, but also for interpretation of results enhancing strategic management.

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**Fig. 3: Costs**

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