Research Paper Statistics

Construction of Three Associate Class PBIB Designs with Three Replicates Using Pairing in Triplets System

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ABSTRACT

A Series of three associate class Partially Balanced Incomplete Block (PBIB) designs with three replicates is introduced in this paper. The method of construction of new designs is based on method of pairing in symbols of triplets system. In the construction of three associate class PBIB designs, we take nine blocks with three replicates. We then paired the iterative values of triplets with its initial values under different conditions to get blocks of new designs. Association scheme of new series of designs is also based on occurrence of treatment pairs. Efficiency factors of three kinds of comparisons E1, E2, E3 and overall efficiency factor E along with illustration have also been given in the paper.

KEYWORDS Partially balanced incomplete block designs, triplets, pairing

1.Introduction

In the essence of construction of higher associate class PBIB designs, various authors such as Vartak (1955), Raghavarao and Chandrasekhararao (1964) introduced three associate class association schemes called rectangular association scheme and cubic association scheme respectively. Nair (1951), Tharthare (1965) introduced four associate class PBIB designs called rectangular lattice designs and generalized right angular designs respectively. Similarly, Garg.et.al (2011), Sinha.et.al (2002) also contributes in the construction of higher associate class PBIB designs along with their association schemes. Recently, Garg and Gurinder (2013) constructed a four associate class PBIB designs with two replicates by using pairing in triplets system.

Here, we constructed a new class of three associate PBIB designs with three replicates again by using pairing in triplets system. The construction method of blocks are based on pairing the initial values of triplet (α, β, γ) with their iterative values of triplets under some particular combinations in nine steps along an association scheme based on occurrence of treatment pairs.

2. Series of three associate class PBIB designs

In this series, we introduce a class of three associate PBIB designs having parameters v=6s , b=9, r=3, k= 3s, $\lambda_1 = 3$, $\lambda_2 = 1$, $\lambda_3 = 0$, $n_1 = s-1$, $n_2 = 3s$, $n_3 = 2s$ by using pairing between initial and iterative values of triplets again. The construction method of triplets, blocks, association scheme, P- matrices along with a numerical has been discussed in the following sections.

2.1 Construction of triplets

We apply the same method of construction of triplets that we have already used in the construction of four associate class PBIB designs using pairing in triplets system. For further clarification of this concept, we reproduce the procedure as below.

Let us arrange 6s ($s \geq 2$) symbols represented by (α, β, γ) , where α, β and γ are non- repeated consecutive natural numbers with $0 < \alpha < \beta < \gamma \leq 6s$. Therefore, initial values of (α, β, γ) are 1, 2 and 3 respectively. So, we get in total 2s such triplets. Each triplet either has two odd and one even natural numbers or vice- versa and a triplet leading in even numbers is denoted by E-type (say) and triplet leading in odd numbers is denoted by O-type(say) . So, we categorize 2s triplets as.

One initial triplet (1, 2, 3), 's' triplets belongs to E- type and 's-1' triplets belongs to O- type.

Numbers occurs in these triplets are sub- divided in some categories according to their mathematical properties under binary operator '*' (multiple) defined as:

$\alpha_0 = 1$, $\beta_0 = 2$, $\gamma_0 = 3$ (values of initial triplet)
$\theta_i \in \beta_0' * \gamma_0'$ (a number not multiple of β_0 and γ_0)
$\theta_j \in \beta_0 * \gamma_0$ (a number multiple of β_0 and γ_0)
$\delta_i \in \beta_0 * \gamma_0'$ (a number multiple of β_0 only)
$\delta_j \in \beta_0' * \gamma_0$ (a number multiple of γ_0 only)

2.2. Construction method of blocks of series

To construct the blocks of series of three associate class PBIB designs, we again use method of pairing between initial values with iterative values of triplets in nine different steps. The main point is that we take only one value from each triplet at a time in every step. Particular combinations for construction of blocks for each step are given in tabular form as :

Steps	Initial Value	Triplet Type(E)	Triplet Type(O)	Possible Pairs
		Iterative type	Iterative Type	
1	α_0	θ_i	θ_i	(α_0, θ_i)
2	α_0	θ_i	θ_i	(α_0, θ_i) (α_0, θ_i)
3	α_0	δ_i	θ_i	(α_0, δ_i) (α_0, θ_i)
4	β_0	δ_i	δ_i	(β_0, δ_i) (β_0, δ_i)
5	β_0	θ_i	δ_i	(β_0, θ_i) (γ_0, δ_i)
6	β_0	θ_i	δ_i	(β_0, θ_i) (β_0, δ_i)
7	γ_0	θ_i	δ_i	(γ_0, θ_i) (γ_0, δ_i)
8	γ_0	θ_i	δ_i	(γ_0, θ_i) (γ_0, δ_i)
9	γ_0	δ_i	δ_i	(γ_0, δ_i) (γ_0, δ_i)

