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Symmetric Loss Function and Bayesian Estimation of two Parameter Rayleigh Life time Model

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ABSTRACT

In this paper, we have proposed a change point model related to the Two Parameter Rayleigh Distribution, where λ is a scale parameter and μ is a location parameter. In the next section, we have obtained the Bayes Estimates and posterior densities of λ_1, λ_2 and 'm'. Then we have derived the Bayes Estimates under symmetric loss function. After that, we have generated a numerical example and then we have studied the sensitivity of Bayes Estimates with respect to the change in the prior of the parameters for the proposed model. In the final section, we have given the conclusions on the basis of the numerical study.

KEYWORDS

Two Parameter Rayleigh Distribution, Two Parameter Rayleigh Life Time Model, Scale Parameter, Location Parameter, Bayes Estimates, Change Point, Symmetric Loss Function, Square Error Loss Function

1. INTRODUCTION

It was Lord Rayleigh who introduced the Rayleigh Distribution in the year 1880. It was studied and considered for the first time when there was some problem related to the acoustics field. The hazard function is an increasing function of time in case of Rayleigh Distribution which is a very significant characteristic to be considered as far as our study is concerned. According to this characteristic, the aging process or the deterioration process of the equipments or items start occurring in a very intense manner. It follows the Rayleigh Law when the failure times are distributed and above mentioned process takes place. We must not forget to mention the invaluable contributions of several researchers who studied the concept of Rayleigh Distribution and gave the conclusions on the basis of their studies. Among those research scholars, Johnson, Kotz and Balakrishnan studied the Rayleigh Law and made a study related to the excellent exposure of the Rayleigh Distribution in the year 1994. Their research work was studied and further carried out by the team of three research scholars named Abd-Elfattah, Hassan and Ziedean in the year 2006. The research was taken further by the joint efforts of Dey and Das in the year 2007 and Dey gave some latest results and conclusions in the year 2009 on the basis of the research work carried out earlier by the above mentioned scholars in the field of Statistics, which includes estimations, predictions and inferential issues for One Parameter Rayleigh Distribution. The Compound Rayleigh Model with a unimodal hazard function was obtained by Mostert, Roux and Bekker in the year 1991 and the Generalized Compound Rayleigh Model was studied by them in the year 2001 from the Bayesian perspective. The main application of the Compound Rayleigh Distribution is for the modelling of the survival times of patients which show the characteristics of a random hazard rate. Here, we shall consider the case of Generalization of the Two Parameter Rayleigh Life Time Distribution. The survival distribution and hazard function of this distribution are given as under:

$f(x_i, \lambda, \mu) = 2\lambda\,(x_i - \mu)\, e^{-\lambda\,(x_i - \mu)^2}$; $x_i > \mu, \lambda > 0$ and i= 1, 2, 3,…,m

$S(t) = e^{-\lambda\,(t-\mu)^2}, \qquad t > \mu$

$h(t) = 2\lambda\,(t - \mu), \qquad t > 0 \qquad (1)$

We shall note that the phenomenon of change point is observed in several situations when we study a life time model. At some point of time, we observe instability in the sequence of life time under observation and study. Our study is mainly focused to find that change point, where we need to find out at what time and at which point, the changes begin to occur. This entire phenomenon is called the change point inference problem. Here, we have clearly proposed the Bayesian Estimation Method as a strong and valid alternative to the method of Classical Estimation. Thus, our purpose is to study the Two Parameter Rayleigh Model with a change point from the Bayesian point of view.

Here, we have proposed a change point model related to the Two Parameter Rayleigh Distribution, where λ is a scale parameter and μ is a location parameter. In the next section, we have obtained the Bayes Estimates and posterior densities of λ_1, λ_2 and ‘m’. Then we have derived the Bayes Estimates under symmetric loss function. After that, we have generated a numerical example and then we have studied the sensitivity of Bayes Estimates with respect to the change in the prior of the parameters for the proposed model. Finally, we have given conclusions on the basis of the numerical study.

