



Feebly Regular Interior Point, Feebly Regular Exterior Point and Feebly Regular Frontier Point in Topological Space

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ABSTRACT	In this paper, we extend the study of [2] and introduce feebly regular interior point, feebly regular exterior point, feebly regular frontier point with the feebly regular cluster point, feebly regular adherent point and feebly regular isolated point and also given the properties of them and analyzed characterization and relationship between them.
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KEYWORDS	feebly regular open, feebly regular closed, feebly regular interior point, feebly regular exterior point, feebly regular frontier point, feebly regular cluster point, feebly regular adherent point, feebly regular isolated point.
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**1.Introduction :** In topological space, the concept of feebly open and feebly closed sets are introduced by S.N. Maheswari and P.C.Jain in 1982. In this concept, further works are developed by many researchers. Throughout this paper, the feebly closure of A and the feebly interior of A is denoted by  $f.cl(A)$  and  $f.int(A)$  where A is the subset of X.

**2.Preliminaries:**

**Definition 2.1:** [5] Let X be a topological space and A be a subset of X. It is said to be semi-regular open if  $A=s\ int(s\ cl(A))$  and also defined on other hand, it is said to be semi-regular open if both semi open (if  $A\subset cl(int(A))$  [3]) and semi closed (if  $int(cl(A))\subset A$ ).

**Definition 2.2:** [4] A subset A of a topological space X is said to be feebly open (resp. feebly closed) if  $A\subset s\ cl(int(A))$  (resp.  $s\ int(cl(A))\subset A$ ).

**Definition 2.3:**[6] A map  $f:X\rightarrow Y$  is said to be feebly closed (resp. feebly open) if the image of each closed set (resp. open set) in X is feebly closed (resp. feebly open) set in Y.

**Remark 2.4:** [1] (i) Every open set is feebly open (ii) Every closed set is feebly closed.

**Definition 2.5:** [2] A subset A of X is said to be feebly regular open(briefly F.reg.open) if  $A=f.int(f.cl(A))$ .

**Definition 2.6:** [2] A subset A of X is said to be feebly regular closed if  $A=f.cl(f.int(A))$  (briefly F.reg.closed).

**Remark 2.7:** [2] The feebly regular open set is analyzed in the way if A is both feebly open and feebly closed.

**Definition 2.8:** [2] A subset A of X is said to be feebly regular clopen if  $A=f.int(f.cl(f.int(A)))$ . On the other hand, if A is F.reg.open and F.reg.closed.

**Definition 2.9: [2]** Let  $A$  be subset of  $X$ . The regular closure of  $A$  (briefly  $F.reg.cl(A)$ ) is the intersection of all feebly regular closed set containing  $A$  and  $F.reg.int(A)$  is the union of all feebly regular open set contained in  $A$ .

**Recall the concepts: 2.10: [2]** (i) The complement of feebly regular open set is feebly regular closed.

(ii) If  $A$  and  $B$  are  $F.reg.closed$  sets then  $A \cup B$  is  $F.reg.closed$ .

(iii) If  $A$  and  $B$  are  $F.reg.open$  then  $A \cap B$  is  $F.reg.open$ .

(iv) Let  $A$  be a subset of  $X$ . For an element  $x \in X$ ,  $x \in F.reg.closed$  set of  $A$  if and only if  $U \cap A \neq \emptyset$  for every  $F.reg.open$  set  $U$  containing  $x$ .

### 3. Feebly regular interior point, feebly regular exterior point and feebly regular frontier point with feebly regular cluster point, feebly regular adherent point and feebly regular isolated point

**Definition 3.1:** Let  $(X, \tau)$  be a topological space and let  $A \subset X$ . A point  $x \in X$  is said to be a feebly regular interior point of  $A$  if there exist a feebly regular open set  $G$  such that  $x \in G \subset A$ . The set of all feebly regular interior points of  $A$  is called the feebly regular interior of  $A$  and is denoted by  $F.reg.int(A)$ . Evidently  $A$  contains all its interior points, that is,  $F.reg.int(A) \subset A$ .

**Definition 3.2:** Let  $(X, \tau)$  be a topological space and let  $A$  be a subset of  $X$ . A point  $x \in X$  is called a  $F.reg.cluster$  point of  $A$  if  $[N - \{x\}] \cap f.int(f.cl(A)) \neq \emptyset$  for every  $\tau$ -neighbourhood  $N$  of  $x$ .

**Remark 3.3:** The point  $x$  is not a  $F.reg.cluster$  point of  $A$  if there exists a neighbourhood  $N$  of  $x$  such that  $N \cap f.int(f.cl(A)) = \emptyset$  or  $N \cap f.int(f.cl(A)) = \{x\}$ .

**Definition 3.4:** Let  $X$  be a topological space and  $A \subset X$ . A point  $x \in X$  is called  $F.reg.adherent$  point of  $A$  if there exist neighbourhood  $N$  of  $x$  such that  $N \cap f.int(f.cl(A)) \neq \emptyset$ . The set of all  $F.reg.adherent$  points of  $A$  is denoted by  $F.reg.adherent(A)$ .

**Definition 3.5:** Let  $X$  be a topological space and  $A \subset X$ . There exists neighbourhood  $G$  of  $x$  such that  $[G - \{x\}] \cap f.int(f.cl(A)) = \emptyset$  or  $G \cap f.int(f.cl(A)) = \{x\}$ . Then  $x \in A$  is called  $F.reg.isolated$  point of  $A$ . The set of all  $F.reg.isolated$  points of  $A$  is denoted by  $F.reg.isolated(A)$ .

**Remark 3.6:** A  $F.reg.adherent$  point  $x \in A$  is called  $F.reg.isolated$  point of  $A$  if  $x$  is not a  $F.reg.cluster$  point of  $A$  where  $A$  is a subset of the topological space  $X$ .

**Definition 3.7:** Let  $A$  be a subset of a topological space  $X$ . A point  $x \in X$  is said to be a  $F.reg.exterior$  point of  $A$  if there exists a feebly regular open set  $G$  such that  $x \in G \subset A'$  where  $A'$  is the complement of  $A$ . The set of all  $F.reg.regular$  exterior points of  $A$  is denoted by  $F.reg.ext(A)$ .

**Remark 3.8:** Let  $A$  be a subset of a topological space  $X$ .

(i) A point  $x \in X$  is a  $F.reg.interior$  point of the complement  $A'$  of  $A$ .

(ii) A point  $x$  belongs to feebly regular open set  $G$  and if  $G \cap A = \emptyset$  then it is  $F.reg.exterior$  point of  $A$ .

(iii)  $F.reg.ext(A) = F.reg.int(A')$ .

(iv)  $F.reg.ext(A') = F.reg.int(A'') = F.reg.int(A)$ .

(v)  $A \cap F.reg.ext(A) = \emptyset$ .

