Research Paper Mathematics

Unsteady MHD Flow of a Casson Fluid in a Parallel Plate Channel With Heat And Mass Transfer of Chemical Reaction

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ABSTRACT

This paper presents the study of unsteady magneto hydrodynamic flow of Casson fluid accompanied by heat transfer through parallel plate in the presence of chemical reaction. Casson fluid model is used to characterize the non-Newtonian fluid behavior. The present fluid model is working to suggest rheological liquids encountered in biotechnology (medical creams) and chemical engineering. This rheological model introduces additional terms into the momentum equation. The basic equations governing the flow, heat transfer, and concentration are reduced to a set of ordinary differential equations by using appropriate transformation for variables. It is recognized that the Schmit number (Sc) and Chemical reaction parameter (Cr) have similar effects on the velocity profile.

KEYWORDS Heat Source, Magnetic Field, Casson fluid, Mass transfer and Chemical reaction.

INTRODUCTION

The analysis of boundary layer flow of viscous and non-Newtonian fluids has been the focus of extensive research by various scientists due to its importance in continuous casting, glass blowing, paper production, polymer extrusion, aerodynamic extrusion of plastic sheet and several others. Crane [1] initiated a study on the boundary layer flow of a viscous fluid towards a linear stretching sheet. Afterwards, many investigations were made to examine flow over a stretching/shrinking sheet under different aspects of MHD, suction/injection, heat and mass transfer etc. [2–9]. In these attempts, the boundary layer flow, due to stretching/shrinking, has been analyzed. Combined heat and mass transfer problems with chemical reaction are of importance in many processes and have, therefore, received a considerable amount of attention in recent years. Examples of this phenomenon are evaporation of water, separation of chemicals in distillation processes, natural or artificial sources etc. In addition, mass transfer with chemical reaction has special significance in chemical and hydrometallurgical industries. Alam et.al., [10] studied the problem of free convection heat and mass transfer flow past an inclined semi-infinite heated surface of an electrically conducting and steady viscous incompressible fluid in the presence of a magnetic field and heat generation. Chamkha [11] investigated unsteady convective heat and mass transfer past a semi-infinite porous moving plate with heat absorption. Hady et.al., [12] studied the problem of free convection flow along a vertical wavy surface embedded in electrically conducting fluid saturated porous media in the presence of internal heat generation or absorption effect. The study of heat and mass transfer with chemical reaction is of great practical importance to engineers and scientists because of its almost universal occurrence in many branches of science and engineering. Muthucumaraswamy and Ganesan [13] analyzed the effect of a chemical reaction on the unsteady flow past an impulsively started vertical plate which is subjected to uniform mass flux and in the presence of heat transfer. Seddeek et.al., [14] analyzed the effects of chemical reaction, radiation and variable viscosity of hydromagnetic mixed convection heat and mass transfer for Hiemenz flow through porous media. Ibrahim et.al., [15] analyzed the effects of the chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi-infinite vertical permeable moving plate with heat source and suction.

Previous studies on the topic show that little work is presented regarding the effect of mass transfer on the MHD flows of non-Newtonian fluids in the presence of chemical reaction. Constitutive equations of the Casson fluid model [16-20] are employed in the mathematical modeling. An exact solution for the dimensionless governing equations has been obtained. The effects of velocity, temperature and concentration are studied for different parameters like magnetic field parameter, Prandtl number, Schmidt number, chemical reaction parameter and heat source parameter.

Mathematical Formulation of the problem

Consider laminar boundary layer two-dimensional flow and heat transfer of an incompressible, conducting non-Newtonian Casson fluid over an unsteady stretching sheet. The unsteady fluid and heat flows start at *t* = 0. The mass transfer phenomenon with chemical reaction is also retained. The flow is subjected to a constant applied magnetic field in the *y* direction.

