Research Paper — Mathematics

Rayleigh-Bénard Convection With Temperature Dependent Variable Viscosity

S. Harisingh Naik

Department of Mathematics, University College of Science, Osmania University, Hyderabad-500007, Telangana.

ABSTRACT

The problem of natural convection flow between two horizontal plates with variable viscosity is investigated by linear stability analysis. The transformed governing equations are numerically solved by using a Galerkin method. The results obtained are compared for special cases with earlier papers and are found to be in excellent agreement. In linear stability analysis I consider stationary convection. The results of linear stability analysis are presented for various boundary conditions viz. rigid-rigid, rigid-free, free-rigid and free-free. I have also obtained the vertical velocity eigenfunctions and tempera¬ture distribution. It is observed that the velocity of the fluid particle where it takes the maximum value is near the bottom of the fluid layer where the fluid is less viscous. Also the temperature perturbations confined to the bottom of the fluid layer.

KEYWORDS Variable Viscosity, Exponential Fluid, Galerkin Method.

1. Introduction

Lord Rayleigh [1] was the first one to solve the problem of the onset of thermal convection in a plane horizontal layer of the fluid heated from below. His linear analysis, is generalized by Pellew and Southwell [2]. Rayleigh theory assumes buoyancy induced convective flows, while Pearson [3] has neglected gravity and used surface tension effect to explain the Bénard convection. This is called Marangoni convection. Manyresearchers yet working on Rayleigh-Bénardconvection(RBC) model with constant fluid properties. However, in this chapter we confine to buoyancy induced Rayleigh-Bénard convection with variable viscosity.

For fluids with strongly temperature dependent viscosity such as those occur in many industrial and geophysical context, the infinitesimal amplitude convection assumed by linear theory should not exist. Instead the convection jumps to a finite-amplitude state and the subcritical convection is possible (Busse [4]). Although linear theory is expected to predict critical Rayleighnumber *(R_c)* accurately and few experimental results also exist.

Convection in a fluid with uniform material properties is governed by two non-dimensional numbers, namely, the Rayleigh number, the Prandtl number. The thermal Rayleigh number can be thought of as the non-dimensional temperature drop across the layer. When the $R_T$exceeds a critical value, R_C, the fluid will began to convect. The Prandtl number is the measure of the importance of inertia in the flow.

In large Prandtl number fluid, thermal boundary layers may exist but velocity boundarylayersmay not. Convection in an extremely simple laboratory fluid can be investigated as a function of R_T.This simple approach fails, however, to explain one of the most basic experimental observations, the occurrence of hexagonal cells at low values of R_T. If the theory is made to include the temperature dependence of the material properties the occurrence of hexagonal cells can be explained. For most real fluids, of all the material properties, it is the viscosity which is most strongly temperature dependent.

Thus in addition to the R_T, the functional dependence of the viscosity on temperature is important. Fortunately,anexponential temperature dependence closely approximates the actual dependence in many fluidssuch as glycerol,golden syrup etc. in those cases the second parameter, which is necessary to describe the convention can be taken to the coefficient of the exponential temperature dependence, $c = \log(v_{max}/v_{min})$,where v_{max} and v_{min} are the viscosities at the upper and lower fluid boundaries, respectively. Note that c is also referred as the ratio of the layer depth to the scale depth for viscosity variation along the conductive temperature profile. It is important to establish a suitable choice of kinematic viscosity v.

Booker [5] studied the RBC with strongly temperature dependent viscosity. He found, experimentally, that the convective heat transport in a fluid layer decreases significantly as the ratio of the viscosity *(c),* at the top and bottom boundaries, increases. Booker and Stengel [6] have shown that this decrease in the heat transport can be entirely as a result of an increase in the $R_c$with variable viscosity. Stengel et al. [7] studied the linear stability analysis of RBC with the temperature dependent viscosity, and compared their theoretical results with the experimental results. In their study, they reported that the R_c is nearly constant for lower value of *c,* then increases, reaches a maximum in the vicinity of *c*=8 and decreases for large *c.*

In the studies related to the variable viscosity effect, glycerol and its aqueous solutions have often been used [8], [9]. Yen Ming Chen and Pearlstein [10] considered the RBC with variable viscosity, and in order to determine the error associated with those commonly used approximate viscosity variations, they compared their predicted critical Rayleigh numbers with those obtained by using the simple fits (Arrhenius and exponential approximations). Sekharand Jayalatha [11] also considered the temperature dependent variable viscosity along with two types of liquids-namely, Maxwell-Jeffery and RivilinErickson liquids- for the RBC problem.