**Definition 3.9:** A point  $x$  of a topological space  $X$  is said to be a F.reg.frontier point of a subset  $A$  of  $X$  if it is neither a F.reg.interior nor a F.reg.exterior point of  $A$ . The set of all F.reg.frontier points of  $A$  is called the F.reg.frontier of  $A$  and is denoted by  $F.reg.Fr(A)$ .

**Theorem 3.10:**  $F.reg.int(A) = \bigcup \{G : G \text{ is feebly regular open, } G \subset A\}$ .

**Proof:**  $x \in F.reg.int(A) \Leftrightarrow A$  is a neighbourhood of  $x \Leftrightarrow$  there exists a feebly regular open set  $G$ , such that  $x \in G \subset A \Leftrightarrow x \in \bigcup \{G : G \text{ is feebly regular open, } G \subset A\}$ . Hence  $F.reg.int(A) = \bigcup \{G : G \text{ is feebly regular open } G \subset A\}$ .

**Corollary 3.11 :**  $F.reg.clopen int(A) = \bigcup \{G : G \text{ is feebly regular open, } G \subset A, \text{ where } A \text{ is a feebly regular clopen set of } x\}$ .

**Theorem 3.12:** Let  $(X, \tau)$  be a topological space and let  $A$  be a subset of  $X$ . Then

- (i)  $F.reg.int(A)$  is a feebly regular open set
- (ii)  $F.reg.int(A)$  is the largest feebly regular open set contained in  $A$
- (iii)  $A$  is feebly regular open if and only if  $F.reg.int(A) = A$

**Proof:** (i) Let  $x$  be any arbitrary point of  $F.reg.int(A)$ . Then  $x$  is a F.reg.interior point of  $A$ . Hence by definition,  $A$  is a neighbourhood of  $x$ . Then there exists a feebly regular open set  $G$  such that  $x \in G \subset A$ . Since  $G$  is feebly regular open, it is a neighbourhood of each of its points and so  $A$  is also a neighbourhood of each point of  $G$ . It follows that every point of  $G$  is a F.reg.interior point of  $A$  so that  $G \subset F.reg.int(A)$ . Thus it is shown that to each  $x \in F.reg.int(A)$ , there exists a feebly regular open set  $G$  such that  $x \in G \subset F.reg.int(A)$ . Hence  $F.reg.int(A)$  is a neighbourhood of each of its points and consequently  $F.reg.int(A)$  is feebly regular open.

(ii) Let  $G$  be any subset of  $A$  and let  $x \in G$  so that  $x \in G \subset A$ . Since  $G$  is feebly regular open,  $A$  is a neighbourhood of  $x$  and consequently  $x$  is a feebly regular interior point of  $A$ . Hence  $x \in F.reg.int(A)$ . Thus we have shown that  $x \in G \Rightarrow x \in F.reg.int(A)$  and so  $G \subset F.reg.int(A) \subset A$ . Hence  $F.reg.int(A)$  contains every feebly regular open subset of  $A$  and it is therefore the largest feebly regular open subset of  $A$ .

(iii) Let  $A = F.reg.int(A)$ , by (i)  $F.reg.int(A)$  is a feebly regular open set and therefore  $A$  is also feebly regular open. Conversely  $A$  be a feebly regular open. Then  $A$  is surely identical with the largest feebly regular open subset of  $A$ . But by (ii),  $F.reg.int(A)$  is the largest feebly regular open subset of  $A$ . Hence  $A = F.reg.int(A)$ .

**Theorem 3.13:** Let  $(X, \tau)$  be a topological space and let  $A$  be a subset of  $X$ . Then  $F.reg.int(A)$  equals the set of all those points of  $A$  which are not F.reg.cluster points of  $A'$ .

**Proof:** Let  $x$  be a point of  $A$  which is not a F.reg.cluster point of  $A'$ . Then there exists a neighbourhood  $N$  of  $x$  which contains no points of  $A'$  and so  $N \subset A$ . This implies that  $A$  is also a neighbourhood of  $x$  and so  $x \in F.reg.int(A)$ . Consequently let  $x \in F.reg.int(A)$ . since  $F.reg.int(A)$  is feebly regular open. Also  $F.reg.int(A)$  contains no point of  $A'$ . It follows that  $x$  is not a F.reg.cluster point of  $A'$ . Thus no point of  $F.reg.int(A)$  is F.reg.cluster point of  $A'$ . Hence  $F.reg.int(A)$  consists of precisely those points of  $A$  which are not F.reg.cluster points of  $A'$ .

**Remark 3.14:** Since  $F.reg.ext(A)$  is the  $F.reg.int(A')$ , it follows from theorem 3.12 that  $F.reg.ext(A)$  is feebly regular open and is the largest feebly regular open set contained in  $A'$ .

**Theorem 3.15:** Let  $(X, \tau)$  be a topological space and  $A \subset X$ . Then  $F.reg.ext(A) = \bigcup \{G \in \tau : G \subset A'\}$ .

**Proof:** By definition,  $F.reg.ext(A) = F.reg.int(A')$ . But by theorem 3.10.

$F.\text{reg.int}(A') = \cup \{G \in \tau : G \subset A'\}$ . Hence  $F.\text{reg.ext}(A) = \cup \{G \in \tau : G \subset A'\}$ .

**Theorem 3.16:** Let  $A$  be a subset of a topological space  $X$ . Then a point  $x \in X$  is a  $F.\text{reg.exterior}$  point of  $A$  if and only if  $x$  is not a feebly regular adherent point of  $A$ , that is, if and only if  $x \in A'$ .

**Proof:** Let  $x$  be a feebly regular exterior point of  $A$ . Then  $x$  is a feebly regular interior point of  $A'$  so that  $A'$  is a neighbourhood of  $x$  containing no points of  $A$ . It follows that  $x$  is not a feebly regular adherent point of  $A$ , that is,  $x \in F.\text{reg.cl}(A')$ . Conversely, suppose that  $x$  is not a feebly regular adherent point of  $A$ . Then there exists a neighbourhood  $N$  of  $x$  which contains no points of  $A$ . This implies that  $x \in N \subset A'$ . It follows that  $A'$  is a neighbourhood of  $x$  and consequently  $x$  is a feebly regular interior point of  $A'$ . That is,  $x$  is a feebly regular exterior point of  $A$ .

**Corollary 3.17:** It follows from the above theorem that  $F.\text{reg.ext}(A) = (F.\text{reg.cl}(A))'$ . From this, we conclude that  $F.\text{reg.int}(A) = F.\text{reg.ext}(A') = (F.\text{reg.cl}(A'))'$ .

**Theorem 3.18:** Let  $X$  be a topological space and  $A \subset X$ . Then a point  $x$  in  $X$  is a feebly regular frontier point of  $A$  if and only if every neighbourhood of  $x$  intersects both  $A$  and  $A'$ .