The procedure to construct blocks from above table will start to select an initial value from second column and paired this value with a particular type of iterative value of each E- type triplet in third column and O- type triplet from fourth column and collect all possible pairs from all E- type and O- type triplets under the above combinations in fifth column for all nine steps.

In this way, pairs occur in each step are written in a set one by one and omit the repeating of α_0 , β_0 and γ_0 in these sets. Now, we recognize all initial values as well as all iterative values as a natural number and from PBIB designs point of view, every natural number considered as a treatment. Therefore, these nine sets represent blocks of a series of PBIB designs with parameters v=6s , b=9, r=3, k= 3s, λ_1 = 3, λ_2 = 1, λ_3 = 0, n_1 = s-1 , n_2 = 3s , n_3 = 2s following the association scheme (2.3) defined below.

2.3. Association scheme of series

Let us we take a particular treatment say 'θ'. Treatment pairs type (θ θ_i) occurs three times in these blocks, where i =1,2,… ,s-1 are 1st associate of 'θ' and n_1 = s-1.

Treatment pairs type (θ θ_j) occurs only once in these blocks , where j= 1,2,…,3s are 2nd associate of 'θ' and n_2 =3s.

Remaining treatments which does not occurs with 'θ' in any block or in other words treatment pair type (θ θ_k) does not occurs in these blocks. All these φ_k 's where k = 1,2,…,2s are 3rd associates of θ and n_3 =2s.

2.4. P- matrices of the association scheme

$$P_1= \begin{bmatrix} s-2 & 0 & 0 \\ 0 & 3s & 0 \\ 0 & 0 & 2s \end{bmatrix}$$

$$P_2= \begin{bmatrix} 0 & s-1 & 0 \\ s-1 & 0 & 2s \\ 0 & 2s & 0 \end{bmatrix}$$

$$P_3= \begin{bmatrix} 0 & 0 & s-1 \\ 0 & 3s & 0 \\ s-1 & 0 & s \end{bmatrix}$$

2.5. Illustration

Let us take s=3, we have a PBIB design having parameters v=18, b=9, r=3, k=6, λ_1 = 3, λ_2 = 1, λ_3 = 0, n_1 = 2 , n_2 = 9 , n_3 = 6. The construction method of triplets, blocks, association scheme, P- matrices has been discussed in the following sections.

2.5.1. Construction of triplets

For s=3, let us arrange 6s=18 symbols represent by (α, β, γ) and α, β and γ are non- repeated consecutive natural numbers with $0 < \alpha < \beta < \gamma \le 6s$. Therefore , initial triplet is (α_0 =1 , β_0=2, γ_0=3) and we get 2s=6 triplets in total and under the condition of non repeated consecutive and $\alpha < \beta < \gamma$, the remaining triplets with iterative values are (4,5,6) (7,8,9) (10,11,12) (13,14,15) (16,17,18). Clearly each triplet either leading two odd and one even natural numbers or vice- versa. So, we categorize 6 triplets as .

Initial triplet : (1,2,3) ; 's=3' E- type triplets : (4,5,6) (10,11,12) (16,17,18) and

's-1 = 2 ' O- type triplets : (7,8,9) (13, 14 ,15)

Numbers occurs in these triplets are sub- divided in some categories according to their mathematical properties under binary operator '*'(multiple) defined as:

α_0 = 1, β_0 = 2, γ_0 =3 (values of initial triplet)
$\theta_i \in \beta_0^{/} * \gamma_0^{/}$ (θ_i's are 5,7,11,13,17)
$\theta_j \in \beta_0 * \gamma_0$ (θ_j's are 6,12,18)
$\delta_i \in \beta_0 * \gamma_0^{/}$ (δ_i's are 4,8,10,14,16)
$\delta_j \in \beta_0^{/} * \gamma_0$ (δ_i's are 9,15)

2.5.2. Construction method of blocks

Now, we pair initial values with particular types of iterative values for all E- type and O-type triplets to construct nine sets using nine steps one by one which represent nine blocks under various combinations as given in tabular form .