They observed that the effect of variable viscosity parameter destabilizes the system. Rajagopal et al. [12] investigated the RBC in which the viscosity is a function of both temperature and pressure. They studied both linear and non-linear stabilities of the problem, and showed that the principle of exchange of instability holds. Vanishree and Siddheshwar [13] considered the non-Boussinisq approximation for an anisotropic rotating porous medium, occupied by the inversely proportional temperature dependent viscosity, to study the onset of stationary convection. AwangKechil et al. [14] investigated the effect of a feedback control strategy on the onset of oscillatory convection, in an infinite horizontal layer of fluid with temperature dependent viscosity. It is shown that the small viscosity variations stabilize the fluid layer. In the case of non-deformable surface, large control gain and large viscosity variation induce the oscillatory instability. They demonstrated that *Pr*plays a key role in destabilizing the liquid layer.

In this paper we study the RBC with variable viscosity physical model with infinite Prandtl number and Boussinesq approximation. In Section 2.2, the mathematical model is formulated.

In Section 2.3,we study the linear stability analysis by using Galerkin technique for various boundary conditions namely, rigid-rigid (R/R) , rigid-free (R/F) , free-rigid (F/R) and free-free (F /F) [16]. Finally in Section 2.4, we have presented the numerical discussion.

2. Mathematical formulation

Convective motion in a horizontal fluid layer heated from below is described by the continuity equation, the momentum equations and the heat equation. We have used thes

boundaries of the layer respectively.

The governing equations in dimensional form are, the equation of continuity

$$\nabla.\vec{V} = 0, \qquad (2.3)$$

The momentum equation

[

$$\rho_0\left(\frac{\overrightarrow{\partial V'}}{\partial t'} + \left(\vec{V'}.\vec{V'}\right)\vec{V'}\right) = -\nabla' P' + K_1\vec{i} + K_2\vec{j} + K_3\vec{k} + \rho g, \qquad (2.4)$$

and the heat equation

$$\frac{\partial T'}{\partial t'} + \left(\vec{V'}.\nabla'\right)T' = \kappa\nabla'^2 T', \qquad (2.5)$$

Where

$$K_1 = 2\mu\frac{\partial^2 u'}{\partial x'^2} + \frac{\partial}{\partial y'}\left[\mu\left(\frac{\partial u'}{\partial y'} + \frac{\partial v'}{\partial x'}\right)\right] + \frac{\partial}{\partial z'}\left[\mu\left(\frac{\partial w'}{\partial x'} + \frac{\partial u'}{\partial z'}\right)\right],$$

$$K_2 = 2\mu\frac{\partial^2 v'}{\partial y'^2} + \frac{\partial}{\partial z'}\left[\mu\left(\frac{\partial v'}{\partial z'} + \frac{\partial w'}{\partial y'}\right)\right] + \frac{\partial}{\partial x'}\left[\mu\left(\frac{\partial u'}{\partial y'} + \frac{\partial v'}{\partial x'}\right)\right],$$

$$K_1 = \frac{\partial}{\partial z'}\left(2\mu\frac{\partial w'}{\partial z'}\right) + \frac{\partial}{\partial x'}\left[\mu\left(\frac{\partial w'}{\partial x'} + \frac{\partial u'}{\partial z'}\right)\right] + \frac{\partial}{\partial y'}\left[\mu\left(\frac{\partial u'}{\partial z'} + \frac{\partial v'}{\partial y'}\right)\right].$$

.

The conduction state is characterized by

$$\vec{V'}_s = \vec{0},$$

$$T = T'_0 - \beta z',$$

$$P'_s = P'_0 + z'g + \alpha\beta g\frac{z'^2}{2}.$$

and the convection state is characterized by

$$\vec{V'} = \vec{V}'_s + \vec{V}'^*, \vec{T}' = \vec{T}'_s + \theta'^*, P' = P'^*. \qquad (2.6)$$

Using the small perturbations given in eq. (2.6), the perturbed governing equations for Rayleigh-Benard convection with variable viscosity along with Boussinesq approximation in dimensional form are given by:

the equation of continuity:

$$\nabla'.\vec{V}' = 0, \qquad (2.7)$$

The momentum equation:

$$\left(\frac{\partial}{\partial t} + (\vec{V}'.\nabla')\right)\vec{V}' = \frac{-\nabla' P'}{\rho_0} + K_1\vec{i} + K_2\vec{j} + K_3\vec{k} + \alpha g\theta'\hat{e}_z, \qquad (2.8)$$

and heat equation:

$$\frac{\partial\theta'}{\partial t'}+\left(\vec{V}'.\nabla'\right)\theta'=\beta\omega'+k\nabla'^2\theta'. \qquad (2.9)$$

The eq.(2.8) describes temporal behavior of perturbation of velocity field$\vec{V}'$.Next equation (2.9) describes the behavior of temperature field perturbationθ'in time.

