Volume : 3 | Issue : 5 | May 2014 ISSN - 2250-1991

Research Paper Mathematics

On The Stability of Rayleigh – Benard Convection in A Micropolar Fluid

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ABSTRACT

Recent scientific and technological advances have brought a great awakening of interest in constructing different types of fluids and investigating their flow behaviour in continuous as well as porous media under different conditions. There are many physical situations in different branches of engineering and science e.g. chemical, mechanical and petroleum engineering where these fluids either natural or man-made, have vide applications.

KEYWORDS Micropolar Fluids, Stability Theory, Rayleigh-Benard Convection.

0.1. MICROPOLAR FLUID

Micropolar fluids are the fluids with microstructure belonging to a class of fluids with non - symmetrical stress tensor referred to as polar fluids. Physically, they represent non – Newtonian fluids consisting of randomly oriented particles suspended in a viscous medium, when the deformation of fluid particles is ignored i.e. in micropolar fluids, the fluid particles rotate about their centroid. Examples of micropolar fluids are polymers, ferrofluids, blood, bubbly liquids, colloidal suspensions, etc.

Micropolar fluids consist of a dispersed phase suspended in a continuous phase. When it passes through singular points such as elbow corners, separation points of boundary layers, a slip motion of spins of each phase causes an additional diffusive momentum. Micropolar fluids have the micro rotational effects and micro rotational inertia. They can support the couple stress, the body couples, and the non – symmetric stress tensor and possess a rotational field, which is independent of the velocity field. The theory, thus has two independent kinematic variables, namely the velocity vector q and the spin or micro rotation velocity vector independent of q.

Navier Stokes' equations are insufficient to describe the flow of micropolar fluids. A system of equations of motion of an incompressible micropolar fluid was obtained in 1964 by Condiff and Dahler and independently in 1966 by Eringen. Trivially, the theory of micropolar fluids contain the couple stress theory for fluids as well as the classical theory of Newtonian fluids (Navier – Stokes fluids) as special cases. In spite of being complicated, these fluids possess a certain simplicity and elegance in their mathematical formulation, in other words, these fluids are manageable to both mathematicians who study its theory and physicists and engineers who apply it.

0.2. APPLICATIONS OF MICROPOLAR FLUIDS

According to Eringen (1964, 1966), the range of possible materials to be modelled by micropolar fluid theory is very wide. The applications of micropolar fluids with periodic microstructures are extensive and rather successful. The applications of this theory are flow in a non – coaxial plate : plate rheometer, lubrication problem (generalised Reynolds equation), Stokes' law about a sphere, stagnation flow, Taylor – Bernard instability problem, boundary layer flow over a plate, mixed convection in vertical flow, flow in porous media, etc.

0.3. STABILITY THEORY

To analyze the flow behaviour of different types of fluids in different physical situations theoretically, an equivalent mathematical model is constructed. While solving and constructing the mathematical model, because of the complexities of real situations, it becomes to take certain assumptions and approximations and thus the exact solution so obtained may not represent exactly the true happening in the fluid. In other words, the patterns of flow can however be realized only for a certain ranges of parameters characterizing them, and outside these ranges, they cannot be realized. This leads to the origin of Hydrodynamic Stability Theory. This theory helps in realizing the range of parameters in which flow patterns can be realized. In this theory, it is seen that what is the reaction of the system to small perturbations to which any physical system is subjected. When the perturbation in the system gradually decays, we say that the system is stable with respect to that perturbation, but if the disturbances grow in amplitude in such a way that the system progressively departs from the initial state and never reverts to it, we say it is unstable. Clearly, a system must be considered as unstable even if there is only one special mode of disturbance with respect to which it is unstable, and a system cannot be considered stable unless it is stable with respect to every possible disturbance to which it can be subjected or we can say, stability must imply that there exists no mode of disturbance for which it is unstable.

0.4. RAYLEIGH – BENARD CONVECTION

The Rayleigh-Benard (henceforth referred to as RB) system has been studied by researchers for a complete century by now and is still a topic of interest. Rayleigh–Benard convection is a type of natural convection, occurring in a plane horizontal layer of fluid heated from below, in which the fluid develops a regular pattern of convection cells known as Benard cells. Rayleigh–Benard convection is one of the most commonly studied convection phenomena because of its analytical and experimental accessibility. The convection patterns are the most carefully examined example of self-organizing nonlinear systems.