**Proof:** We have  $x \in F.\text{reg.Fr}(A) \Leftrightarrow x \notin F.\text{reg.int}(A)$  and  $x \notin F.\text{reg.ext}(A) = F.\text{reg.int}(A) \Leftrightarrow$  neither  $A$  nor  $A'$  is a neighbourhood of  $x \Leftrightarrow$  no neighbourhood of  $x$  can be contained in  $A$  or in  $A' \Leftrightarrow$  every neighbourhood of  $x$  intersects both  $A$  and  $A'$ .

**Corollary 3.19:**  $F.\text{reg.Fr}(A) = F.\text{reg.Fr}(A')$ .

**Proof:**  $x \in F.\text{reg.Fr}(A) \Leftrightarrow$  every neighbourhood of  $x$  intersects both  $A$  and  $A' \Leftrightarrow$  every neighbourhood of  $x$  intersects both  $(A')'$  and  $A'$ . Since  $(A')' = A' \Leftrightarrow x \in F.\text{reg.Fr}(A')$ .

#### 4. Properties of feebly regular interior, feebly regular exterior and feebly regular frontier

**Theorem 4.1:** [2] For any two subsets  $A$  and  $B$  of  $(X, \tau)$

- (i) If  $A \subset B$ , then  $F.\text{reg.int}(A) \subset F.\text{reg.int}(B)$
- (ii)  $F.\text{reg.int}(A \cap B) = F.\text{reg.int}(A) \cap F.\text{reg.int}(B)$
- (iii)  $F.\text{reg.int}(A) \cup F.\text{reg.int}(A) \subset F.\text{reg.int}(A \cup B)$
- (iv)  $F.\text{reg.int}(X) = X$
- (v)  $F.\text{reg.int}(\emptyset) = \emptyset$

**Lemma 4.2:** [2] Let  $A$  be a subset of  $X$

- (i)  $(F.\text{reg.int}(A))' = F.\text{reg.cl}(A')$  (ii)  $(F.\text{reg.cl}(A))' = F.\text{reg.int}(A')$

**Remark 4.3:** (i)  $F.\text{reg.int}(A) \cup F.\text{reg.int}(B) \neq F.\text{reg.int}(A \cup B)$

- (ii)  $F.\text{reg.int}(F.\text{reg.int}(A)) = F.\text{reg.int}(A)$  (iii)  $F.\text{reg.int}(A) \subset A'$

**Theorem 4.4:** Let  $(X, \tau)$  be a topological space and let  $A, B$  be subsets of  $X$ . Then

- (i)  $F.\text{reg.ext}(X) = \emptyset$ ,  $F.\text{reg.ext}(\emptyset) = X$
- (ii)  $F.\text{reg.ext}(A) \subset A'$
- (iii)  $F.\text{reg.ext}(A) = F.\text{reg.ext}((F.\text{reg.ext}(A))')$
- (iv)  $A \subset B \Rightarrow F.\text{reg.ext}(B) \subset F.\text{reg.ext}(A)$
- (v)  $F.\text{reg.int}(A) \subset F.\text{reg.ext}(F.\text{reg.ext}(A))$
- (vi)  $F.\text{reg.ext}(A \cup B) = F.\text{reg.ext}(A) \cap F.\text{reg.ext}(B)$

**Proof:** (i)  $F.\text{reg.}\text{ext}(X)=F.\text{reg.}\text{int}(X')=F.\text{reg.}\text{int}(\varnothing)=\varnothing$   $F.\text{reg.}\text{ext}(\varnothing)=F.\text{reg.}\text{int}(\varnothing')$   
 $=F.\text{reg.}\text{int}(X)=X$

(ii)  $F.\text{reg.}\text{ext}(A=F.\text{reg.}\text{ext}(A'))\subset A'$ , by (iii) of remark 4.3

(iii)  $F.\text{reg.}\text{ext}(F.\text{reg.}\text{ext}(A))'=F.\text{reg.}\text{ext}((F.\text{reg.}\text{int}(A'))')=F.\text{reg.}\text{ext}(F.\text{reg.}\text{int}(A'))$   
 $=F.\text{reg.}\text{int}(((F.\text{reg.}\text{int}(A'))')=F.\text{reg.}\text{int}(F.\text{reg.}\text{int}(A'))=F.\text{reg.}\text{int}(F.\text{reg.}\text{int}(A'))$   
 $=F.\text{reg.}\text{int}(A')=F.\text{reg.}\text{ext}(A)$ . since  $A''=A$  for any set  $A$ .

(iv)  $A\subset B\Rightarrow B'\subset A'\Rightarrow F.\text{reg.}\text{int}(B')\subset(F.\text{reg.}\text{int}(A'))\Rightarrow F.\text{reg.}\text{ext}(B)\subset F.\text{reg.}\text{ext}(A)$

(v) By (ii), we have  $F.\text{reg.}\text{ext}(A)\subset A'$ . Then (iv) gives  $F.\text{reg.}\text{ext}(A')\subset F.\text{reg.}\text{ext}(F.\text{reg.}\text{ext}(A))$ .  
 But  $F.\text{reg.}\text{int}(A)=F.\text{reg.}\text{ext}(A')$ . Hence  $F.\text{reg.}\text{int}(A)\subset F.\text{reg.}\text{ext}(F.\text{reg.}\text{ext}(A))$ .

(vi)  $F.\text{reg.}\text{ext}(A\cup B)=F.\text{reg.}\text{int}(A\cup B)'=F.\text{reg.}\text{int}(A'\cap B')=F.\text{reg.}\text{ext}(A)\cap F.\text{reg.}\text{ext}(B)$ .

**Theorem 4.5:** Let  $A$  be any subset of a topological space  $X$ . Then  $F.\text{reg.}\text{int}(A)$ ,  $F.\text{reg.}\text{ext}(A)$  and  $F.\text{reg.}\text{Fr}(A)$  are disjoint and  $X=F.\text{reg.}\text{int}(A)\cup F.\text{reg.}\text{ext}(A)\cup F.\text{reg.}\text{Fr}(A)$ . Further  $F.\text{reg.}\text{Fr}(A)$  is a feebly regular closed set.

**Proof:** By definition,  $F.\text{reg.}\text{ext}(A)=F.\text{reg.}\text{int}(A')$ . Also  $F.\text{reg.}\text{int}(A)\subset A$  and  $F.\text{reg.}\text{int}(A')=A'$ . Since  $A\cap A'=\varnothing$ , it follows that  $F.\text{reg.}\text{int}(A)\cap F.\text{reg.}\text{ext}(A)=F.\text{reg.}\text{int}(A)\cap F.\text{reg.}\text{int}(A')=\varnothing$ .