We have used the Cartesian system of coordinates whose dimensionless horizontal co-ordinates, *x, y* and vertical co-ordinate z are scaled based on, *d*, the depth of the fluid layer. The velocity $\vec{V}'(u,v,\omega)$ the temperature$\vec{\theta}'$, the time t' and the pressure *P* are non-dimensionalized by using the scaling

$$x=\frac{x'}{d},y=\frac{y'}{d},z=\frac{z'}{d},t=\frac{t'}{d^2/_k},u=\frac{u'}{kd^{-1}}$$

$$v=\frac{v'}{kd^{-1}},\omega=\frac{\omega'}{kd^{-1}},\theta=\frac{\theta'}{\Delta T'},p=\frac{p'}{\rho^{k^2}d-2} \qquad (2.10)$$

In a temperature dependent variable viscosity fluid, the three-dimensional flow is unstable near the onset of convection and there exists only a class ofstable two-dimensional roll flows. To study the onset of convection, the y-direction is considered as the axis of roll patterns and the flow is independent of y-direction. The variable viscosity *v*(*z*) is given by$v(z)=v_0f(z)$,where $v_0$is the reference value of the kinematic viscosity evaluated at the mean of the boundary temperatures. Using the reference viscosity we have introduced the dimensionless viscosity ratio as

$f=v/v_0$,which is obtained after using $T'-T'_0=(T'_t-T'_b)z$, where $T'_t and T'_b$ are the temperatures at the top and bottom boundaries, respectively, in eq.(2.1).

Using the scaling given in eq. (2.10) the dimensionless equations for a Rayleigh-Bénard system with variable viscosity in the Boussinisq approximation are given by:

$$\nabla.\vec{V}=0 \qquad (2.11)$$

$$\frac{1}{Pr}\left(\frac{\partial\vec{V}}{\partial t}+\left(\nabla.\vec{V}\right)\vec{V}\right)=-\frac{\nabla P}{Pr}+f\nabla^2\vec{V}+R_T\theta\hat{e}_z$$

$$+\hat{e}_x\left[2f\frac{\partial^2 y}{\partial x^2}+\frac{\partial}{\partial z}\left(f\left(\frac{\partial\omega}{\partial x}+\frac{\partial u}{\partial z}\right)\right)\right]+\hat{e}_y\left[\frac{\partial}{\partial z}\left(f\frac{\partial v}{\partial z}\right)+\frac{\partial}{\partial x}\left(f\frac{\partial v}{\partial x}\right)\right]$$

$$+\hat{e}_z\left[\frac{\partial}{\partial z}\left(2f\frac{\partial\omega}{\partial z}\right)+\frac{\partial}{\partial z}\left(f\left(\frac{\partial\omega}{\partial x}+\frac{\partial u}{\partial z}\right)\right)\right], \qquad (2.12)$$

$$\frac{\partial}{\partial t}+\left(\vec{V}.\nabla\right)\theta=\omega+\nabla^2\theta. \qquad (2.13)$$

The dimensionless parameters in the above equations are Reyleigh number, $R_T(=\alpha g\Delta T\, d^3/kv)$,Prandtl number $Pr(=v/k)$ an d viscosity ratio *c*.We have considered the convection in (*x*,*z*)-plane hence $\partial/_{\partial y}=0$.the *z* –component of curl of momentum equation (2.12) is given by

$$\left(\frac{1}{Pr}\frac{\partial}{\partial t}-f\nabla^2\right)\nabla^2\omega-2\frac{df}{dz}\nabla^2\frac{\partial\omega}{\partial z}+\frac{d^2f}{dz^2}\left(\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial z^2}\right)\omega+R_T\nabla_h^2\theta$$

$$=\hat{e}_z.\left(\frac{1}{Pr}\left(\nabla X\left[\vec{V}.\nabla\right]-\vec{V}X[\vec{\omega}.\nabla]\vec{V}\right)\right) \qquad (2.14)$$

where$\nabla^2=\partial^2/\partial x^2+\partial^2/\partial y^2+\partial^2/\partial z^2,\nabla_h^2=\partial^2/\partial x^2$and *w* is the *z*-component of velocity.

Since Rayleigh-Bénard model is a single diffusive system, to studythe marginal stability of the physical system we have considered the linear parts of the equations (2.13) and (2.14) also by dropping the time dependent, we get

$$\frac{d^2f}{dz^2}\left(\frac{d^2}{dx^2}-\frac{d^2}{dz^2}\right)\omega-2\frac{df}{dz}\nabla^2\frac{\partial\omega}{\partial z}-f\nabla^4\omega+R_T\nabla_h^2\theta=0 \qquad (2.15)$$

$$\nabla^2\theta+\omega=0. \qquad (2.16)$$

We have considered the following non-dimensional viscosity variation:

$$f=\exp\left(c(T_0-T)\right), \qquad (2.17)$$

Where

$$0\le\gamma\le 1, c=log\frac{v_{max}}{v_{min}}=log\left(\frac{1+\gamma}{1-\gamma}\right) \qquad (2.18)$$

And T_0 is the mean of the boundary temperature. The function given in eq. (2.17) is used by Torrence and Turcotte [15] in their numerical study of finite amplitude convection, and is called as an exponential fluid.