Steps	Initial Value	Triplet Type(E) Iterative type	Triplet Type(O) Iterative Type	Possible Pairs E-Type O-Type
1	α_0=1	θ_i	θ_i	(1,5) (1,7) (1,11) (1,13) (1,17)
2	α_0 =1	θ_i	θ_i	(1,6) (1,12) (1,18) (1,7) (1,13)
3	α_0 =1	δ_i	θ_i	(1,4) (1,10) (1,16) (1,7) (1,13)
4	β_0=2	δ_i	δ_i	(2,4) (2,8) (2,10) (2,14) (2,16)
5	β_0=2	θ_i	δ_i	(2,6) (2,12) (2,18) (2,8) (2,14)
6	β_0=2	θ_i	δ_i	(2,5) (2,11) (2,17) (2,8) (2,14)
7	γ_0=3	θ_i	δ_i	(3,6) (3,12) (3,18) (3,9) (3,15)
8	γ_0=3	θ_i	δ_i	(3,5) (3,11) (3,17) (3,9) (3,15)
9	γ_0=3	δ_i	δ_i	(3,4) (3,10) (3,16) (3,9) (3,15)

Now, we explain the construction methodology of blocks from the above table. Firstly, we select an initial value from 2nd column and we select a particular type of iterative value for all E- type triplets in 3rd column and O- type triplets from 4th column and then we paired initial value with particular iterative types for all E-type and O- type triplets. We take only one value from each triplet at a time and consider each initial value thrice and write all existing pairs in 5th column for all nine steps.

Thereafter, pairs occurs in each step are written in a set for each step one by one and we omit the repeating of initial values 1 , 2 and 3 in these sets, then we get nine sets as

(1,5,7,11,13,17)	(1,6,7,12,13,18)	(1,4,7,10,13,16)
(2,4,8,10,14,16)	(2,6,8,12,14,18)	(2,5,8,11,14,17)
(3,6,9,12,15,18)	(3,5,9,11,15,17)	(3,4,9,10,15,16)

In this way, these sets constitute a PBIB design with parameters v=18 , b=9, r=3 , k= 6 , λ_1 = 3, λ_2 = 1, λ_3 = 0, n_1 = 2 , n_2 = 9 , n_3 = 6 follow an association scheme (2.5.3) defined below.

2.5.3. Association scheme

Let us we take a particular treatment say 'θ=1'. Treatment pairs type (θ,θ_i) which are (1,7) (1,13) occurs three times in these blocks are 1st associate of '1' and n_1 = 2.

Treatment pairs type (θ θ_j) which are (1,4) (1,5) (1,6) (1,10) (1,11) (1,12) (1,16) (1,17) (1,18) occurs only once in these blocks are 2nd associate of '1' and n_2 =9.

Remaining treatments which does not make pair with '1' are 2,3,8,9,14,15 are 3rd associates of '1'and n_3=6.

2.5.4. P- Matrices

$$P_1= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$P_2= \begin{bmatrix} 0 & 2 & 0 \\ 2 & 0 & 6 \\ 0 & 6 & 0 \end{bmatrix}$$

$$P_3= \begin{bmatrix} 0 & 0 & 2 \\ 0 & 9 & 0 \\ 2 & 0 & 3 \end{bmatrix}$$

Table -1 Series of three associate class PBIB designs with r=3, k ≤ 20

s	v	b	r	k	n_1	n_2	n_3	E_1	E_2	E_3	E
2	12	9	3	4	1	6	4	1	.75	.666	.733
3	18	9	3	6	2	9	6	1	.818	.75	.809
4	24	9	3	8	3	12	8	1	.857	.80	.851
5	30	9	3	10	4	15	10	1	.88	.83	.878
6	36	9	3	12	5	18	12	1	.90	.857	.897

7	42	9	3	14	6	21	14	1	.913	.875	.911
8	48	9	3	16	7	24	16	1	.92	.88	.92
9	54	9	3	18	8	27	18	1	.93	.90	.929
10	60	9	3	20	9	30	20	1	.937	.909	.936

Conclusion

In this paper, we used the concept of pairing in initial and iterative values of triplets system for construction of three associate class PBIB design. Here, we have restricted number of replications in the new series of designs. The fixation of replication size to three will be economical as well as time saving in the use of new designs. Also, we have kept fixed number of blocks as a result when number of treatments as well as block size increases rapidly as compare to number of replicates, the efficiencies of designs in the new series increases, which is a significant feature.

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