Again by definition of feebly regular frontier, we have  $x\in F.\text{reg.}\text{Fr}(A)\Leftrightarrow x\notin F.\text{reg.}\text{int}(A)$  and  $x\notin F.\text{reg.}\text{ext}(A)\Leftrightarrow x\notin F.\text{reg.}\text{int}(A)\cup F.\text{reg.}\text{ext}(A)\Leftrightarrow x\in (F.\text{reg.}\text{int}(A)\cup F.\text{reg.}\text{ext}(A))'$ . Thus

$F.\text{reg.}\text{Fr}(A)\Leftrightarrow (F.\text{reg.}\text{int}(A)\cup F.\text{reg.}\text{ext}(A))'\rightarrow(1)$ . It follows that  $F.\text{reg.}\text{Fr}(A)\cap F.\text{reg.}\text{int}(A)=\varnothing$  and  $F.\text{reg.}\text{Fr}(A)\cap F.\text{reg.}\text{ext}(A)=\varnothing$  and  $X=F.\text{reg.}\text{int}(A)\cup F.\text{reg.}\text{ext}(A)\cup F.\text{reg.}\text{Fr}(A)$ . Since  $F.\text{reg.}\text{int}(A)$  and  $F.\text{reg.}\text{ext}(A)$  are open sets, we see from (1) that  $F.\text{reg.}\text{Fr}(A)$  is a feebly regular closed set.

## 5. Characterization of a topological space in terms of feebly regular interior and feebly regular exterior operators

**Definition 5.1:** Let  $(X,\tau)$  be a topological space. Then feebly regular interior operator on  $X$  is a function  $F.\text{reg.}i:P(X)\rightarrow P(X)$  satisfying the following conditions, usually known as feebly regular interior axioms: **(A<sub>1</sub>)** :  $F.\text{reg.}i(X)=X$ , **(A<sub>2</sub>)**:  $F.\text{reg.}i(A)\subset A$ , **(A<sub>3</sub>)**:  $F.\text{reg.}i(A\cap B)=F.\text{reg.}i(A)\cap F.\text{reg.}i(B)$  and **(A<sub>4</sub>)**:  $F.\text{reg.}i(F.\text{reg.}i(A))=F.\text{reg.}i(A)$  where  $A, B$  are subsets of  $X$ .

**Theorem 5.2:** Let  $X$  be any set and let  $F.\text{reg.}i$  the feebly regular interior operator on  $X$ , that is, a function  $F.\text{reg.}i:P(X)\rightarrow P(X)$  such that **(A<sub>1</sub>)** :  $F.\text{reg.}i(X)=X$  **(A<sub>2</sub>)**:  $F.\text{reg.}i(A)\subset A$  **(A<sub>3</sub>)**:  $F.\text{reg.}i(A\cap B)=F.\text{reg.}i(A)\cap F.\text{reg.}i(B)$  **(A<sub>4</sub>)**:  $F.\text{reg.}i(F.\text{reg.}i(A))=F.\text{reg.}i(A)$  for any subset  $A, B$  of  $X$ . Then there exists a unique topology on  $X$  such that for each  $A\subset X$ ,  $F.\text{reg.}i(A)$  coincides with  $\tau$ - $F.\text{reg.}$ interior of  $A$ .

**Proof:** Let  $\tau$  be the collection of all those subsets  $G$  of  $X$  for which  $F.\text{reg.}i(G)=G$ . Then we shall show that  $\tau$  is a topology on  $X$  such that the  $\tau$ - $F.\text{reg.}$ interior of  $A\subset X$  is  $F.\text{reg.}i(A)$ .

**[T<sub>1</sub>]:** By **(A<sub>1</sub>)**,  $F.\text{reg.}i(X)=X$ . Therefore  $X\in\tau$ . Also by **(A<sub>2</sub>)**  $F.\text{reg.}i(\varnothing)\subset\varnothing$ . But  $\varnothing\subset F.\text{reg.}i(\varnothing)$ . Hence  $F.\text{reg.}i(\varnothing)=\varnothing$ . Therefore  $\varnothing\in\tau$ .

**[T<sub>2</sub>]:** Let  $G_1\in\tau$  and  $G_2\in\tau$ . Then  $F.\text{reg.}i(G_1)=G_1$  and  $F.\text{reg.}i(G_2)=G_2\rightarrow(1)$ . By **(A<sub>3</sub>)**,  $F.\text{reg.}i(G_1\cap G_2)=F.\text{reg.}i(G_1)\cap F.\text{reg.}i(G_2)$ , by(1). Hence  $G_1\cap G_2\in\tau$ .

[T<sub>1</sub>]: By (A<sub>1</sub>),  $F.reg.i(X)=X$ . Therefore  $X \in \tau$ . Also by (A<sub>2</sub>)  $F.reg.i(\varnothing) \subset \varnothing$ . But  $\varnothing \subset F.reg.i(\varnothing)$ . Hence  $F.reg.i(\varnothing)=\varnothing$ . Therefore  $\varnothing \in \tau$ .

[T<sub>2</sub>]: Let  $G_1 \in \tau$  and  $G_2 \in \tau$ . Then  $F.reg.i(G_1)=G_1$  and  $F.reg.i(G_2)=G_2 \rightarrow (1)$ . By (A<sub>3</sub>),  $F.reg.i(G_1 \cap G_2)=F.reg.i(G_1) \cap F.reg.i(G_2)$ , by (1). Hence  $G_1 \cap G_2 \in \tau$ .