3. Linear Stability Analysis

In linear stability theory, the perturbations are assumed to be arbitrarily small and hence those terms in the governing equations which are product of the perturbations and their derivatives are neglected. Thus a system of homogeneous linear differential equations (2.15) and (2.16) with homogeneous boundary conditions is obtained. This analysis is called linearization. Therefore in the linear stability theory, the perturbations can either grow exponentially or the magnitude of the perturbations remains constant. If the perturbations grow exponentially then the system is said to be unstable and if the magnitude remains constant then the system is said to be in the marginal state. By using the normal mode method, the stability of the convective system is analyzed.

Hence the perturbed functions are expressed as

$$\omega(x,z,t) = W(z)\exp(iqx+pt) \text{ and } \theta(x,z,t) = \Theta(z)\exp(iqx+pt). \qquad (3.19)$$

Using the solutions (3.19), into the linearized equations (2.15) and (2.16), we get the following set of ordinary differential equations.

$$(D^2)(D^2+q^2)W + 2(Df)(D^2-q^2)DW + f(D^2-q^2)W - R_T q^2\Theta = 0. \quad 3.20)$$

$$(D^2-q^2)\Theta + W = 0, \qquad (3.21)$$

Where D denotes the differentiation with respect to the vertical co-ordinate z, q are the non-dimensional wave number.

3.1. Boundary conditions

The physical model given in eqs. (3.20)-(3.21) are analyzed by considering the following four types of boundary conditions namely, R/R, R/F, F/R and F/F. In the abbreviations R/R, R/F, F/R and F/F, the numerator and denominator denote the upper and lower boundaries, respectively.

(i)No slip condition on the top and bottom boundaries (R/R). i.e,

$$W = DW = \Theta = 0 \text{ at } z = 1/2, z = -1/2. \qquad (3.22)$$

(ii)Stress-free condition at the top (z = 1/2) and no slip condition at the bottom(z = -1/2) boundaries (F/R). i.e.,

$$W = D^2W = \Theta = 0 \text{ at } z = 1/2,$$

$$W = DW = \Theta = 0 \text{ at } z = -1/2 \qquad (3.23)$$

(iii)no slip condition at the top (z = 1/2) and stress free condition at thbottom (z = -1/2) boundaries (R/F). i.e,

$$W = DW = \Theta = 0 \text{ at } z = 1/2,$$

$$W = D^2W = \Theta = 0 \text{ at } z = -1/2. \qquad (3.24)$$

(iv) Stress-free condition at the top and bottom boundaries (F/F) i.e,

$$W = D^2W = \Theta = 0 \text{ at } z = 1/2, z = -1/2. \qquad (3.25)$$

In the above equations (3.22)-(3.25), D denotes differentiation with respect to the vertical coordinate z.

In this section the Galerkin method is applied to examine the possibility of occurrence of

(1) stationary convection,

(2) narmalized vertical velocity eigenfunctions,

(3) temperatureeigenfunctions.

To study the above, we choose suitable trial functions for the z-components of velocity and temperature that satisfy the homogeneous boundary conditions (3.22)-(3.25) and are in the form

$$W(z) = \sum_1^\infty a_n W_n(z) \text{ and } \Theta(z) = \sum_1^\infty b_n \Theta(z), \qquad (3.26)$$

Where W_n and Θ_n are the functions that satisfy the boundary conditions (3.22)-(3.25) and a_n, b_n are unknown constants. Corresponding to each boundary conditions the trial functions vary and they are discussed below.
(1) The trial functions that satisfy no slip boundary conditions (3.22) are given by

$$W_n(z) = \begin{cases} \frac{\cosh(\alpha_n z)}{\cosh(\alpha_n/2)} - \frac{\cos(\alpha_n z)}{\cos(\alpha_n/2)}, \text{n odd}, \\ \frac{\sinh(\alpha_n z)}{\sin(\alpha_n/2)} - \frac{\sin(\alpha_n z)}{\sin(\alpha_n/2)}, \quad \text{n even.} \end{cases} \tag{3.27}$$

The constantsα_n are zeros of

$$\tanh(\alpha_n/2) + \tan(\alpha_n/2) = 0, \text{n is odd},$$

$$\coth(\alpha_n/2) + \cot(\alpha_n/2) = 0, \text{n is even},$$

and the functionΘ_n is given by

$$\Theta_n(z) = \begin{cases} \cos(n\pi z), \text{n is odd}, \\ \sin(n\pi z), \text{n is even}. \end{cases} \tag{3.28}$$

