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Strongly μ-s- Regular Spaces in Generalized Topology

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ABSTRACT

The aim of this paper is to introduce and study the class of strongly μ-s-regular spaces in generalized topology which lies entirely between the class of μ -regular spaces and the class of μ-s-regular spaces. Furthermore, some basic properties of μ-semiopen sets are investigated and relationships between μ -dense set, μ-semiopen set and μ-regular spaces are investigated.

KEYWORDS μ- open , μ- semiopen, μ- dense, μ-s-regular ,strongly μ-s-regular .

1.Introduction:

In the past decade,Csaszar [1] and others have been considering generalized topological spaces and developing a theory for generalized topology.The aspects of generalized topology and generalized continuity are given in [1].This paper is concerned with the adaptation of the change of topology approach from topological topic to aspects of the theory of generalized topological spaces.This shows that "change of generalized topology" exhibits some characteristic analogous to change of topology in the topological category.A common application of the change of generalized topologyapproach occurs whenthe spaces are ordinary topological spaces.In this case,the generalized topologies are families of distinguished subsets of a topological spaces which are not topologies but are generalized topologies. The introduction and development of generalized topology based on the above concept and is introduced and initiated in [1].

2.Preliminaries:

Let X be a set. A subset μ of exp X is called a generalized topology on Xand (X, μ) is called a generalized topological space [1] (abbr.GTS) if μ has the following properties:
(i) $\emptyset \in \mu$,
(ii)Any union of elements of μ belongs to μ.
Generalized topological space is an important generalization of topological spaces, and many interesting results have been obtained. Throughout this paper, a space (X, μ) or simply X for short, will always mean a strong generalized topological spaces with strong generalized topology μ unless stated otherwise explicitly. A generalized topology μ is said to be strong [2] if X$\in \mu$. For the space (X, μ), the elements of $\mu$are calledμ- open sets and the complement of μ- open sets are calledμ-closed sets. For A $\subset$ X, we denote by c_μ(A) the intersection of all μ- closed sets containing A, that is the smallest μ- closed set containing A, and by i_μ(A), the union of all μ- open sets contained in A, that is the largest μ- open set contained in A. It is easy to observe that $c_\mu$and i_μ are idempotent and monotonic, where γ:exp X$\to$exp X is said to idempotent if and only if A $\subset B \subset$ X implies $\gamma(\gamma(A)) = \gamma(A)$ and monotonic if and only if A $\subset B \subset$ X implies $\gamma(A) \subset \gamma(B)$. It is also well known that from [3,4] that if $\mu$is a generalized topology on X and A $\subset$X, $x \in$X then $x \in c_\mu$(A) if and only if $x \in M \in \mu \Rightarrow M \cap A \neq \emptyset$ and $c_\mu(X-A) = X- i_\mu(A)$.

Let B $\subset$ exp X and $\emptyset \in$ B. Then B is called a base [2] for μ if $\{\cup$ B': B'$\subset$ B$\}= \mu$. Wealso say that μ is generated by B. A point $x\in$ X is called a μ-cluster point of B if U $\cap$ (B$-\{x\}) \neq \emptyset$ for each U $\in \mu$with$x\in$U. The set of all μ-cluster point of B is denoted by d(B).The omission or end of the proof denoted by ■.

3.Properties of μ- semiopen sets in generalized topology : This section purely based on [6]

Definition 3.1: A set A in a GTS X will be termed μ-semiopen (abbr.μ-s.o) if and only if there exists μ-open set U such that U$\subset$ A $\subset c_\mu$(U).The complement of μ-s.o set is μ-semiclosed.The μ-semiclosure of A is the intersection of all μ-semiclosed set containing A (abbr. sc_{μ}(A)).

Theorem 3.2: A subset A in a GTS(X, μ) is μ-semiopen (μ-s.o) if and only if A $\subset$ $c_{\mu}i_{\mu}$(A)

Proof: Sufficiency : Let A $\subset c_{\mu}i_{\mu}$(A) and let U be μ-open in X.Then for some U =i_{μ}(A),we have U $\subset$ A $\subset c_{\mu}$(U).
Necessity : Let A be μ-s.o.Then U$\subset$ A $\subset$ c_{μ}(U) for some μ-open set U.But U $\subset i_{\mu}$(A) and thus c_{μ}(U) $\subset$ $c_{\mu}i_{\mu}$(A).Hence, A $\subset$ c_{μ}(U) $\subset c_{\mu}i_{\mu}$(A) ■.

