



Mathematical Model of Piston of Four Stroke Diesel Engine

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ABSTRACT

This thesis work is concerned with temperature distribution and thermal stress [1] analysis in a four stroke diesel engine piston. The finite element approach is applied for the above analysis. The piston selected for the above objective is having diameter 195 mm. Although temperature of hot gases (T_g) and coefficient of heat transfer (h_g) between hot gases and piston top surface vary cyclically, but the problem has been simplified by giving them constant values.

KEYWORDS

Diesel engine, Mathematical model, Piston, FEM, Thermal Stress

STATEMENT OF THE PROBLEMS

The instantaneous values of heat transfer coefficient H_g on the gases face at crank angle (ϕ) is given by Eichelberg[2] as:

$$H_g(\phi) = 2.44(S)^{1/3} [P_g(\phi) T_g(\phi)]^{0.5} \text{ W/m}^2\text{K}$$

Where,

S = the mean piston speed m/s

$P_g(\phi)$ = gas pressure in N/m²

$T_g(\phi)$ = gas temperature in °C

Where ϕ denotes the crank angle measured from TDC position as '0'.

The mean heat transfer coefficient over the cycle is given by:

$$H_{gm} = (1/\phi_0) \int H_g(\phi) d\phi$$

The resultant gas temperature is given by:

$$T_{gr} = (1/h_{gm}) \int H_g(\phi) T_g(\phi) d\phi$$

Φ = angle covered

The value of the H_{gm} and T_{gm} are function of the break mean effective pressure and the load on the engine.

Here piston is of Aluminum. Piston ring is the cast steel and cylinder wall is of cast iron. The cooling water temperature and air temperature are 85°C, 80°C respectively. The coefficient of heat transfer at cooling water side and air side of the piston are 1860 W/m²k and 175 W/m²k respectively.

The constant heat transfer coefficient between the piston and ring is given as:

$$h_{c1} = 35000 \text{ W/m}^2\text{k}, h_{c2} = 20 \text{ W/m}^2\text{k}, h_{c3} = 5810 \text{ W/m}^2\text{k}, h_{c4} = 5810 \text{ W/m}^2\text{k}, h_{c5} = 1745 \text{ W/m}^2\text{k}$$

The present analysis has been carried out at four different

loading given as:

(i) $T_g = 1000^\circ\text{C}$, $h_g = 290 \text{ W/m}^2\text{k}$ (ii) $T_g = 800^\circ\text{C}$, $h_g = 262 \text{ W/m}^2\text{k}$

(iii) $T_g = 600^\circ\text{C}$, $h_g = 232 \text{ W/m}^2\text{k}$ iv) $T_g = 400^\circ\text{C}$, $h_g = 230 \text{ W/m}^2\text{k}$

1.2 METHOD OF SOLUTION

The problem is solved by Finite Element Analysis [3] method. For solution of the problem, the piston along with the rings and wall has been divided into 196 tri-angular elements having 164 nodes. The breakup of the elements and nodes for different part is as given below:

Part	Element		Nodes	
	From	To	From	To
Piston body	1	151	1	107
Piston ring	152	161	108	127
Cylinder wall	162	196	128	164

This is the case of two dimensional steady state heat conduction problems in a cylindrical surface. The co-ordinate has been found for each node. The analysis presented in this paper is divided into two sections, the field distribution and the thermal stresses. The Finite Element Technique with tri-angular element is used to reduce the variational formulation [4] to a set of algebraic equations. The expressions to calculate nodal temperature and the corresponding thermal stresses at every element are divided. The construction of finite element approach starts from the variational statement of the problem and then using proper shape function a number of algebraic equations are developed which equal the number of nodal elements in the problem domain. The necessary boundary conditions imposed are convective on all the three sides' i. e. air, water and gas side of the piston and the contact boundary condition between piston ring clearances. Then the principle of minimization of variational integral is carried out and the elemental equations developed above are summed up in order to get a set of simultaneous equations for whole of the

piston body. These simultaneous equations are solved by using computer. Computational algorithm and a FORTRAN program code is developed to solve these equations in order to find the unknown parameters i. e. temperature at different nodal points [5] of the piston.