A Galerkin method, is used to obtain an infinite set of linear homogeneous algebraic equations for the unknown coefficients a_n andb_n. In this method in order to obtain a finite number of linear homogeneous algebraic equations, the expansions W(z) and $\Theta(z)$ in eqs. (3.26) are truncated at M and N terms respectively. After introducing the representations (3.26) in eqs.(3.20) and(3.21); multiplying the resultant equations of (3.20) and (3.21) by $W_j\ 1 \le j \le M;$ and $\Theta_j, 1 \le j \le N$,respectively and averaging the result over the fluid layer from z = —1/2 to z = 1/2 by allowing *f(z)* to depend arbitrarily on temperature, the matrix eigen value problem is obtained by

$$UX = 0, \tag{3.29}$$

where$U$ and X are the matrices and are given by

$$U = \begin{pmatrix} A_{ji} & B_{ji} \\ C_{ji} & D_{ji} \end{pmatrix}, X = \begin{pmatrix} a_n \\ b_n \end{pmatrix}, \tag{3.30}$$

The elements in the above eq.(3.30) are defined as

$$A_{ji} = \langle (D^2 f)(W_j D^2 W_i + q^2 W_j W_i) + (2Df)(W_j D^3 W_i - q^2 W_j DW_i) + f(W_j D^4 W_i - 2q^2 W_j D^2 W_i + q^4 W_j W_i)\rangle,$$

$$B_{ji} = -R_T q^2 \langle W_j T_i \rangle;\ C_{ji} = \langle T_j W_i \rangle;\ D_{ji} = \langle T_j T_i'' - q^2 T_j T_i \rangle,$$

If *det(A)* = 0 in eq.(3.29), non-trivial solutions and also a polynomial in terms of the Rayleigh number R_T, wave number q and viscosity ratio c are obtained. The Rayleigh number $R_T$for the onset of instabilities depends on the physical parameters, q and c. At each q the number of eigenvalues depends on the umbers M and N of trial functions used in (3.26). Only those $R_T$that are real, positiveand finite are physically meaningful.

For any value of q we take the smallest of these meaningful R_T. The critical value of $R_T$is obtained by iterating q until the marginal stable $R_T$is minimized. The convective system can be unstable to either stationary convection or oscillatory convection at the onset of instability. For this physical model only stationary convection which depends on the physical parameters R_T, c and q, has been obtained.

4. Numerical results and discussions

In this section, attention has been paid to the effect of viscosity variation c on the threshold values of $R_T$and q for different boundary conditions. In particular, for the R/R boundary conditions the effect of viscosity variation, c on theeigenfunctions namely,W(z) and $\Theta(z)$are plotted and discussed. For large *Pr,* at the onset of convection we get only stationary convection. The critical Rayleigh number and critical wave numbers are denoted by $R_{sc}$and q_{sc}respectively.

Depending on the different ranges of the viscosity ratio c, three different regions can be distinguished;

(i) low viscosity ratio region, when $0 \le c \le 1.5$; $0 \le V_{max}/V_{min} \le 5$;
(ii) moderate viscosity ratio, when $1.5 \le c \le 8$; $5 \le V_{max}/V_{min} \le 3000$;
(iii) large viscosity ratio, when $c > 8$.

Figure 1, shows the effect of c on $R_{sc}$for mentioned boundary conditions. For R/R boundary conditions, it can be noted that when the viscosity variation c is low, $R_{sc}$is nearly a constant. For the moderate viscosity variation as c increases R_{sc}also increases. This implies that the effect of moderate viscosity variation stabilizes the onset of convection. For large viscosity variation as c increases R_{sc}decreases. This implies that the effect of large viscosity variation destabilizes the convective system. The decrease of R_{sc}in the region of sufficiently large viscosity variation is related to the fact that a transition takes place in the fluid region, from motions involving the whole layer thickness to those located in a certain low viscosity sub-layer. It is noticed that c = 0 corresponds to constant viscosity.

From this Figure 1 we can observe that the R_{sc}obtained for a moderate viscosity ratio c is greater than the R_{sc}obtained for a value of low viscosity ratio *c*. We can observe from this figure that the remaining boundary conditions show similar effect onR_{sc}. We can also observe that at $c = 0$ the $R_{sc}$is same for both R/F and F/R boundary conditions. The role of number of rigid boundaries show the impact on R_{sc}. This implies that as number of rigid boundaries increases R_{sc}increases.

Figure 2, shows the effect of *c* on q_{sc}. For R/R boundary conditions, when the value of *c* increases in low viscosity region, $q_{sc}$is almost constant. For the moderate viscosity variation, as *c* increases q_{sc}also increases. For large values of viscosity ratio *c,* as *c* increases q_{sc}increases. From this figure we can observe that the remaining boundary conditions show similar results. We can also observe that at $c = 0$, the $q_{sc}$is same for both R/F and F/R boundary conditions