The rheological equation of state for an isotropic and incompressible flow of a Casson fluid as follows:

$$\tau_{ij} = \begin{cases} \left(\mu_B + \dfrac{p_y}{\sqrt{2\pi}}\right) 2e_j, & \pi > \pi_c, \\ \left(\mu_B + \dfrac{p_y}{\sqrt{2\pi_c}}\right) 2e_j & \pi < \pi_c \end{cases}$$

where t_{ij} is the (i, j)-th component of the stress tensor, p = e_{ij} e_{ij} and e_{ij} are the (i, j)-th component of the deformation rate, p is the product of the component of deformation rate with itself, p_c is a critical value of this product based on the non-Newtonian model, m_B is plastic dynamic viscosity of the non-Newtonian fluid , and p_y is the yield stress of the fluid.

Considering u and v as velocity components in the directions of x and y respectively (axial and normal respectively) at time t in the flow field, we may write the two dimensional boundary layer equations in presence of transverse magnetic field as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$\frac{\partial u}{\partial t} + \frac{1}{\rho}\frac{\partial p}{\partial x} = \nu\left(1 + \frac{1}{\gamma}\right)\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 u}{\rho} + g\beta(T - T_0) + g\beta^*(C - C_0) \qquad (2)$$

$$\frac{\partial T}{\partial t} = \frac{K'}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho C_p}(T - T_0)$$

(3)

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial y^2} - K_r(C - C_0) \quad (4)$$

where ν is the kinematic fluid viscosity, ρ is the fluid density, γ = μβ√2πc/Pyis the Casson parameter, σ is the electrical conductivity of the fluid, and H_0 is the strength of magnetic field applied in the y direction. r and m is the density and viscosity of the blood while p_* stands for pressure. K′ is thermal conductivity; C_p is the specific heat at constant pressure. Q is the quantity of heat, T is the temperature, β is the volumetric coefficient of thermal expansion, β* is the volumetric concentration coefficient, D is the mass diffusion, C the concentration field and K_r the chemical reaction parameter.

The boundary conditions for velocity, temperature and species concentration are taken as:

$$u = e^{-\lambda^2 t},\ \theta = e^{-\lambda^2 t},\ C = e^{-\lambda^2 t} \text{ at } y = -1$$

$$u = 0,\ \theta = 0,\ C = 0 \text{ at } y = 1$$

Let us introduce the non-dimensional variables,

$$x^* = \frac{x}{h},\ y^* = \frac{y}{h},\ u^* = \frac{u}{m/2\rho h},$$

$$v^* = \frac{v}{m/2\rho h}\ \ t^* = \frac{t}{\rho h^2/\mu},$$

$$p^*(x,t) = \frac{dp/dx}{\mu m/2\rho^2 h^3},\ \theta^* = \frac{\theta}{\mu m/2\rho^2 h^3},$$

$$C^* = \frac{C}{\mu m/2\rho^2 h^3} \quad (5)$$

Equations (1)-(4) can be made dimensionless by using (5), we get

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (6)$$

$$\frac{\partial u}{\partial t} + p = \left(1 + \frac{1}{\gamma}\right)\frac{\partial^2 u}{\partial y^2} - Ha^2 u + g\beta\theta + g\beta^* C \quad (7)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\upsilon P_r}\frac{\partial^2 \theta}{\partial y^2} + \frac{S}{\upsilon P_r}\theta \quad (8)$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} + C_r C \quad (9)$$

where the heat source parameter, $S = \frac{Qh^2}{K_T}$,

Prandtl number, $P_r = \frac{\mu C_p}{K_T}$,

dimensionless chemical reaction parameter, $C_r = \frac{K_r h^2}{\upsilon}$,

Schmidt number, $S_c = \frac{D}{\nu}$.