[T<sub>3</sub>]: We first show  $A \subset B \Rightarrow F.reg.i(A) \subset F.reg.i(B) \rightarrow (2)$ . Now  $A \subset B \Rightarrow A \cap B = A \Rightarrow F.reg.i(A \cap B) = F.reg.i(A) \Rightarrow F.reg.i(A) \cap F.reg.i(B) = F.reg.i(A) \Rightarrow F.reg.i(A) \subset F.reg.i(B)$ . by (A<sub>3</sub>). Now let  $G_\lambda \in \tau$  for every  $\lambda \in \Lambda$  where  $\Lambda$  is an arbitrary set. Then by definition of  $\tau$ ,  $F.reg.i(G_\lambda)=G_\lambda$  for every  $\lambda \in \Lambda \rightarrow (3)$ . By (A<sub>2</sub>),  $F.reg.i(\cup \{G_\lambda : \lambda \in \Lambda\}) \subset \cup \{G_\lambda : \lambda \in \Lambda\} \rightarrow (4)$ . To prove the other way inclusion, we have  $G_\lambda \subset \cup \{G_\lambda : \lambda \in \Lambda\}$  for every  $\lambda \in \Lambda \Rightarrow F.reg.i(G_\lambda) \subset F.reg.i(\cup \{G_\lambda : \lambda \in \Lambda\})$  for every  $\lambda \in \Lambda \Rightarrow G_\lambda \subset F.reg.i(\cup \{G_\lambda : \lambda \in \Lambda\})$  for every  $\lambda \in \Lambda$  by (3)  $\Rightarrow \cup \{G_\lambda : \lambda \in \Lambda\} \subset F.reg.i(\cup \{G_\lambda : \lambda \in \Lambda\}) \rightarrow (5)$ . From (4) and (5), we get  $F.reg.i(\cup \{G_\lambda : \lambda \in \Lambda\}) = \cup \{G_\lambda : \lambda \in \Lambda\}$ . Hence  $\cup \{G_\lambda : \lambda \in \Lambda\} \in \tau$ . It follows that  $\tau$  is a topological on  $X$ . We shall now show that the  $\tau$ - $F.reg.$ interior of any subset  $A$  of  $X$  is  $F.reg.i(A)$ . Since  $F.reg.i(F.reg.i(A))=F.reg.i(A)$ , we have  $F.reg.i(A) \in \tau$ . Also by (A<sub>2</sub>),  $F.reg.i(A) \subset A$ . Thus  $F.reg.i(A)$  is a  $\tau$ -feebly regular open set contained in  $A$ . Let  $B$  be any  $\tau$ -feebly regular open subset of  $A$ . Since  $B \in \tau$ ,  $F.reg.i(B)=B$  and hence  $B \subset A$ , we have  $F.reg.i(B) \subset F.reg.i(A)$ . It follows that  $B \subset F.reg.i(A)$ . thus  $F.reg.i(A)$  contains any feebly regular open set contained in  $A$  so that  $F.reg.i(A)$  is the largest feebly regular open set contained in  $A$  so that  $F.reg.i(A)$  is the largest feebly regular open set contained in  $A$ . Therefore  $\tau$ -feebly interior of  $A=F.reg.i(A)$ .

**Definition 5.3:** Let  $(X, \tau)$  be a topological space. Then feebly regular exterior operator on  $X$  is a function  $F.reg.e:P(X) \rightarrow P(X)$  satisfying the following conditions, usually know as feebly regular exterior axioms: (E<sub>1</sub>) :  $F.reg.e(X)=\varnothing$ ,  $F.reg.e(\varnothing)=X$  (E<sub>2</sub>):  $F.reg.e(A) \subset A'$ , (E<sub>3</sub>):  $F.reg.e(A)=F.reg.(F.reg.(A))'$  and (E<sub>4</sub>):  $F.reg.e(A \cup B)=F.reg.e(A) \cap F.reg.e(B)$  where  $A, B$  are subsets of  $X$ .

**Theorem 5.4:** Let  $X$  be any set and let  $F.reg.e$  be feebly regular exterior operator on  $X$ , that is, a function  $F.reg.e:P(X) \rightarrow P(X)$  such that (E<sub>1</sub>) :  $F.reg.e(X)=\varnothing$ ,  $F.reg.e(\varnothing)=X$  (E<sub>2</sub>):  $F.reg.e(A) \subset A'$  (E<sub>3</sub>):  $F.reg.e(A)=F.reg.(F.reg.(A))'$  (E<sub>4</sub>):  $F.reg.e(A \cup B)=F.reg.e(A) \cap F.reg.e(B)$  for any subsets  $A, B$  of  $X$ . then there exists a unique topology on  $X$  such that for each  $A \subset X$ ,  $F.reg.e(A)$  coincides with  $\tau$ - $F.reg.$ exterior of  $A$ .

## 6. Relation between feebly regular closure, feebly regular interior and feebly regular frontier

**Theorem 6.1:** Let  $(X, \tau)$  be a topological space and let  $A \subset X$ . Then

- (i)  $F.reg.int(A)=(F.reg.cl(A'))'$
- (ii)  $F.reg.cl(A')=(F.reg.int(A))'$
- (iii)  $F.reg.cl(A)=(F.reg.int(A'))'$

**Proof:** (i) By using corollary 3.17, the result (i) follows.

(ii) Taking complements in (i),  $(F.reg.int(A))'=(F.reg.cl(A'))''=F.reg.cl(A')$ . Taking complements again,  $(F.reg.int(A))'=(F.reg.cl(A'))'$ . That is,  $F.reg.int(A)=(F.reg.cl(A'))'$ . Since  $S''=S$  for any set  $S$ .

(iii) By (ii)  $F.reg.cl(A')=(F.reg.int(A))'$ . Replacing  $A$  by  $A'$  in this, we get  $F.reg.cl(A'')$



$= (F.\text{reg.int}(A'))' \text{ or } F.\text{reg.cl}(A'') = (F.\text{reg.int}(A'))'$ . Hence  $F.\text{reg.cl}(A) = (F.\text{reg.int}(A'))'$ .

**Theorem 6.2:** Let  $(X, \tau)$  be a topological space and let  $B \subset X$ . Then

$$F.\text{reg.cl}(A) = F.\text{reg.int}(A) \cup F.\text{reg.Fr}(A).$$

**Proof :** By definition of  $F.\text{reg.cl}(A)$ , we have

$F.\text{reg.cl}(A) = \bigcap \{F : F \text{ is feebly regular closed and } F \supset A\}$ . Then by De-Morgan law.  $(F.\text{reg.cl}(A))' = \bigcup \{F' : F' \text{ is feebly regular open and } F' \subset A'\} = F.\text{reg.ext}(A)$ . Taking complements, we get  $(F.\text{reg.cl}(A))'' = (F.\text{reg.ext}(A))' = F.\text{reg.int}(A) \cup F.\text{reg.Fr}(A)$ , by theorem 4.5. Hence  $F.\text{reg.cl}(A) = F.\text{reg.int}(A) \cup F.\text{reg.Fr}(A)$ .

**Corollary 6.3:**  $F.\text{reg.Fr}(F.\text{reg.cl}(A)) \subset A$ .

**Corollary 6.4:**  $F.\text{reg.cl}(A) = A \cup F.\text{reg.Fr}(A)$ .

**Proof:** Since  $A \subset F.\text{reg.cl}(A)$  and  $F.\text{reg.Fr}(A) \subset F.\text{reg.cl}(A)$ ,

we have  $A \cup F.\text{reg.Fr}(A) \subset F.\text{reg.cl}(A) \rightarrow (1)$ . Also  $F.\text{reg.Fr}(A) = (F.\text{reg.int}(A) \cup F.\text{reg.ext}(A))'$

$= (F.\text{reg.int}(A))' \cap (F.\text{reg.ext}(A))'$ . Again since  $F.\text{reg.int}(A) \subset A$  and

$F.\text{reg.cl}(A) = F.\text{reg.int}(A) \cup F.\text{reg.Fr}(A)$ , it follows that  $F.\text{reg.cl}(A) \subset A \cup F.\text{reg.Fr}(A) \rightarrow (2)$ . From (1) and (2), we get  $F.\text{reg.cl}(A) = A \cup F.\text{reg.Fr}(A)$ .