Figure 1 shows that for large values of viscosity variation the values of R_{sc}obtained from R/R boundary conditions coincide with the values of R_{sc}obtained from F/R boundary condition. Again the values of R_{sc}obtained from R/F boundary conditions coincide with those values of R_{sc} obtained from F/F boundary conditions. In Figure 2 we can see that the value of q_{sc}obtained from R/R, R/F, F/R and F/F coincide for large values of viscosity variation *c.* Figure1 shows that for large values of viscosity variation the values of R_{sc}obtained from R/R boundary conditions coincide with the values of R_{sc}obtained from F/R boundary condition. Again the values of R_{sc}obtained from R/F boundary conditions coincide with those values of R_{sc} obtained from F/F boundary conditions. In Figure 2 we can see that the value of q_{sc}obtained from R/R, R/F, F/R and F/F coincide for large values of viscosity variation *c.*

Once the critical Rayleigh number $R_{sc}$and critical wave number $q_{sc}$are calculated one can compute the vertical velocity and temperature eigenfunctionsfrom the eigenvalue problem which are the marginal modes in the bifurcation. The condition $det(M(R_{sc}, q_{sc})) = 0$ indicates that there exists non-trivial solutions of the system

$$M(R_c, q_c)X = 0. \qquad (4.31)$$

The numerical values of the linear combination (3.26) in the case of different values of the viscosity ratio *c* are computed for R/R boundary conditions. For a given $c = 3$ the critical values of $(q_{sc}, R_{sc}) = (3.084, 1889.678992)$. The corresponding numerical values of the linear combination for vertical velocity and temperature, are given by the following eqs. (4.32) and (4.33), respectively.

$$(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)=\xi\,(0.9999,-0.2979495146,0.07599825841,-0.02542744054,0.009897555175,-0.004475329123,0.002101074728,-0.0008093584776),\ \xi\epsilon\mathcal{R}, \qquad (4.32)$$

and

$$(b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8) = \xi\,(0.07283870744,-0.8550136027\times10^{-2}, 0.1246998257\times10^{-2},-0.3127123140\times10^{-3},-0.2258731284\times10^{-3}, 0.6996984137\times10^{-4}, 0.5081197646\times10^{-4},- 0.2007343754\times10^{-4}),\ \xi\epsilon\mathcal{R}, \qquad (4.33)$$

The eigenfunctions$W(z)$ and $\Theta(z)$ are numerically calculated at the critical bifurcation point (q_{sc}, R_{sc}) for $c = 3$ given as,

$$W(z)=(0.9999)W_1(z)+(-0.0085501360)\,W_2(z)+(0.075998258)\,W_3(z) +(-0.025427440)\,W_4(z)+(0.009897551)\,W_5(z)+(-0.0044753291)\,W_6(z) +(0.0021010747)\,W_7(z)+(-0.00080935847)\,W_8(z).$$

$$\Theta(z)=(0.072838707)\Theta_1(z)+(-0.0085501360)\,\Theta_2(z)+(0.0012469982)\,\Theta_3(z) +(-0.00031271231)\,\Theta_4(z)+(-0.00022587312)\,\Theta_5(z)+(0.69969841\times10^{-4})\,\Theta_6(z) +(0.50811976\times10^{-4})\,\Theta_7(z)+(0.20073437\times10^{-4})\,\Theta_8(z)$$

Figure 3 displays different velocity eigenfunctions as a function of depth *z*at the threshold values of $R_{sc}$and $q_{sc}$for different values of viscosity variation *c*. From this figure we can observe that that with increasing *c,* the points where the maximum value of $W(z)$ takes place is near the bottom of the layer where the fluid is less viscous. Figure 4, shows the effect of *c* on $\Theta(z)$. Here $\Theta(z)$ is the first order temperature perturbation and independent of platform. Like all the temperature perturbation functions, $\Theta(z)$ must vanish on the boundary of the fluid layer since the boundaries are isothermal. It is observed that for $c = 0$, $\Theta(z)$ is symmetric about the midpoint of the layer. As *c* increases, the temperature perturbation becomes confined to a small region near the bottom of the layer where the fluid is less viscous.

5. Conclusions

A thermally conducting exponential fluid is considered between two infinite horizontal boundaries, which is heated from below. The non-dimensional linearized perturbed governing equations are solved by using the Galerkinmethod for four different types of boundaries. The occurrence of stationary/oscillatory convection depends only on the physical parameters and noton the boundary conditions. The viscosity variation *c,*has been studied on the onset of convection for *R/R, R/F, F/R* and *F/F* boundary conditions. The onset of convection has been understood with the help of critical Rayleigh number R_{sc}Some important observations are listed below:

For finite Prandtl number, we have obtained the frequency $\omega^2 < 0$ for *R/R* boundary conditions. Thus, it is observed that we don't get oscillatory convection for this physical model. Also, for large Prandtlnumber, at the onset we get only stationary convection. Thus, we have studied the effect of physical parameters on the onset of stationary convection.