Analytical Solution of the Problem

With the above discussions in the previous section, let us choose the solutions of the equations (5) – (7) respectively as

$$u(y,t) = F(y)e^{-\lambda^2 t} \quad (10)$$

$$v(y,t) = G(y)e^{-\lambda^2 t} \quad (11)$$

$$\theta(y,t) = H(y)e^{-\lambda^2 t} \quad (12)$$

$$C(y,t) = I(y)e^{-\lambda^2 t} \quad (13)$$

along with the boundary conditions

$$F = 1,\quad H = 1,\quad I = 1, \text{ at } y = -1$$

$$F = 0\quad H = 0,\quad I = 0, \text{ at } y = 1 \quad (14)$$

By virtue of (10) – (13), we obtain the equations (6) – (9) respectively as

$$F''(y) + \frac{\gamma}{1+\gamma}\left(\lambda^2 - Ha^2\right)F(y) \quad (15)$$

$$= \frac{\gamma}{1+\gamma}\left(p - g\beta H(y) - g\beta^* I(y)\right)$$

$$G = C \quad (16)$$

$$H''(y) + \left(S + \lambda^2 P_r \upsilon\right)H(y) = 0 \quad (17)$$

$$I''(y) + S_c\left(\lambda^2 - C_r\right)I(y) = 0 \quad (18)$$

Solution of equation (17) and (18) using the boundary condition (14) is as follows

$$H(y) = \frac{1}{2\cos\delta}\cos(\delta y) - \frac{1}{2\sin\delta}\sin(\delta y) \quad (19)$$

$$I(y) = \frac{1}{2\cos\xi}\cos(\xi y) - \frac{1}{2\sin\xi}\sin(\xi y) \quad (20)$$

where $\delta = \sqrt{S + \lambda^2 P_r \upsilon}$, $\xi = \sqrt{S_c\left(\lambda^2 - C_r\right)}$

From (12), (13), (19) and (20) the temperature distribution is given by

$$\theta(y,t) = \left(\frac{1}{2\cos\delta}\cos(\delta y) - \frac{1}{2\sin\delta}\sin(\delta y)\right)e^{-\lambda^2 t} \quad (21)$$

$$C(y,t) = \left(\frac{1}{2\cos\xi}\cos(\xi y) - \frac{1}{2\sin\xi}\sin(\xi y)\right)e^{-\lambda^2 t} \quad (22)$$

Using the equations (19) and (20) into equation (15) we get

$$F''(y) + \frac{\gamma}{1+\gamma}\left(\lambda^2 - Ha^2\right)F(y)$$

$$= \frac{\gamma}{1+\gamma}\left(\begin{array}{l} p_1 - g\beta\left(\frac{1}{2\cos\delta}\cos(\delta y) - \frac{1}{2\sin\delta}\sin(\delta y)\right) \\ - g\beta^*\left(\frac{1}{2\cos\xi}\cos(\xi y) - \frac{1}{2\sin\xi}\sin(\xi y)\right)\end{array}\right) \quad (23)$$

From equation (10) and (23) the velocity of the flow of the fluid parallel to the direction of the channel is obtained as,

$$u(y,t) = \left(\begin{array}{l} c(c_1 + c_2\cos\delta y + c_3\sin\delta y + c_4\cos\xi y + c_5\sin\xi y) \\ + c_6\cos\alpha y + c_7\sin\alpha y \end{array}\right)e^{-\lambda^2 t} \quad (24)$$

where $p_1 = \frac{p^*}{e^{-\lambda^2 t}}$, $c = \frac{\gamma}{1+\gamma}$, $\alpha = \sqrt{c\left(\lambda^2 - Ha^2\right)}$,

$$c_1 = \frac{p_1}{\alpha^2},\ c_2 = -\frac{g\beta}{2\left(\alpha^2 - \delta^2\right)\cos\delta},$$

$$c_3 = \frac{g\beta}{2\left(\alpha^2 - \delta^2\right)\sin\delta}$$

$$c_4 = -\frac{g\beta^*}{2\left(\alpha^2 - \xi^2\right)\cos\xi},\ c_5 = \frac{g\beta^*}{2\left(\alpha^2 - \xi^2\right)\sin\xi},$$

$$c_6 = \frac{1 - 2c(c_1 - (c_2\cos\delta + c_4\cos\xi))}{2\cos\alpha},$$

$$c_7 = -\frac{1 + 2(c_2\sin\delta + c_5\sin\xi)}{2\sin\alpha}$$

From equations (11) and (16), the velocity of the fluid flow perpendicular to the direction of the channel is given by

$$v(y,t) = Ce^{-\lambda^2 t} \quad (25)$$

where *C* is an arbitrary constant.