**Theorem 6.5:** Every feebly regular closed subset of a topological space is the disjoint union of its feebly regular interior and feebly regular frontier.

**Proof:** Let  $A$  be a feebly regular closed subset of a topological space  $X$ , so that  $F.\text{reg.cl}(A) = A$ . Hence by theorem 6.2,  $A = F.\text{reg.int}(A) \cup F.\text{reg.Fr}(A)$ . Also we get  $F.\text{reg.int}(A) \cap F.\text{reg.Fr}(A) = \emptyset$ .

**Theorem 6.6:** Let  $(X, \tau)$  be a topological space and let  $A, B$  be subset of  $X$ . Then

$$(i) F.\text{reg.Fr}(A) = F.\text{reg.cl}(A) \cap A' - F.\text{reg.int}(A)$$

$$(ii) F.\text{reg.int}(A) = A - F.\text{reg.Fr}(A)$$

$$(iii) (F.\text{reg.Fr}(A))' = F.\text{reg.int}(A) \cup F.\text{reg.int}(A')$$

$$(iv) F.\text{reg.Fr}(F.\text{reg.int}(A)) \subset F.\text{reg.Fr}(A)$$

$$(v) F.\text{reg.Fr}(F.\text{reg.cl}(A)) \subset F.\text{reg.Fr}(A)$$

$$(vi) F.\text{reg.Fr}(A \cup B) \subset F.\text{reg.Fr}(A) \cup F.\text{reg.Fr}(B)$$

$$(vii) F.\text{reg.Fr}(A \cap B) \subset F.\text{reg.Fr}(A) \cup F.\text{reg.Fr}(B)$$

**Proof:** (i) We have  $F.\text{reg.Fr}(A) = (F.\text{reg.int}(A) \cup F.\text{reg.ext}(A))'$

$= (F.\text{reg.int}(A))' \cap (F.\text{reg.ext}(A))'$  by De-Morgan law

$$= (F.\text{reg.cl}(A'))' \cap (F.\text{reg.cl}(A))''$$

$$= (F.\text{reg.cl}(A'))' \cap (F.\text{reg.cl}(A)), \text{ by 3.17. Now } F.\text{reg.cl}(A) \cap F.\text{reg.cl}(A')$$

$$= F.\text{reg.cl}(A) - (F.\text{reg.cl}(A'))' = F.\text{reg.cl}(A) - F.\text{reg.int}(A), \text{ by 3.17. Hence}$$

$$F.\text{reg.Fr}(A) = F.\text{reg.cl}(A) \cap A = F.\text{reg.cl}(A) - F.\text{reg.int}(A).$$

$$(ii) A - F.\text{reg.Fr}(A) = A - (F.\text{reg.cl}(A) - F.\text{reg.int}(A)), \text{ by (i)}$$

$$= F.\text{reg.int}(A) \text{ since } F.\text{reg.int}(A) \subset A$$

$$(iii) \text{ We have } (F.\text{reg.Fr}(A))' = (F.\text{reg.cl}(A) - F.\text{reg.int}(A))' \text{ by (i)}$$

$$= F.\text{reg.cl}(A') \cup (F.\text{reg.cl}(A'))' \text{ using De-Morgan law and by corollary 3.17,}$$

$$(F.\text{reg.cl}(A'))' = F.\text{reg.int}(A) \text{ and so } F.\text{reg.int}(A') = (F.\text{reg.cl}(A'))' = (F.\text{reg.cl}(A'))' = (F.\text{reg.cl}(A))'$$

$$\text{since } A'' = A. \text{ Therefore } (F.\text{reg.Fr}(A))' = F.\text{reg.int}(A') \cup F.\text{reg.int}(A) = F.\text{reg.int}(A) \cup F.\text{reg.int}(A').$$

(iv)  $F.\text{reg. Fr}((F.\text{reg. int}(A)) = F.\text{reg. cl}(F.\text{reg. int}(A)) \cap F.\text{reg. cl}(F.\text{reg. int}(A')), \text{ by (i)}$   
 $= F.\text{reg. cl}(F.\text{reg. int}(A)) \cap F.\text{reg. cl}(F.\text{reg. cl}(A'))'$   
 $= F.\text{reg. cl}(F.\text{reg. int}(A)) \cap F.\text{reg. cl}(A') \subset F.\text{reg. cl}(A) \cap F.\text{reg. cl}(A')$   
 $= F.\text{reg. Fr}(A) \text{ by (i). Thus } F.\text{reg. Fr}(F.\text{reg. int}(A)) \subset F.\text{reg. Fr}(A).$   
(v)  $F.\text{reg. Fr}(F.\text{reg. cl}(A)) = F.\text{reg. cl}(A) \cap F.\text{reg. cl}(F.\text{reg. cl}(A')), \text{ by (i)}$   
 $= F.\text{reg. cl}(F.\text{reg. cl}(A)) \cap F.\text{reg. cl}(F.\text{reg. cl}(A'))$ . Now  $A \subset F.\text{reg. cl}(A)$   
 $\Rightarrow F.\text{reg. cl}(F.\text{reg. cl}(A')) \subset F.\text{reg. cl}(A')$ . Hence  
 $F.\text{reg. Fr}(A) \subset F.\text{reg. cl}(A) \cap F.\text{reg. cl}(A') = F.\text{reg. Fr}(A).$   
(vi)  $F.\text{reg. Fr}(A \cap B) = F.\text{reg. cl}(A \cap B) \cap F.\text{reg. cl}(A \cup B)', \text{ by (i)}$   
 $= (F.\text{reg. cl}(A) \cup F.\text{reg. cl}(B)) \cap (A' \cap B A'), \text{ by using De-Morgan law}$   
 $= (F.\text{reg. int}(A))' \cap (F.\text{reg. ext}(A))'$ . Again since  $F.\text{reg. int}(A) \subset A$  and  
 $F.\text{reg. cl}(A) = F.\text{reg. int}(A) \cup F.\text{reg. Fr}(A)$ , it follows that  $F.\text{reg. cl}(A) \subset A \cup F.\text{reg. Fr}(A) \rightarrow (2)$ . From (1)  
and (2), we get  $F.\text{reg. cl}(A) = A \cup F.\text{reg. Fr}(A)$ .

**Theorem 6.5:** Every feebly regular closed subset of a topological space is the disjoint union of its feebly regular interior and feebly regular frontier.

**Proof:** Let  $A$  be a feebly regular closed subset of a topological space  $X$ , so that  $F.\text{reg. cl}(A) = A$ . Hence by theorem 6.2,  $A = F.\text{reg. int}(A) \cup F.\text{reg. Fr}(A)$ . Also we get  $F.\text{reg. int}(A) \cap F.\text{reg. Fr}(A) = \emptyset$ .

