Research Paper Mathematics

ON TERNARY QUADRATIC EQUATION $11x^2+3y^2=14z^2$

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ABSTRACT

The Ternary quadratic Diophantine equation given by $11x^2+3y^2=14z^2$ is analyzed for its pattern of non-zero integral solutions. A few interesting relations between the solutions and special polygonal numbers are exhibited

KEYWORDS Ternary, Quadratic, integral solutions, special numbers

Introduction:
The ternary of Diophantine equation offers an unlimited field for research because of their varìely [1,2]. For an extensive review of various problems, one may refer [3-23]. This communication concerns with yet another interesting ternary quadratic equation $11x^2+3y^2=14z^2$ for determining its infinitely many non-zero distinct integral solutions. Also a few interesting relations among the solutions have been presented.

Notations used:

- $T_{3,n}$-Triangular number of rank n
- P_n^3 -Tetrahedral number of rank n
- P_n^4 -Square pyramidal number of rank n
- P_n^5-Pentagonal pyramidal number of rank n

Method of Analysis
The Ternary Quadratic Diophantine Equation to be solved for its non-zero distinct integral solution is

$$11x^2 + 3y^2 = 14z^2 \quad (1)$$

Pattern – I

On substitution of linear transformations $(u \neq v \neq 0)$

$$x = u + 3v, y = u - 11v \quad (2)$$

In (1) leads to $u^2 + 33v^2 = z^2$ (3)

The corresponding solutions of (3) is the form

$$\left.\begin{aligned} z &= 33m^2 + n^2 \\ v &= 2mn \\ u &= 33m^2 - n^2 \end{aligned}\right\} \quad (4)$$

In view of (4), the solution of (1) can be written as

$$x = 33m^2 - n^2 + 6mn$$

$$y = 33m^2 - n^2 - 22mn$$

$$z = 33m^2 + n^2$$

Instead of (2) using the transformations $x = u - 3v$, $y = u + 11v$, in (1), we get again (3) only, Thus, the integer solutions of (1) are obtained as

$$x = 33m^2 - n^2 - 6mn$$

$$y = 33m^2 - n^2 + 22mn$$

$$z = 33m^2 + n^2$$

A few interesting properties observed are as follows:

(i) $x(m,1) - 2t_{3,n} \equiv -3 \pmod 5$

(ii) $y(1,n) - 44t_{3,n} \equiv 1 \pmod{11}$

(iii) $y(m,1) - 2t_{3,n} \equiv 10 \pmod{23}$

(iv) $y(2,n) + 66t_{3,n} \equiv 4 \pmod{11}$

(v) $x(m,2) - 2t_{3,n} \equiv 0 \pmod{11}$

(vi) $y(m,2) - 2t_{3,n} \equiv 42 \ (mod\ 45)$

(vii) $x(m,3) - 2t_{3,n} \ -11m \equiv 0 \ (mod\ 3)$

(viii) $y(m,3) - 2t_{3,n} \equiv 29 \ (mod\ 67)$

(ix) $y(3,n) + 66t_{3,n} \equiv 9 \ (mod\ 33)$

Pattern – II

Equation (3) is equivalent to

$u^2 = z^2 - 33v^2$

Assume that $u = a^2 - 33b^2$ (5)

Substituting (5) in the above equation

$(z + \sqrt{33}\,v) - (z - \sqrt{33}\,v) = (a + \sqrt{33}\,b)^2 (a - \sqrt{33}\,b)^2$ (6)

Equating the rational and irrational factors in (6), we get

$z = z\,(a,b) = a^2 + 33b^2$

$v = v(a,b) = 2ab$

From which we obtained

$x = a^2 + 6ab \ - 33b^2$

$y = a^2 - 33b^2 - 22ab$

$z = a^2 + 33b^2$

A few interesting properties observed are as follows

1. $x\,(A, A+1) - y\,(A, A+1) - \ 56T_{3,A}$

2. $x\,(A, (A+1)(A+2)) - 168P_A^3 \equiv 0$

3. $y\,(A, A(A+1) - x(A, (A+1)) - P_A^5 \equiv 0$

4. Each of the following expression represents a nasty numbers.

a) $x\,(a,b) - y(a,b)$

b) $y\,(a,b) - z\,(a,b)$

Pattern – III

Equation (3) can be written as

$u^2 + 33v^2 \ = \ z^2 \ * 1$ (7)

Assume that $z = a^2 + \ 33b^2$ (8)

Write 1 as 1

$$= \frac{(4 + i\,\sqrt{33})(4 - i\sqrt{33})}{49} \quad (9)$$

Use (8) and (9) in (7) and employing the method of factorization. Define

$(u + i\sqrt{33}\,v)\,(u - i\sqrt{33}\,v)$

$$= \frac{(4 + i\,\sqrt{33})(4 - i\sqrt{33})}{49}$$

$u + i\sqrt{33}\,v = \frac{1}{7}\{(4 + i\sqrt{33})(a + i\sqrt{33}b)^2\}$ (10)

Equating the real and imaginary parts in (10)

$u = \frac{1}{7}(4a^2 - 66ab - 132b^2)$ (11)

$v = \frac{1}{7}(a^2 + 8ab - 33b^2)$ (12)

Our interest is to obtain the integer solutions, so that the values of u and v are integers for suitable choices of the parameters a and b.

(iv) $y\,(5,B) + 66t_{3,n} \equiv -25\ (mod\ 143)$

(v) $y\,(A,6) - 2t_{3,n} \equiv 33\ (mod\ 35)$

Conclusion

In this paper we have presented three different patterns of non zero distinct integer solutions of the ternary quadratic equation given by $11x^2 + 3y^2 = 14z^2$ To conclude, one may search for other patterns of solutions and their corresponding properties.

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