Equations (21), (22), (24) and (25) show the temperature distribution, the axial velocity and normal velocity respectively.

Results and Discussions

In this section, we discuss the different physical parameters, such as heat source parameter (S), Hartmann number (M), Prandtl number (P_r), Schmidt number (S_c), Chemical reaction parameter (C_r) and decay parameter (l) on temperature distribution, axial velocity and normal velocity. The obtained computational results are presented graphically and the variations in velocity and temperature are discussed.

Effects of different physical parameters on temperature profile

Figure 1 shows the performance of temperature distributions versus y at l = 0.5, P_r = 1, u = 0.5, and t = 1 for different values of heat source parameter (S = 1, 1.75, 2.5, 3.25, 4). We observe that the temperature field decreases with increasing the values of S, for y £ 0.5, and temperature field increases for y ³ 0.5. The maximum effect of heat source is at y = -1.

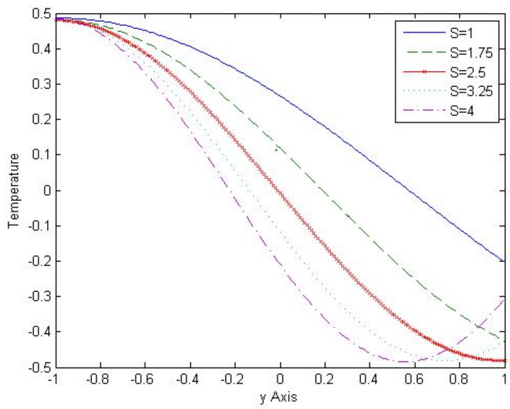

Figure 2 emphasizes that the temperature field distribution for different values of Prandtl number (P_r = 1, 3, 5, 7, 9) at S = 1, l = 0.5, u = 0.2, and t = 1. The effect of Prandtl number on temperature steadily decreases with increasing the values of Prandtl number.

Figure 1. Temperature profile for different values of values of Heat Source Parameter (*S*)

It is clear from Figure 3 that temperature field distribution decreases with increasing the decay parameter up to y £ 0.16 and it increases with increasing the decay parameter for y ³ 0.16, at S = 1, P_r = 1, u = 0.2, and t = 1 for different values of decay parameter (l = 0.5, 0.75, 1, 1.25, 1.5). The maximum effect of decay parameter on the temperature field is between -0.8 £ y £ -0.4.

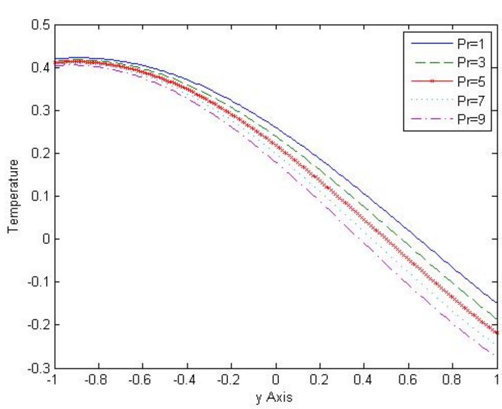

Figure 2. Temperature profile for different values of values of Prandtl Numer (P_r)