**Theorem 6.6:** Let  $(X, \tau)$  be a topological space and let  $A, B$  be subset of  $X$ . Then

- (i)  $F.\text{reg. Fr}(A) = F.\text{reg. cl}(A) \cap A' - F.\text{reg. int}(A)$
- (ii)  $F.\text{reg. int}(A) = A - F.\text{reg. Fr}(A)$
- (iii)  $(F.\text{reg. Fr}(A))' = F.\text{reg. int}(A) \cup F.\text{reg. int}(A')$
- (iv)  $F.\text{reg. Fr}(F.\text{reg. int}(A)) \subset F.\text{reg. Fr}(A)$
- (v)  $F.\text{reg. Fr}(F.\text{reg. cl}(A)) \subset F.\text{reg. Fr}(A)$
- (vi)  $F.\text{reg. Fr}(A \cup B) \subset F.\text{reg. Fr}(A) \cup F.\text{reg. Fr}(B)$
- (vii)  $F.\text{reg. Fr}(A \cap B) \subset F.\text{reg. Fr}(A) \cup F.\text{reg. Fr}(B)$

**Proof:** (i) We have  $F.\text{reg. Fr}(A) = (F.\text{reg. int}(A) \cup F.\text{reg. ext}(A))'$   
 $= (F.\text{reg. int}(A))' \cap (F.\text{reg. ext}(A))'$  by De-Morgan law  
 $= (F.\text{reg. cl}(A'))'' \cap (F.\text{reg. cl}(A))''$   
 $= (F.\text{reg. cl}(A'))'' \cap (F.\text{reg. cl}(A))$ , by 3.17. Now  $F.\text{reg. cl}(A) \cap F.\text{reg. cl}(A')$   
 $= F.\text{reg. cl}(A) - (F.\text{reg. cl}(A'))' = F.\text{reg. cl}(A) - F.\text{reg. int}(A)$ , by 3.17. Hence  
 $F.\text{reg. Fr}(A) = F.\text{reg. cl}(A) \cap A = F.\text{reg. cl}(A) - F.\text{reg. int}(A).$   
(ii)  $A - F.\text{reg. Fr}(A) = A - (F.\text{reg. cl}(A) - F.\text{reg. int}(A))$ , by (i)  
 $= F.\text{reg. int}(A)$  since  $F.\text{reg. int}(A) \subset A$   
(iii) We have  $(F.\text{reg. Fr}(A))' = (F.\text{reg. cl}(A) - F.\text{reg. int}(A'))$  by (i)  
 $= F.\text{reg. cl}(A') \cup (F.\text{reg. cl}(A'))'$  using De-Morgan law and by corollary 3.17,  
 $(F.\text{reg. cl}(A'))' = F.\text{reg. int}(A)$  and so  $F.\text{reg. int}(A') = (F.\text{reg. cl}(A'))' = (F.\text{reg. cl}(A''))' = (F.\text{reg. cl}(A))'$   
since  $A'' = A$ . Therefore  $(F.\text{reg. Fr}(A))' = F.\text{reg. int}(A') \cup F.\text{reg. int}(A) = F.\text{reg. int}(A) \cup F.\text{reg. int}(A')$ .  
(iv)  $F.\text{reg. Fr}(F.\text{reg. int}(A)) = F.\text{reg. cl}(F.\text{reg. int}(A)) \cap F.\text{reg. cl}(F.\text{reg. int}(A'))$ , by (i)  
 $= F.\text{reg. cl}(F.\text{reg. int}(A)) \cap F.\text{reg. cl}(F.\text{reg. cl}(A'))'$   
 $= F.\text{reg. cl}(F.\text{reg. int}(A)) \cap F.\text{reg. cl}(A') \subset F.\text{reg. cl}(A) \cap F.\text{reg. cl}(A')$   
 $= F.\text{reg. Fr}(A) \text{ by (i). Thus } F.\text{reg. Fr}(F.\text{reg. int}(A)) \subset F.\text{reg. Fr}(A).$



(v)  $F.reg.Fr(F.reg.cl(A))=F.reg.cl(A)\cap F.reg.cl(F.reg.cl(A'))$ , by (i)  
 $=F.reg.cl(F.reg.cl(A))\cap F.reg.cl(F.reg.cl(A'))$ . Now  $A\subset F.reg.cl(A)$   
 $\Rightarrow F.reg.cl(F.reg.cl(A'))\subset F.reg.cl(A')$ . Hence  
 $F.reg.Fr(A)\subset F.reg.cl(A)\cap F.reg.cl(A')=F.reg.Fr(A)$ .

(vi)  $F.reg.Fr(A\cap B)=F.reg.cl(A\cap B)\cap F.reg.cl(A\cup B)'$ , by (i)  
 $=(F.reg.cl(A)\cup F.reg.cl(B))\cap (A'\cap B')$ , by using De-Morgan law  
 $\subset (A\cup B) \cap (F.reg.cl(A') \cap F.reg.cl(B'))$   
 $= F.reg.cl(A)\cap (F.reg.cl(A')\cap F.reg.cl(B')) \cup (F.reg.cl(B) \cap (F.reg.cl(A')\cap F.reg.cl(B'))$   
 $= ((F.reg.cl(A) \cap (F.reg.cl(A')\cap F.reg.cl(B')))\cup ((F.reg.cl(B)\cap (F.reg.cl(A')\cap F.reg.cl(B'))$   
 $= (F.reg.Fr(A)\cap F.reg.cl(B'))\cup (F.reg.Fr(B)\cap A')$ , by (i)  $\subset F.reg.Fr(A)\cup F.reg.Fr(B)$   
(vii)  $F.reg.Fr(A\cap B)=F.reg.cl(A\cap B)\cap F.reg.cl(A\cap B)'$ , by (i)  
 $\subset (F.reg.cl(A)\cap F.reg.cl(B))\cap F.reg.cl(A'\cup B')=(F.reg.cl(A)\cap F.reg.cl(B))\cap (F.reg.cl(A')\cup F.reg.cl(B'))$   
 $= (F.reg.Fr(A)\cap F.reg.cl(B))\cup (F.reg.cl(A)\cap F.reg.Fr(B))$ , by (i)  
 $\subset F.reg.Fr(A)\cup F.reg.Fr(B)$ .

**Remark 6.7:** Let  $A$  be subset of a topological space  $X$ . Then

- (i) if  $A$  is feebly regular open then  $F.reg.Fr(A)=F.reg.cl(A)-A$ .
- (ii)  $F.reg.Fr(A)=\emptyset$  if and only if  $A$  is as well as feebly regular closed.
- (iii)  $A$  is feebly regular open if and only if  $A\cap F.reg.Fr(A)=\emptyset$ , that is, if and only if  $F.reg.Fr(A)\subset A'$ .
- (iv)  $A$  is feebly regular closed if and only if  $F.reg.Fr(A)\subset A$ .