- The variation of viscosity, *c,* affects the onset of convection. Based on the effect of c on R_{sc} the range of c is divided as low viscosity region, moderate viscosity region and large viscosity region.
- In the low viscosity region, is observed that R_{sc} is almost constant. This implies that the effect of viscosity variation in the low viscosity region does not show any impact on the onset of stationary convection.
- In the moderate viscosity region, R_{sc} increases with c and R_{sc} attains a maximum value for $c \approx 8$. The effect of viscosity variation in the moderate viscosity ratio region stabilizes the onset of convection.
- The effect of large viscosity variation destabilizes the onset of convection. In the large viscosity ratio region as c increases R_{sc} decreases and the onset of convection is governed by a sublayer.

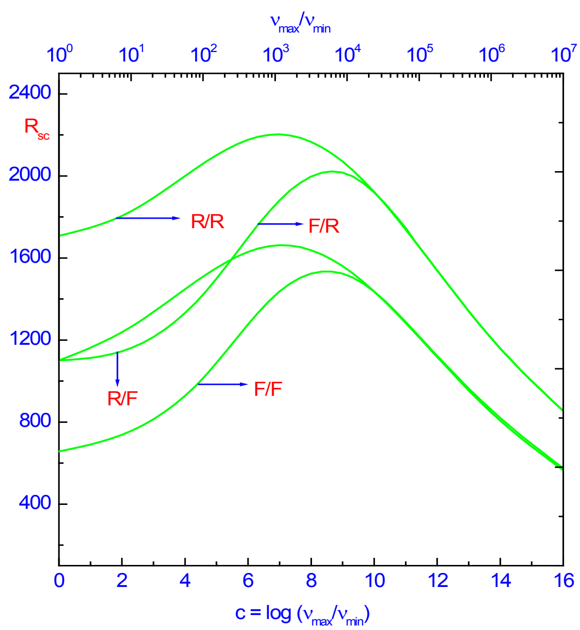

Figure 1:Critical wave number R_{sc} versus viscosity ratio c, which is the natural logarithm of the ratio of maximum cell viscosity to minimum cell viscosity for the exponential fluid for various boundary conditions viz., R/R, R/F, F/R and F/F.

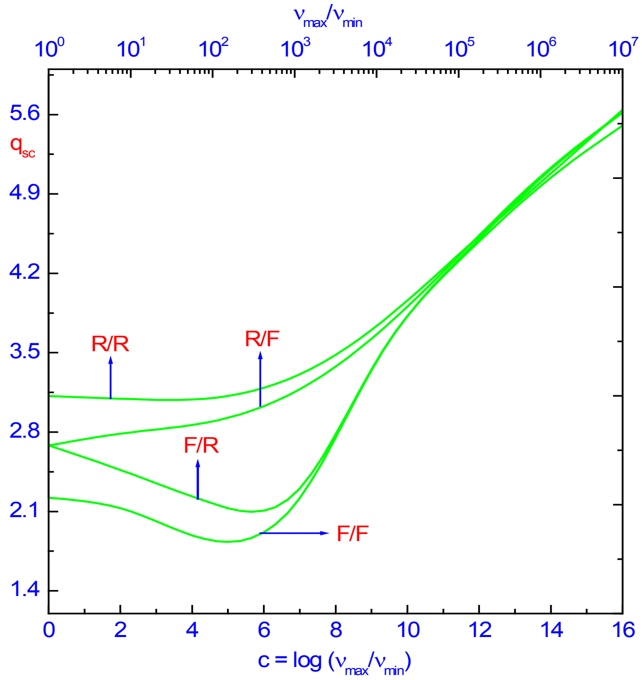

Figure2: Critical wave number q_{sc} versus viscosity ratio c, which is the natural logarithm of the ratio of maximum cell viscosity to minimum cell viscosity for the exponential fluid for various boundary conditions viz., R/R, R/F, F/R and F/F.

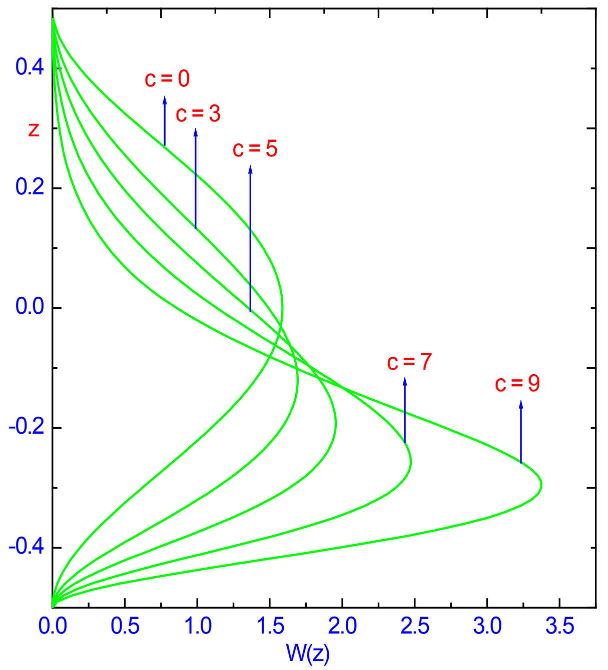

Figure 3:Vertical velocity eigenfunctions W(z) as a function of depth z and viscosity variation c for R/R boundary conditions.