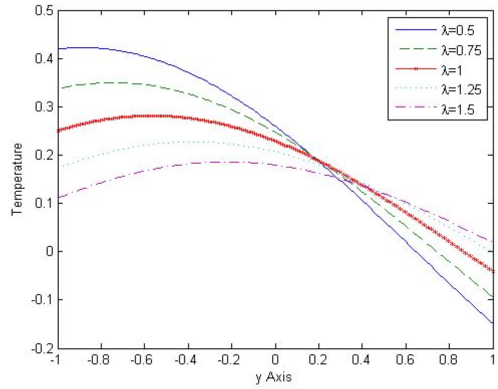

Figure 3. Temperature profile for different values of values of Prandtl Numer (Pr)

- **Effects of different physical parameters on concentration profile**

Figures 4 – 6 shows the concentration profiles obtained for various values of Sc, Cr and l. As shown

Concentration profile decreases as Sc (Schimdt number) and l (Decay parameter) increases, but concentration profile increases as Cr (Dimensionles chemical reaction parameter) increases.

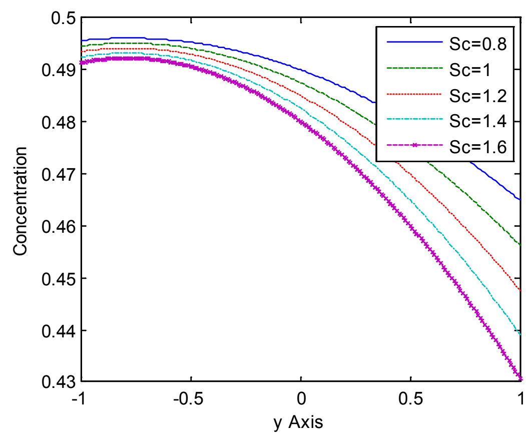

Figure 4. Concentration profile for different values of values of Schmidt Numer (Sc)

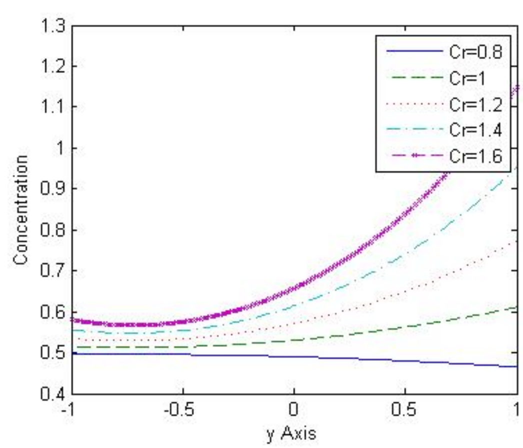

Figure 5. Concentration profile for different values of values of Chemical reation parameter (Cr)

- **Effects of different physical parameters on velocity fields**

Figure 7 indicates the effect of the axial velocity for different values of Chemical reaction parameter (C_r = 1, 2, 3, 4, 5) at

S_c = 0.8, S = 1, l = 0.25, P_r = 1, u = 0.2, t = 1, g = 1, g = 9.8, b=0.5, b* = 0.5, p = 0.5, M = 2. It is observed that the axial velocity increases with increasing the chemical reaction parameter up to y £ -0.2 and for y ³ -0.2 axial velocity decreases with increasing the chemical reaction parameter.

Figure 8 shows the axial velocity profiles for several values of Schimdt number (S_c = 0.2, 0.4, 0.6, 0.8, 1) at S_c = 0.2, C_r = 3, S = 1, l = 0.25, P_r = 1, u = 0.2, t = 1, g = 1, g = 9.8, b = 0.5, b* = 0.5, p = 0.5, M = 2. It is noticed that the axial velocity increases with increasing the Schimdt number (S_c) up to y £ -0.2 and for y ³ -0.2 axial velocity decreases with increasing the Schimdt number (S_c). Effect of Schimdt number and chemical reaction parameter over the axial velocity profile are same.