2. PROPOSED CHANGE POINT MODEL

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Let $X_1, X_2, X_3,\ldots, X_n$ (n ≥ 3) be a sequence of random lifetimes. Let first 'm' observations be coming from Two Parameter Rayleigh Distribution with the parameters $(\lambda_{1,}\mu)$. So the probability density function is given by:

$f\ (x_i\,,\lambda_1,\mu) = 2\lambda_1\,(x_i\ -\mu)\ e^{-\lambda_1\,(x_i-\mu)^2}$; (2)

where $x_i\ > \mu, \lambda_1 > 0$ and i= 1, 2, 3,…,m with the survival distribution function $S_1(t)$ and hazard function $h_1(t)$given by,

$$S_1(t) = e^{-\lambda_1\,(t-\mu)^2} \quad ; \qquad t > \mu \qquad (3)$$

$$h_1(t) = 2\lambda_1\,(t-\mu)\ ; \qquad t > 0 \qquad (4)$$

Later n-m observations are coming from the Two Parameter Rayleigh Distribution $(\lambda_{2,}\mu)$ probability density function is given by,

$f(x_i\,,\lambda_2,\mu) = 2\lambda_2\,(x_i\ -\mu)\ e^{-\lambda_2(x_i-\mu)^2}$; $\quad x_i\ > \mu, \lambda_2 > 0$ and i=m+1,…, n (5)

with the survival distribution function $S_2(t)$ and hazard function $h_2(t)$given by,

$$S_2(t) = e^{-\lambda_2\,(t-\mu)^2} \quad ; \qquad t > 0 \qquad (6)$$

$$h_2(t) = 2\lambda_2\,(t-\mu) \quad ; \qquad t > 0 \qquad (7)$$

For the given sample information T= $(T_1,\ldots, T_m, T_{m+1}\ . . . , T_n)$, the likelihood function will be as under:

$$L(\lambda_1,\lambda_2,\mu,m|\underline{T}) = 2^n\ U\ {\lambda_1}^m e^{-\lambda_1 T_1}\ {\lambda_2}^{n-m} e^{-\lambda_2\,T_2} \qquad (8)$$

where,

$T_1 = T_1(m) = \sum_{i=1}^{m}(x_i\ -\mu)^2$,

$T_2 = T_2(m) = \sum_{i=m+1}^{n}(x_i - \mu)^2$,

U $=\prod_{i=1}^{n}(x_i - \mu)$ (9)

3. POSTERIOR DENSITIES

We suppose the marginal prior distribution of 'm' to be discrete uniform over the set {1, 2, 3,..., n-1}.

$$g(m) = \frac{1}{n-1} \quad (10)$$

We also suppose a discrete prior distribution on the parameter μ considering the reference of the research work of Soland carried out in the year 1969. Further, we make an assumption that let the parameter μ be restricted to finite number of values say, $\mu_1, \mu_2, \dots, \mu_w$ in the interval (0,1).

Pr (μ)=ξ_j,

$$\sum_{j=1}^{w}\xi_j = 1, \quad 0\leq \xi_j \leq 1, \quad j=1, 2,\dots, w \quad (11)$$

Let us now suppose the conditional gamma prior on λ_1 and λ_2 given μ=ξ_j where,

j=1, 2, . . .,w which leads to tractable mathematics, viz.

$$g(\lambda_1 | \xi_j) = \frac{{b_1}^{a_1}}{\Gamma a_1}{\lambda_1}^{a_1-1}e^{-b_1\lambda_1}, \qquad \lambda_1 > 0,\ a_1, b_1 > 0$$

$$g(\lambda_2 | \xi_j) = \frac{{b_2}^{a_2}}{\Gamma a_2}{\lambda_2}^{a_2-1}e^{-b_2\lambda_2} \qquad \lambda_2 > 0,\ a_2, b_2 > 0 \quad (12)$$

Let the prior information be given respectively in terms of prior means $\mu_1 and\ \mu_2$ and coefficient of variations $\Phi_1 and\ \Phi_2$. Then, we have:

$\mu_1 = \frac{a_1}{b_1}$,

$\Phi_1 = \frac{1}{\sqrt{a_1}}$,

$\mu_2 = \frac{a_2}{b_2}$ and

$$\Phi_2 = \frac{1}{\sqrt{a_2}} \qquad (13)$$

Thus, if we have prior knowledge of μ_1, μ_2, Φ_1 and Φ_2, then gamma parameter can be

obtained by $a_1 = \frac{1}{{\Phi_1}^2}$,

$b_1 = \frac{1}{\mu_1 {\Phi_1}^2}$,

$$a_2 = \frac{1}{{\Phi_1}^2} \text{ and } b_2 = \frac{1}{\mu_2 {\Phi_2}^2} \qquad (14)$$

Hence, the joint prior density will be:

$$g(\lambda_1, \lambda_2, \mu, m) = \frac{1}{n-1} \xi_j \frac{{b_1}^{a_1}}{\Gamma a_1} \frac{{b_2}^{a_2}}{\Gamma a_2} {\lambda_1}^{a_1 - 1} e^{-b_1 \lambda_1} {\lambda_2}^{a_2 - 1} e^{-b_2 \lambda_2} \qquad (15)$$

The joint posterior density of parameters $\lambda_1, \lambda_2, \mu$ and $'m'$ is obtained using the likelihood function and the joint prior density of the parameters as under:

$$g(\lambda_1, \lambda_2, \mu, m | T_{1j}, T_{2j}, \xi_j) = \frac{L(\lambda_1, \lambda_2, \mu, m | T_{1j}, T_{2j}, \xi_j) g(\lambda_1, \lambda_2, \mu, m)}{h(T)}$$

$$= k_1 \xi_j c^n U_j {\lambda_1}^{a_1 + m - 1} e^{-\lambda_1 (T_1 + b_1)} {\lambda_2}^{a_2 + n - m - 1} e^{-\lambda_2 (T_2 + b_2)} / h(\underline{T}) \qquad (16)$$

where, $k_1 = \frac{1}{n-1} \frac{{b_1}^{a_1}}{\Gamma a_1} \frac{{b_2}^{a_2}}{\Gamma a_2}$, $U_j = \prod_{i=1}^{n} (x_i - \mu_j)$, $T_{1j} = \sum_{i=1}^{m} (x_i - \mu_j)^2$

and $T_{2j} = \sum_{i=m+1}^{n}(x_i - \mu_j)^2$ (17)

and $h(\underline{T})$ is the marginal posterior density of $\underline{T}$.

$$h(\underline{T}) = \sum_{m=1}^{n-1}\sum_{j=1}^{w}\int_0^\infty\int_0^\infty k_1\xi_j\, U_j\, \lambda_1^{a_1+m-1}e^{-\lambda_1(T_{1j}+b_1)}\lambda_2^{a_2+n-m-1}e^{-\lambda_2(T_{2j}+b_2)}$$

$$=\sum_{m=1}^{n-1}\sum_{j=1}^{w} k_1\xi_j\, U_j\int_0^\infty \lambda_1^{a_1+m-1}e^{-\lambda_1(T_{1j}+b_1)}d\lambda_1\int_0^\infty \lambda_2^{a_2+n-m-1}e^{-\lambda_2(T_{2j}+b_2)}\,d\lambda_2$$

$$= \sum_{m=1}^{n-1}\sum_{j=1}^{w} k_1\xi_j\, U_j\frac{\Gamma m+a_1}{(T_{1j}+b_1)^{m+a_1}}\frac{\Gamma n-m+a_2}{(T_{2j}+b_2)^{n-m+a_2}} \quad (18)$$

We shall apply the discrete version of Bayes theorem to obtain the marginal posterior probability distribution of the $\mu = \mu_j$ as under:

$$P_j = P_r\left(\mu = \mu_j|T_j\right) \propto$$

$$\sum_{m=1}^{n-1} k_1\xi_j U_j\int_0^\infty \lambda_1^{a_1+m-1}e^{-\lambda_1(T_{1j}+b_1)}d\lambda_1\int_0^\infty \lambda_2^{a_2+n-m-1}e^{-\lambda_2(T_{2j}+b_2)}\,d\lambda_2 h^{-1}(\underline{T})$$

which further reduces to

$$P_j = \sum_{m=1}^{n-1} k_1\xi_j U_j\,\frac{\Gamma m+a_1}{(T_{1j}+b_1)^{m+a_1}}\frac{\Gamma n-m+a_2}{(T_{2j}+b_2)^{n-m+a_2}}h^{-1}(\underline{T}) \text{ for j=1, 2,…,}w \quad (19)$$