Theorem 3.3: Let $\{A_{\alpha}\}_{\alpha\in J}$ be a collection ofμ-s.o sets in a GTS (X, μ).Then $\bigcup_{\alpha\in J} A_{\alpha}$ is μ-s.o .
Proof: For each $\alpha \in$ J, we have an U_{α} such that $U_{\alpha} \subset A_{\alpha} \subset c_{\mu}(U_{\alpha})$.Then $\bigcup_{\alpha\in J} U_{\alpha} \subset \bigcup_{\alpha\in J} A_{\alpha} \subset \bigcup_{\alpha\in J} c_{\mu}(U_{\alpha}) \subset c_{\mu} \bigcup_{\alpha\in J} U_{\alpha}$.Hence let U=$\bigcup_{\alpha\in J} U_{\alpha}$.Then, U $\subset \bigcup_{\alpha\in J} A_{\alpha} \subset c_{\mu}$(U).This implies that $\bigcup_{\alpha\in J} A_{\alpha}$ is μ-s.o ■.

Theorem 3.4: Let A be μ-s.o set in the GTS X such that A $\subset$ B $\subset c_{\mu}$(A).Then Bis μ-s.o
Proof: Since A is μ-s.o ,there exists μ-open set U such that U$\subset$ A $\subset c_{\mu}$(U) and since A $\subset$ B $\subset c_{\mu}$(A),we have U$\subset$ B. But c_{μ}(A) $\subset$ c_{μ}(U)and thus B $\subset$ c_{μ}(U). Hence U$\subset$ B $\subset$ c_{μ}(U) and so B is μ-s.o■.

Remark3.5: (i) If Uis μ-open in X ,thenU is μ-s.o in X.
(ii) The class of all μ-s.o sets in X denoted by μ-s.o(X) .

Theorem 3.6: Let μ be the class of μ-open sets in the GTS X.Then (i) $\mu \subset \mu$-s.o(X) and (ii) for A $\in$ μ-s.o(X) and A $\subset$ B $\subset$ c_{μ}(A), then B $\in$ μ-s.o(X)
Proof: This follows from Theorem 3.4 and Remark 3.5 ■.

Theorem 3.7: Let $\mathcal{B}=\{B_{\alpha}\}$be a collection of sets in (X, μ) such that (i) $\mu \subset \mathcal{B}$ and (ii) if B $\in \mathcal{B}$ and B $\subset$ D $\subset c_{\mu}(B)$ then D $\in \mathcal{B}$.Then μ-s.o(X) $\subset \mathcal{B}$. Thus μ-s.o(X) is the smallest class of sets in (X, μ) satisfying (i) and (ii) .
Proof: Let A $\in \mu$-s.o(X).Then U$\subset$ A $\subset$ c_{μ}(U) for some U $\in \mu$ .Then U $\in \mathcal{B}$ by (i) and thus A $\in \mathcal{B}$ by (ii)■.

Theorem 3.8: Let A $\subset$ Y $\subset$ X where X is a generalized topology and Y is a subspace. Let A $\in \mu$-s.o(X).Then A $\in \mu$-s.o(Y) .
Proof: Let A $\in \mu$-s.o(X).Then U $\subset$ A $\subset$ $c_{\mu(X)}$(U) ,where U is μ-open in X and $c_{\mu(X)}$denotes the closure operator in X. Now, U$\subset$ Y and thus U=U$\cap$Y $\subset$ A$\cap$Y $\subset$ Y$\cap c_{\mu(X)}$(U) or U $\subset$ A $\subset$ $c_{\mu(Y)}$(U).Since U=U$\cap$Y , U is μ-open in Y (by subspace topology) and the theorem is proved ■.

Recall that ,A space X is *μ-dense* [5] in itself if no point of X is μ-open.

Definition 3.9: A set A in a generalized topology X is *nowhere μ-dense* if $i_{\mu}c_{\mu}$(A)= $\emptyset$.

Lemma 3.10: Let U be μ-open in X. Then c_{μ}(U) $-$U is nowhere μ-dense in (X, μ).