Figure 9 indicates the effect of magnetic field on the axial velocity for different values of Hartmann number (M = 1, 1.25, 1.5, 1.75, 2) at S_c = 0.8, C_r = 0.2, S = 1, l = 0.5, P_r = 1, u = 0.2, t = 1, g = 1, g = 9.8, b = 0.5, b* = 0.5, p = 0.5, M = 1. It is clear that the axial velocity decreases with increasing the magnetic field up to y £ -0.2 and for y ³ -0.2 axial velocity increases with increasing the magnetic field.

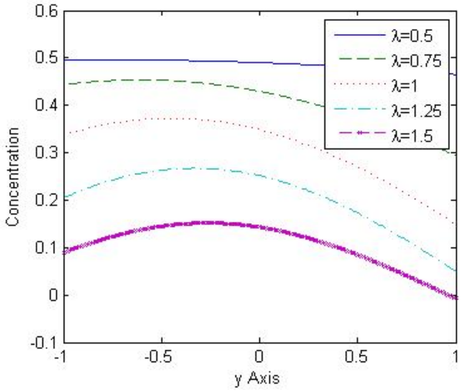

Figure 6. Concentration profile for different values of values of Decay parameter (l)

Effects of Casson parameter g on velocity profiles for unsteady motion are clearly exhibited in Figure 10 the behavior of velocity with increasing g is noted at S_c = 0.8, C_r = 0.2, S = 1, l = 0.5, P_r = 1, u = 0.2, t = 1, g = 1, g = 9.8, b = 0.5, b* = 0.5, p = 0.5, M = 2. It is observed that the axial velocity decreases with increasing the magnetic field up to y £ -0.2 and for y ³ -0.2 axial velocity increases with increasing the magnetic field.

Figure 11 defines the effect of decay parameter on the axial velocity for different values of decay parameter (l = 0.5, 0.75, 1, 1.25, 1.5) at S_c = 0.8, C_r = 0.8, S = 0.5, l = 0.25, P_r = 1, u = 0.2, t = 1, g = 1, g = 9.8, b = 0.5, b* = 0.5, p = 0.5, M = 2. It is illustrated that the axial velocity increases with increasing the decay parameter.

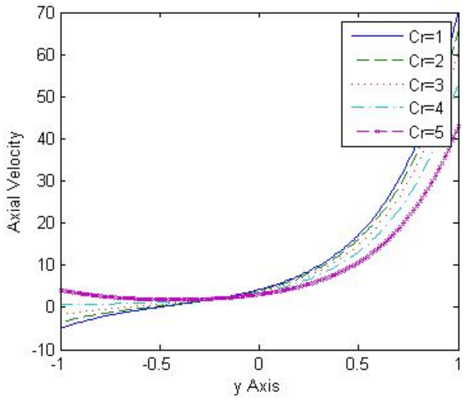

Figure 7. Axial velocity for different values of values of Chemical reaction parameter(*Cr*)

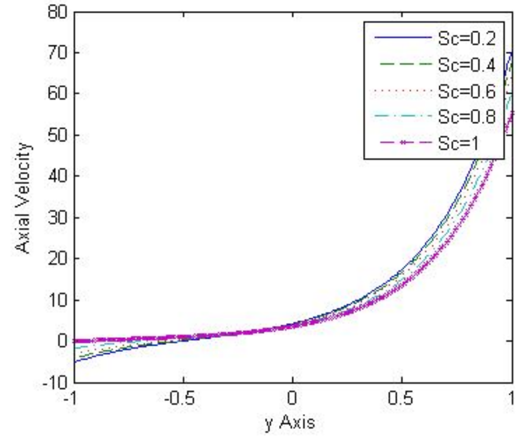

Figure 8. Axial velocity profile for different values of values of Schmidt Numer (Sc)