Then, the temperature of the bottom plane is increased slightly yielding a flow of thermal energy conducted through the liquid. The system will begin to have a structure of thermal conductivity: the temperature, and the density and pressure with it, will vary linearly between the bottom and top plane. A uniform linear gradient of temperature will be established. Once conduction is established, the microscopic random movement spontaneously becomes ordered on a macroscopic level, forming Benard convection cells, with a characteristic correlation length.

1.5 THE RAYLEIGH – BENARD INSTABILITY

Since there is a density gradient between the top and the bottom plate, gravity acts trying to pull the cooler (or denser) liquid from the top to the bottom. This gravitational force is opposed by the viscous damping force in the fluid. The balance of these two forces is expressed by a non-dimensional parameter called the Rayleigh number. The Rayleigh Number is defined as:

$$Ra = \frac{g\beta (T_u - T_b) L^3}{\nu\alpha}$$

where, Tu and Tb are the Temperatures of the top and the bottom plate, L is the height of the container, g is the acceleration due to gravity, being the kinematic viscosity, α is the Thermal diffusivity, β being the Thermal expansion coefficient. As the Rayleigh number increases, the gravitational forces become more dominant. At a critical Rayleigh number of 1708, the instability sets in, and convection cells appear. The critical Rayleigh number can be obtained analytically for a number of different boundary conditions by doing a perturbation analysis on the linearized equations in the stable state.

1. REVIEW OF LITERATURE

During the review of literature it has been found that the micropolar fluids originated by Eringen in 1964 and nanofluids introduced by Choi et al. in 1995, and became the topics of concern of of many researchers.

Eringen (1964), introduced micropolar fluid theory in order to deal with a class of fluids which do not satisfy the Navier – Stokes equations. The onset of convection instabilities in a horizontal layer of a micropolar fluid heated from below has been considered first by Ahmadi (1976). A solution was obtained in the case of free boundaries and it was demonstrated that the micropolar fluids are more stable than the Newtonian one. The same problem was extended by Rama Rao (1979) to study the onset of convection of heat conducting micropolar fluid layer confined between two horizontal rigid boundaries. They also concluded that micropolar fluids heated from below are more stable when compared to the pure viscous fluid situation.

During last two dacades, Qin and Kaloni (1992) studied thermal instability of in a rotating micropolar fluid heated from below. They have pointed out that the micropolar parameters contribute to the condition deciding whether stationary convection or oscillatory convection will prevail.

The effect of medium permeability on thermal convection in micropolar fluids have been considered by Sharma and Gupta (1995). It was found that the presence of coupling between thermal and micropolar effects may introduce oscillatory motions and the increase in Rayleigh number for the stationary convection and the decrease in Rayleigh number for overstability with the increase in permeability. Rayleigh- Taylor instability of fluids in porous medium is studied by Sharma and Kumar (1995) in the presence of suspended particles and variable horizontal magnetic field. An account of the Mathematical details of Micropolar fluid theory and some of its applications has been given by Lukaszewich (1999).

More recently, A linear stability analysis is performed by Irdis et. al. (2009) to study the effect of non – uniform basic temperature gradients on the onset of Benard – Marangoni convection in a micropolar fluid. It has been found that the presence of micron – sized suspended particles delays the onset of convection.

Recently, Pranesh and Kiran (2010) investigated the onset of Rayleigh-Benard

convection in a micropolar fluid by the substitution of substitution of Maxwell-Cattaneo law instead of the Classical Fourier heat conduction law. Eigenvalues are obtained using Galerkin method for various boundary combinations of velocity and temperature. Most recently, Srinivasacharya and Ch. Ram Reddy (2011) have studied the Soret & Dufor effects on mixed convection in a Non-Darcy porous medium saturated with micropolar fluid.

2. CONCLUSION & PROPOSED FUTURE WORK :

The characterstic features : microrotation and microinertia of micropolar fluids provides a wide spectrum of new and critical applications in the theory of fluids. The suitable applications of different micropolar effects will range from cooling densely packed integrated circuits at the small scale to heat transfer in nuclear reactors at the large scale. These will further have applications in electronics, nuclear and biomedical instrumentation and equipments, transportation and industrial cooling and heat management in various critical applications, as well as environmental control and clean up. The objective of the proposed work is to develop the mathematical models and analyze their stability for different physical situations In view of the characterstics of micropolar, the demand of the new world, it is planned to examine the stability of the thermal convection of a micropolar fluid under different physical situations. Due to the importance of flows in porous media in diverse areas as rheology, geophysics, chemical and petroleum industries, stability of micropolar effects will be investigated through porous media as well. Anisotropicity is the most natural property of porous media, therefore study will also be extended to anisotropic porous media.

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