The joint posterior density will be:

$$g\left(\lambda_1,\lambda_2\,|\mu_j,\underline{T}\right)=\sum_{m=1}^{n-1} k_1\xi_j\, U_j\,\lambda_1^{a_1+m-1}e^{-\lambda_1(T_{1j}+b_1)}\lambda_2^{a_2+n-m-1}e^{-\lambda_2(T_{2j}+b_2)}h^{-1}(\underline{T})$$

Marginal posterior density $g\left(\lambda_1\middle|\mu_j,\underline{T}\right)$ and $g(\lambda_2|\mu_j,\underline{T})$ will be:

$$g(\lambda_1|\mu_j,\underline{T}) = \sum_{m=1}^{n-1}\int_0^\infty g(\lambda_1,\lambda_2|\mu_j,\underline{T})\,d\lambda_2$$

$$= \sum_{m=1}^{n-1} k_1\xi_j\, U_j\lambda_1^{a_1+m-1}e^{-\lambda_1(T_{1j}+b_1)}\,\frac{\Gamma n-m+a_2}{(T_{2j}+b_2)^{n-m+a_2}}h^{-1}(\underline{T}) \quad (20)$$

and $g(\lambda_2|\mu_j, \underline{T}) = \sum_{m=1}^{n-1}\int_0^\infty g_1(\lambda_1, \lambda_2|\mu_j, \underline{T})\, d\lambda_1$

$$= \sum_{m=1}^{n-1} k_1\xi_j\, U_j\lambda_2^{a_2+n-m-1} e^{-\lambda_2(T_{2j}+b_2)} \frac{\Gamma m+a_1}{(T_{1j}+b_1)^{m+a_1}} h^{-1}(\underline{T}) \qquad (21)$$

Combining (19) and (20), marginal posterior density of λ_1 say $g(\lambda_1|\underline{T})$ will be as under:

$$g(\lambda_1|\underline{T}) = \sum_{m=1}^{n-1}\sum_{j=1}^{w} \frac{\boldsymbol{P}_j(T_{1j}+b_1)^{m+a_1}\ \lambda_1^{a_1+m-1} e^{-\lambda_1(T_{1j}+b_1)}}{\Gamma m+a_1} \qquad (22)$$

Combining (19) and (21), marginal posterior density of λ_1 say $g(\lambda_2|\underline{T})$ will be:

$$g(\lambda_2|\underline{T}) = \sum_{m=1}^{n-1}\sum_{j=1}^{w} \frac{\boldsymbol{P}_j(T_{2j}+b_2)^{n-m+a_2}\ \lambda_2^{a_2+n-m-1} e^{-\lambda_2(T_{2j}+b_2)}}{\Gamma n-m+a_2} \qquad (23)$$

The marginal posterior density of change point 'm' will be:

$$g\left(m \,\middle|\mu_j, \underline{T}\right) = B(m)/h(\underline{T}) \text{ where } j=1, 2,\ldots,w \qquad (24)$$

where $B(m) = k_1\xi_j U_j \int_0^\infty \lambda_1^{a_1+m-1}\, e^{-\lambda_1(T_{1j}+b_1)}\, d\lambda_1 \int_0^\infty \lambda_2^{a_2+n-m-1}\, e^{-\lambda_2(T_{2j}+b_2)} d\lambda_2$

$$= k_1\xi_j U_j \quad \frac{\Gamma m+a_1 \Gamma n-m+a_2}{(T_{1j}+b_1)^{m+a_1}(T_{2j}+b_2)^{n-m+a_2}} \qquad (25)$$