**Conclusion 6.8:** The concepts of full paper are analyzed in an example. Let  $X=\{a,b,c,d,e\}$  be a topological space with topology  $\tau=\{\emptyset, \{b\}, \{c,d\}, \{b,c,d\}, \{a,c,d\}, \{a,b,c,d\}, X\}$ . We know that the closure of the subset  $A$  of the topological space  $X$  is the intersection of all closed sets containing  $A$  and the interior of the subset of the topological space  $X$  is the union of all open sets contained in  $A$ . That is the subset  $A$  of  $(X,\tau)$ ,  $cl(A)=\cap\{\text{all closed sets } \supset A\}$  and  $int(A)=\cup\{\text{all open sets } \subseteq A\}$ . Also we know that the members of  $\tau$  are open sets. Here open sets are  $\emptyset, \{b\}, \{c,d\}, \{b,c,d\}, \{a,c,d\}, \{a,b,c,d\}, X$  and closed sets are  $X, \{a,c,d,e\}, \{a,b,e\}, \{a,e\}, \{b,e\}, \{e\}, \emptyset$ . The definition of the semi open and semi closed sets are  $A\subset cl(int(A))$  and  $int(cl(A))\subset A$ . Here  $A$  is the subset of  $X$ . Here the subsets of  $(X,\tau)$  are  $\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{e\}, \{a,b\}, \{b,c\}, \{c,d\}, \{d,e\}, \{a,e\}, \{a,c\}, \{a,d\}, \{b,e\}, \{c,e\}, \{a,b,c\}, \{b,c,d\}, \{c,d,e\}, \{a,d,e\}, \{a,c,d\}, \{b,d,e\}, \{b,c,e\}, \{a,b,e\}, \{a,b,d\}, \{a,b,c,d\}, \{a,c,d,e\}, \{a,b,d,e\}, \{a,b,c,e\}, \{b,c,d,e\}, X$ . First we take the subsets are  $\emptyset, \{a\}$ .  $int(\{\emptyset\})=\cup\{\{\emptyset\}\subseteq\{\emptyset\}\}=\{\emptyset\}$  and  $cl(\{\emptyset\})=\cap\{\text{all closed sets of } X\supset\{\emptyset\}\}=\emptyset$ . That is  $cl(int(\{\emptyset\}))=\emptyset$ . This implies that  $\{\emptyset\}\subset cl(int(\{\emptyset\}))=\{\emptyset\}\Rightarrow\{\emptyset\}\subset\{\emptyset\}$ . Therefore  $\{\emptyset\}$  is semi open set of  $(X,\tau)$ . Now  $int(\{a\})=\cup\{\{\emptyset\}\subseteq\{a\}\}=\{\emptyset\}$  and  $cl(\{a\})=\cap\{\text{all closed sets of } X\supset\{a\}\}=\{\emptyset\}$ . That is  $cl(int(\{a\}))=\{\emptyset\}$ . This implies that  $\{a\}\not\subset\{\emptyset\}$ . Therefore  $\{a\}$  is not a semi open sets of  $X$ . Proceeding in this way we get, semi open sets are  $\{\emptyset\}, \{b\}, \{d\}, \{c,d\}, \{b,e\}, \{b,c,d\}, \{a,c,d\}, \{a,b,c,d\}, \{a,c,d,e\}, \{b,c,d,e\}, X$ . The complement of semi open sets, that is semi closed sets of  $X$ ,  $\{a,c,d,e\}, \{a,b,c,e\}, \{a,b,e\}, \{a,c,d\}, \{a,e\}, \{b,e\}, \{e\}, \{b\}, \{a\}, \{\emptyset\}$ . Here  $\{a,c,d\}, \{a,c,d,e\}, \{b\}$  are semi clopen sets in  $(X,\tau)$ . Feebly open sets and feebly closed sets are defined  $A\subset s\,cl(int(A))$  and  $s\,int\,cl(A)\subset A$ . We gets, these sets are  $\{\emptyset\}, \{b\}, \{c,d\}, \{b,c,d\}, \{a,c,d\}, \{a,b,c,d\}, \{b,c,d,e\}, X$  and  $X, \{a,c,d,e\}, \{a,b,e\}, \{a,e\}, \{b,e\}, \{e\}, \{a\}, \{\emptyset\}$ . And also we work out that feebly regular open and feebly regular closed sets.

The definitions of these are  $A=f.int(f.cl(A))$  and  $A=f.cl(f.int(A))$  and the sets are  $\{\varnothing\}$ ,  $\{b\}$ ,  $\{a,c,d\}$ ,  $X$  and  $X$ ,  $\{a,c,de\}$ ,  $\{b,e\}$ ,  $\{\varnothing\}$ . Another results are analyzed (i) open sets  $\longrightarrow$  semi open  
Feebly open  $\nearrow$

(ii) every feebly regular open set (resp. feebly regular closed) is open (resp. closed). In the following table we find feebly regular interior, feebly regular exterior and feebly regular frontier for a subset A of X.

Subset A of X is	F.reg.int(A) is	F.reg.ext(A) is	F.reg.Fr(A) is
{a}	{j}	{b}	{a,c,d,e}
{b}	{b}	{a,c,d}	{e}
{c}	{j}	{b}	{a,c,d,e}
{d}	{j}	{b}	{a,c,d,e}
{e}	{j}	{a,b,c,d}	{e}
{a,b}	{b}	{j}	{a,c,d,e}
{b,c}	{b}	{j}	{a,c,d,e}
{c,d}	{j}	{b}	{a,c,d,e}
{d,e}	{j}	{b}	{a,c,d,e}
{a,e}	{j}	{b}	{a,c,d,e}
{a,c}	{j}	{b}	{a,c,d,e}
{a,d}	{j}	{b}	{a,c,d,e}
{b,e}	{b}	{a,c,d}	{e}
{c,e}	{j}	{b}	{a,c,d,e}
{a,b,c}	{b}	{j}	{a,c,d,e}
{b,c,d}	{b}	{j}	{a,c,d,e}
{c,d,e}	{j}	{b}	{a,c,d,e}
{a,d,e}	{j}	{b}	{a,c,d,e}
{a,c,d}	{a,c,d}	{b}	{e}
{b,d,e}	{b}	{j}	{a,c,d,e}
{b,c,e}	{b}	{j}	{a,c,d,e}
{a,b,e}	{b}	{j}	{a,c,d,e}
{a,b,d}	{b}	{j}	{a,c,d,e}
{a,b,c,d}	{a,b,c,d}	{j}	{e}
{a,c,d,e}	{a,c,d}	{b}	{e}
{a,b,d,e}	{b}	{j}	{a,c,d,e}
{a,b,c,e}	{b}	{j}	{a,c,d,e}
{b,c,d,e}	{b}	{j}	{a,c,d,e}
{a,b,c,d,e}	{a,b,c,d,e}	{j}	-
{j}	{j}	X	-

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