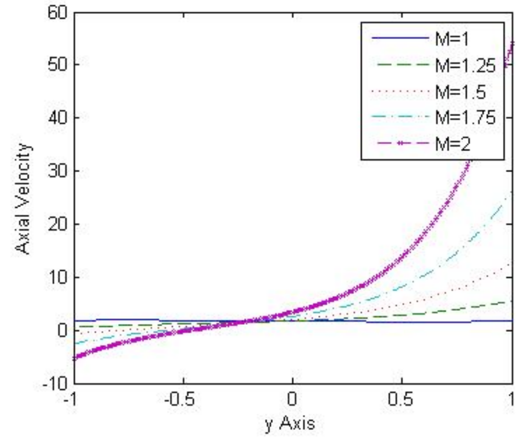

Figure 9. Axial velocity for different values of values of Magnetic Field Parameter (*M*)

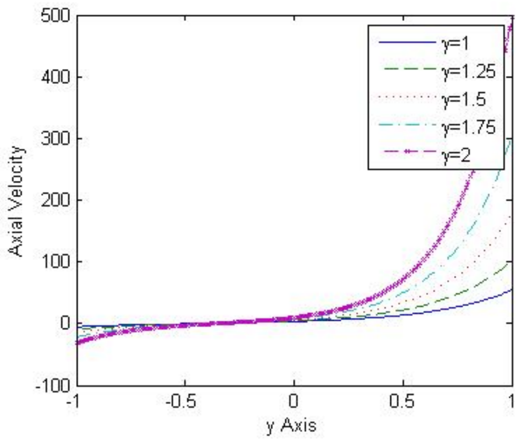

Figure 10. Axial velocity for different values of Casson Parameter (g).

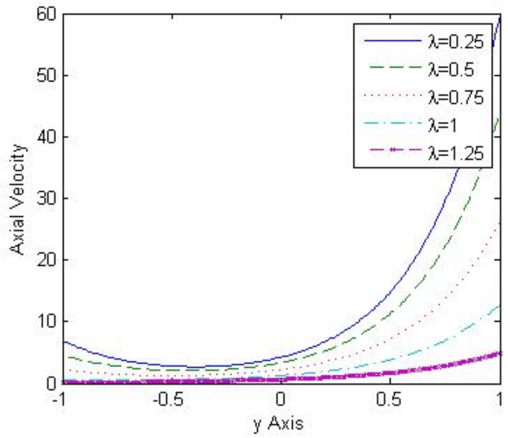

Figure 11. Axial velocity for different values of Decay Parameter (l).

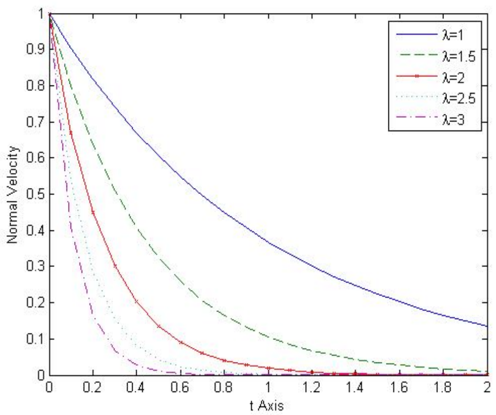

Figure 12. Normal velocity for different values of Decay Parameter (l).

Normal velocity for different values of decay parameter is shown in Figure 12. It is depicted that normal velocity is decreasing with increasing values of l. The normal velocity is decreasing slowly at low values of the decay parameter (l = 1) while it decreases very fast and tends to zero at high values of decay parameter (l = 3).

Conclusion

The present study has been carried out for heat and mass transfer of MHD flow over a Casson fluid in presence of heat source and chemical reaction.

The analytical solutions are obtained for temperature, concentration and velocity profiles. The following interpretations are evident from the graphs,

- **Concentration of the fluid increases when Schmit number (Sc) increases.**
- The Casson parameter (g) and Hartman number (M) have similar effects on the velocity profile.
- The Schmit number (S_c) and Chemical reaction parameter (C_r) have similar effects on the velocity profile.
- The concentration profile increase upon increasing the Chemical reaction parameter (C_r).
- An increase in the Heat source (S) and Prandtl number (P_r) causes the decrease in temperature.

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