We get the marginal posterior density of the change point $'m'$ on combining (19) and (24) as

$$g(m\,|\underline{T}) = \sum_{j=1}^{W} P_j B\,(m)/h(\underline{T}) \qquad (26)$$

4. BAYES ESTIMATES UNDER SYMMETRIC LOSS FUNCTION

In this section, we have obtained the Bayes Estimates of the change point, survival times, hazard rates and parameters λ_1, λ_2 under the symmetric loss function. Earlier in section 3, we have discussed about the Bayes Estimates of the unknown change point 'm', using conditional gamma prior $g(\lambda_1, \lambda_2, m)$. The Bayes estimator of 'm' under Square Error Loss Function is

$$m^* = \sum_{m=1}^{n-1} \sum_{j=1}^{w} P_j \mathrm{m}\, B(m) / h(\underline{\mathrm{T}}) \tag{27}$$

where $B(m)$ and $h(\underline{\mathrm{T}})$ are same as obtained in (18) and (25) respectively.

The Bayes Estimates of λ_1 under Square Error Loss Function (SEL) are:

$$\lambda_{1s}^* = \sum_{m=1}^{n-1} \sum_{j=1}^{w} P_j \int_0^\infty g\left(\lambda_1 \middle| T_{1j}, T_{2j}\right) \lambda_1 \, d\lambda_1$$

$$= \sum_{m=1}^{n-1} \sum_{j=1}^{w} P_j \frac{(T_{1j}+b_1)^{m+a_1}}{\Gamma m+a_1} \int_0^\infty \lambda_1^{a_1+m} e^{-\lambda_1 (T_{1j}+b_1)} d\lambda_1$$

$$= \sum_{m=1}^{n-1} \sum_{j=1}^{w} P_j \frac{(T_{1j}+b_1)^{m+a_1} \Gamma m+a_1+1}{\Gamma m+a_1 (T_{1j}+b_1)^{m+a_1+1}}$$

$$= \sum_{m=1}^{n-1} \sum_{j=1}^{w} P_j \frac{m+a_1}{(T_{1j}+b_1)} \tag{28}$$

The Bayes Estimates of λ_2 under Square Error Loss Function (SEL) are:

$$\lambda_{2s}^* = \sum_{m=1}^{n-1} \sum_{j=1}^{w} P_j \int_0^\infty g\left(\lambda_2 \middle| T_{1j}, T_{2j}\right) \lambda_2 \, d\lambda_2$$

$$= \sum_{m=1}^{n-1} \sum_{j=1}^{w} P_j \frac{(T_{2j}+b_2)^{n-m+a_2}}{\Gamma n-m+a_2} \int_0^\infty \lambda_2^{a_2+n-m} e^{-\lambda_2 (T_{2j}+b_2)} d\lambda_2$$

$$= \sum_{m=1}^{n-1} \sum_{j=1}^{w} P_j \frac{(T_{2j}+b_2)^{n-m+a_2}}{\Gamma n-m+a_2} \frac{\Gamma n-m+a_2+1}{(T_{2j}+b_2)^{n-m+a_2+1}}$$

$$= \sum_{m=1}^{n-1} \sum_{j=1}^{w} P_j \frac{\Gamma n-m+a_2}{T_{2j}+b_2} \tag{29}$$

The Squared Error Loss (SEL) functions for survival distribution functions are respectively given by:

$$s_{1s}^{*} = \sum_{m=1}^{n-1} \sum_{j=1}^{w} P_j \int_0^{\infty} g(\lambda_2 | T_{1j}, T_{2j})\, d\lambda_1$$

$$= \sum_{m=1}^{n-1} \sum_{j=1}^{w} P_j \frac{(T_{1j}+b_1)^{m+a_1}}{\Gamma m+a_1} \int_0^{\infty} \lambda_1^{a_1+m-1} e^{-\lambda_1 (T_{1j}+b_1)} d\lambda_1$$

$$= \sum_{m=1}^{n-1} \sum_{j=1}^{w} P_j \frac{(T_{1j}+b_1)^{m+a_1}}{\Gamma m+a_1} \int_0^{\infty} \lambda_1^{a_1+m-1} e^{-\lambda_1 [(T_{1j}+b_1)} d\lambda_1$$

$$= \sum_{m=1}^{n-1} \sum_{j=1}^{w} P_j \frac{(T_{1j}+b_1)^{m+a_1} \Gamma m+a_1}{\Gamma m+a_1 (T_{1j}+b_1)^{m+a_1}}$$

$$= \sum_{m=1}^{n-1} \sum_{j=1}^{w} P_j \left(1 - \frac{1}{(T_{1j}+a_2)}\right)^{-(m+a_1)} \qquad (30)$$