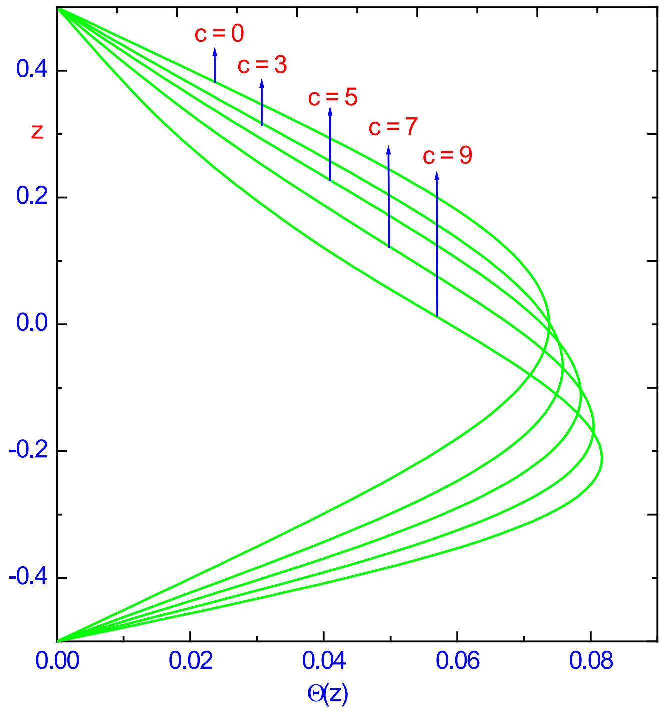

Figure 4:Temperature perturbations $\Theta(z)$*as a functionof depthz and viscosity variation c for R/R boundaryconditions.*

REFERENCES

[1] L. Rayleigh, On convection currents in a horizontal layer of fluid when the higher temperature is on the under side, Phil. Mag. 32(1916), pp. 529. | [2] A.Pellew and R.U.Southwell, On maintained convective motion in a fluid heated from below, Proc.Roy.Soc.A, 176(1940) ,pp.1312. | [3] J.R.A. Pearson, On convection cells induced by surface tension, J. Fluid Mech. 4 (1958), pp. 489. | [4] F.H. Busse, The stability of finite amplitude cellular convection and its relation to an extremum principle, J.Fluid Mech. bf 30(4),(1967), pp. 625. | [5] J.R. Booker, Thermal convection with strongly temperature dependent viscosity, J. Fluid Mech. 76(1976), pp.741. | [6] J.R.Booker and K.C.Stengel, Further thoughts on convective heat trans¬port in variable viscosity fluids, J. Fluid Mech. 86(1978), pp.289. | [7] K. C. Stengel, D. S. Oliver, and J. R. Booker ,Onset of convection in a variable viscosity fluid, J. Fluid Mech. 120(1982), pp.411. | [8] D.R.Jenkins, Rolls verses squares in thermal convection of fluid with temperature dependent viscosity, J. Fluid Mech. 178(1987), pp. 491. | [9] S.Thanham,C.F.Chen, Stability analysis on the convection of a variable viscosity fluid in an infinite vertical slot, Phys. Fluids 29(1986), pp. 1367. | [10] Yen-Ming Chen and Arne J. Pearlstein, Onset of convection in a vari-able viscosity fluids: Assessment of approximate viscosity -temperature relations, Phys of Fluids 31(1988), pp.1380. | [11] G. N.Sekhar, and G.Jayalatha, Elastic Effects on Rayleigh-Benard con¬vection in liquids with temperature dependent viscosity, International Journal of Thermal Sciences 49 (2010), pp.67. | [12] K. R.Rajagopal, G. Saccomandi and L.Vergori, Stability Analysis of the Rayleigh-Benard convection for a fluid with temperature and pressure dependent viscosity, ZAMP 60(2009), pp.739. | [13] R.K.Vanishree, P.G.Siddheswar, Effect of rotation on thermal convec¬tion in an anisotropic porous medium with temperature dependent vis¬cosity, Transp. Porou Med. 81(2010), pp.73. | [14] S.AwangKechil,I.Hashim, Oscillatory Marangoni convection in variable-viscosity fluid layer: The effct of thermal feedback contol, Int. J. Therm. Sci. 48(2009), pp 1102. | [15] K. E.Torrence, and D. L.Turcotte ,Thermal convection with large vis¬cosity variation, J. Fluid Mech. 47(1971), pp.113. | [16] S. Chandrashekar, Hydrodynamic and Hydromagnatic stability, Oxford: Clarendon. (1961). |