This is well known and the proof left to the reader ■.
Theorem 3.11: Let A $\in \mu$-s.o(X) where X is a generalized topology. Then A=U$\cup$B where (i) U $\in \mu$ (ii) U$\cap$B=$\emptyset$ and (iii) B is nowhere

μ-dense .

Proof: Let A $\in$ μ-s.o(X).Then for some μ-open set U in (X, μ) , U$\subset$ A $\subset$ c_μ(U). But A=U $\cup$(A $-$U).Let B=A $-$U.Then B $\subset$ c_μ(U) $-$U and thus is nowhere μ-dense by lemma 3.10.Then A=U$\cap$B ,and (i) and (ii) immediately follows ■.

Definition 3.12: A subset A of a GTS Xis μ-regular open if A= $i_\mu c_\mu$(A). A subset F of a GTS X is said to be μ-regular closed (abbr. μ-rc) if X $-$ F is μ-regular open or equivalently ,if F=$c_\mu i_\mu$(F).

Lemma 3.13: If a singleton is μ-semi open ,then it is μ-open■.

Lemma 3.14: If A is μ-semi open ,thenA $-$ i_μ(A) is nowhere μ-dense in X ■.

Lemma 3.15: A is μ-semiclosed if and only if $i_\mu c_\mu$(A) $\subset$ A ■.

Lemma 3.16: Singletons which areμ-semiclosed in a generalized topologyX are either μ-regular open or nowhere μ-dense .

Proof: Let $\{x\}$be μ-semiclosed in X .By lemma 3.15 ,$i_\mu c_\mu(\{x\}) \subset \{x\}$.If $i_\mu c_\mu(\{x\})=\emptyset$,then $\{x\}$ is nowhere μ-dense.If $i_\mu c_\mu(\{x\})\neq \emptyset$,then $i_\mu c_\mu(\{x\})=\{x\}$.This implies that $\{x\}$ is μ-regular open ■.

4.Stronglyμ-s-regular spaces in generalized topology.

We introduce the class of stronglyμ-s-regular spaces in generalized topology which lies entirely between the class of μ-regular spaces and the class of μ-s-regular spaces.

Definition 4.1: A space (X, μ) is called strongly μ-s-regular if for any μ-closed set A $\subset$ X and any point $x \in$ X $-$ A there is an F $\in$ μ-regular closed set with $x\in$ F and F$\cap$A=$\emptyset$

Definition 4.2: A space (X, μ) is called μ-s-regular if for any μ-closed set A $\subset$ X and any point $x \in$ X $-$ A there exist disjoint μ-semiopen sets U_1 and U_2 such that $x \in U_1$ and A $\subset U_2$

Lemma 4.3: A strongly μ-s-regular space X in GTS is μ-s -regular .

Proof: Let A be a μ-closed set in (X, μ) and let $x \in$ X $-$ A. By hypothesis there is an F belongs to μ-regular closed set of (X, μ) with $x\in$ F and F$\cap$A= $\emptyset$.Clearly F and X $-$F are the desired disjointμ-semiopen sets■.

Lemma 4.4: For a space (X, μ) the following are equivalent.

(i) (X, μ) is strongly μ-s-regular .(ii)For any μ-open set U and any point $x \in$U there is an F $\in$ μ-rc(X, μ) with $x\in$ F $\subset$ U. (iii) Every μ-open set in (X, μ) is the union of μ-regular closed sets. (iv)Every μ-closed set in (X, μ) is the intersection of μ-regular open sets ■.

Theorem 4.5: Let X be strongly μ-s-regular space and let Y be μ-open in (X, μ).Then the subspace (Y, $\mu/_Y$) is strongly μ-s-regular.

Proof: Let A $\subset$ Y be μ-closed in (Y, $\mu/_Y$) and let $y\in$ Y $-$ A .Let A=$B\cap Y$ where B is μ-closed in (X, μ).Since $y\in$ X $-$ B ,there is an F $\in$ μ-rc(X) such that $y\in$ F and F$\cap B$= $\emptyset$.Since F is μ-open in X ,if F'=F $\cap Y$ then it is easily checked that $F' \in$ μ-rc ((Y, $\mu/_Y$)).Clearly $y\in F'$ and $F'\cap$A=$\emptyset$■.

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