Similarly, we get

$$s_{2s}^{*} = \sum_{m=1}^{n-1} \sum_{j=1}^{w} P_j \int_0^{\infty} g(\lambda_2 | T_{1j}, T_{2j})\, d\lambda_2$$

$$= \sum_{m=1}^{n-1} \sum_{j=1}^{w} P_j \frac{(T_{2j}+b_2)^{n-m+a_2}}{\Gamma n-m+a_2} \int_0^{\infty} \lambda_2^{a_2+n-m-1} e^{-\lambda_2 (T_{2j}+b_2)} d\lambda_2$$

$$= \sum_{m=1}^{n-1} \sum_{j=1}^{w} P_j \frac{(T_{2j}+b_2)^{n-m+a_2} \Gamma n-m+a_2}{\Gamma n-m+a_2 (T_{2j}+b_2)^{n-m+a_2}}$$

$$= \sum_{m=1}^{n-1} \sum_{j=1}^{w} P_j \left(1 + \frac{1}{(T_{2j}+a_2)}\right)^{-(n-m+a_2)} \qquad (31)$$

The Bayes Estimates of $h_1(t)$ and $h_2(t)$ under Square Error Loss Functions (SEL) are given by:

$$h_{1s}^{*} = \int_0^{\infty} \sum_{m=1}^{n-1} \sum_{j=1}^{w} P_j\, g(\lambda_1 | T_{1j}, T_{2j})\, d\lambda_1$$

$$= \sum_{m=1}^{n-1} \sum_{j=1}^{w} P_j \frac{(T_{1j}+b_1)^{m+a_1}}{\Gamma m+a_1} \int_0^{\infty} \lambda_1^{a_1+m} e^{-\lambda_1 (T_{1j}+b_1)}\, d\lambda_1$$

$$= \sum_{m=1}^{n-1} \sum_{j=1}^{w} P_j \frac{(T_{1j}+b_1)^{m+a_1} \Gamma m+a_1+1}{\Gamma m+a_1 (T_{1j}+b_1)^{m+a_1+1}}$$

$$=\sum_{m=1}^{n-1}\sum_{j=1}^{w} P_j \frac{(m+a_1)}{(T_{1j}+b_1)} \tag{32}$$

Similarly, we get

$$h_{2s}^{*} = \int_0^{\infty} \sum_{m=1}^{n-1}\sum_{j=1}^{w} P_j\, g(\lambda_2|T_{1j},T_{2j})\, d\lambda_2$$

$$=\sum_{m=1}^{n-1}\sum_{j=1}^{w} P_j \frac{(T_{2j}+b_2)^{n-m+a_2}}{\Gamma n-m+a_2}\int_0^{\infty} \lambda_2^{\,n-m+a_2} e^{-\lambda_2(T_{2j}+b_2)}\, d\lambda_2$$

$$=\sum_{m=1}^{n-1}\sum_{j=1}^{w} P_j \frac{(T_{2j}+b_2)^{n-m+a_2}\Gamma n-m+a_2+1}{\Gamma n-m+a_2(T_{2j}+b_2)^{n-m+a_2+1}}$$

$$=\sum_{m=1}^{n-1}\sum_{j=1}^{w} P_j \frac{(n-m+a_2)}{(T_{2j}+b_2)} \tag{33}$$

5. NUMERICAL STUDY

We have generated 20 random observations from the Two Parameter Rayleigh Model explained earlier. The first ten observations were taken with $\lambda_1 = 0.55$ and the next ten observations were taken with $\lambda_2 = 1.08$ from the same distribution. λ_1 and λ_2 themselves were random observations from gamma distributions with prior means as μ_1=0.55, μ_2=1.08 and coefficient of variations $\Phi_1 = 0.85, \Phi_2 = 0.36$ respectively. The resultant values obtained are given in table 1 for $b_1 = 1.56, b_2 = 2.46, a_1$=1.11 and $a_2 = 3.24$.

TABLE 1

Generated observations from Two Parameter Rayleigh Model

0.2315	0.7731	0.6921	0.1985	0.3170	0.4545	0.2919
0.3927	5.4075	0.2014	0.6870	0.6507	8.2664	0.6894

TABLE 2

Hyper parameter values of the gamma prior and the posterior probabilities

j	**1**	**2**	**3**	**4**
ξ_j	1/13	1/13	1/13	1/13
U_j	0.000198	0.000296	0.000324	1.60639×10^{-8}
b_1	1.08	1.23	1.44	1.53
a_1	1.14	1.16	1.11	1.17
b_2	0.91	1.17	1.62	2.43
a_2	2.07	2.22	2.57	3.24
p_j	0.1757	0.3247	0.4707	0.0261

The results of posterior mean of m, γ_1, γ_2, $s_1(t)$, $s_2(t)$, $h_1(t)$, $h_2(t)$ and posterior median of 'm' which we have calculated are shown in table 3, table 4 and table 5 respectively.

TABLE 3

Bayes Estimates of the change point 'm', λ_1 *and* λ_2 under

Square Error Loss Function

Prior Density	Bayes Estimates of the change point		Bayes Estimates of the Posterior Means of λ_1 and λ_2	
	Posterior Median of 'm'	Posterior Mean of 'm'	Posterior Mean of λ_1	Posterior Mean of λ_2
Informative	8.0	9.0	0.55	1.08

TABLE 4

Bayes Estimates of $s_1(t)$ *and* $s_2(t)$ under Square Error Loss Function

Posterior mean $of\, s_1(t)$	Posterior mean $of\, s_2(t)$
0.28	0.08

TABLE 5

Bayes Estimates of $h_1(t)$ and $h_2(t)$ under Square Error Loss Function

Prior Density	Bayes Estimates of Posterior Means of $h_1(t)$ and $h_2(t)$	
	Posterior Means of $h_1(t)$	Posterior Means of $h_2(t)$
Informative	0.49	0.91

6. SENSITIVITY OF BAYES ESTIMATES

Here, we have studied the sensitivity of the Bayes Estimates obtained earlier in section 2 and section 3 with respect to the change in the prior of the parameter. We have computed m^* and m_E^* for the data given in table 1 considering the different values of (μ_1, μ_2) and results are shown in table 6.

TABLE 6

Bayes Estimates m* and m_E^* for different values of Prior Means

μ_1	μ_2	m^*	m^*_E
0.55	1.98	8	8
0.55	1.17	8	8
0.55	1.25	8	8
0.54	1.08	8	8
0.45	1.08	8	8
0.51	1.08	8	8
0.51	0.85	8	8
0.57	1.43	8	8
0.48	1.35	8	8

TABLE 7

Frequency Distributions of the Bayes Estimates of the change point

Bayes Estimates	% Frequency for		
	01-05	06-10	11-14
Posterior Mean	13	85	09
Posterior Median	14	85	09
m_L^*	27	63	18
m_E^*	39	57	09

7. CONCLUSIONS

On the basis of the numerical studies, we come to the conclusion that the Bayes Estimates of the posterior mean of 'm' and m^*_E are robust in nature with respect to the correct choice of the prior specifications on $\lambda_1(\lambda_2)$ and incorrect choice of the prior specifications on $\lambda_2\ (\lambda_1)$ respectively. It is quite clear that he results are case sensitive in prior specifications on λ_1 and λ_2. Moreover, simultaneous deviations from the true values are clearly seen from the results. Table 6 leads to the conclusion that m^* and m^*_E are robust wit respect to the correct choice of the prior density and are also robust with respect to the change in the shape parameter.

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