Heat balance [6] of the problem is checked for accuracy and satisfied with result by observing that the amount of heat supplied at gas side of the piston is equal to the amount of heat loss to both water and air side of the piston. Isotherm [7] at four different loads are plotted in order to study the thermal analysis i.e. how the temperature is distributed in piston cross-section. Using these temperature fields, first the thermal stresses [7] are calculated.

1.3 FORMULATION FOR STRESS DETERMINATION DEFORMATION AND DISPLACEMENT RELATIONSHIP IN AXI-SYMMETRIC PROBLEM

Let *u* be the displacement in radial direction. *V* be the displacement in axial direction Then,

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Then,

$$\begin{Bmatrix} \epsilon_r \\ \epsilon_\theta \\ \epsilon_z \\ \gamma_{rz} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial r} \\ \frac{u}{r} \\ \frac{\partial v}{\partial z} \\ \frac{\partial u}{\partial z} + \frac{\partial v}{\partial r} \end{Bmatrix}$$

Now, $u(r, z) = c_1 + c_2 r + c_3 z = [1 \quad r \quad z] \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \end{Bmatrix}$

$$\begin{Bmatrix} u_i \\ u_j \\ u_k \end{Bmatrix} = \begin{Bmatrix} 1 & r_i & z_i \\ 1 & r_j & z_j \\ 1 & r_k & z_k \end{Bmatrix} \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \end{Bmatrix} \quad \{u\}^{(e)} = [P]^{(e)} \{c\}^{(e)}$$

$$\{c\}^{(e)} = \{u\}^{(e)} [P]^{(e)-1} = [R]^{(e)} \{u\}^{(e)}$$

$$\{c\}^{(e)} = \begin{Bmatrix} \alpha_1 & \alpha_j & \alpha_k \\ \beta_1 & \beta_j & \beta_k \\ \gamma_1 & \gamma_j & \gamma_k \end{Bmatrix} \begin{Bmatrix} u_i \\ u_j \\ u_k \end{Bmatrix} = [R]^{(e)} \{u\}^{(e)}$$

$$\{u\}^{(e)} = N_1 u_i + N_j u_j + N_k u_k = [N] \{u\}^{(e)} = [N]^{(e)} \{u\}^{(e)}$$

Similarly, $v(r, z) = N_1 v_i + N_j v_j + N_k v_k = [N] \{v\}^{(e)} = [N]^{(e)} \{v\}^{(e)}$

$$\epsilon_r = \frac{\partial u}{\partial r} = \frac{\partial N_1}{\partial r} u_i + \frac{\partial N_j}{\partial r} u_j + \frac{\partial N_k}{\partial r} u_k = b_1 u_i + b_j u_j + b_k u_k = [b]^{(e)} \{u\}^{(e)}$$

$$\epsilon_\theta = \frac{u}{r} = \frac{N_1}{r} u_i + \frac{N_j}{r} u_j + \frac{N_k}{r} u_k$$

$$\epsilon_z = \frac{\partial v}{\partial z} = \frac{\partial N_1}{\partial z} v_i + \frac{\partial N_j}{\partial z} v_j + \frac{\partial N_k}{\partial z} v_k = c_1 v_i + c_j v_j + c_k v_k = [c]^{(e)} \{v\}^{(e)}$$

$$\gamma_{rz} = c_1 u_i + c_j u_j + c_k u_k + b_1 v_i + b_j v_j + b_k v_k$$

$$\{\epsilon\}^{(e)} = [B] \begin{Bmatrix} u_i \\ u_j \\ u_k \end{Bmatrix} = [B]^{(e)} \{u\}^{(e)}$$

$$\epsilon_r = \frac{\alpha_1}{r} - \frac{\partial}{\partial r} \sigma_\theta - \frac{\partial}{\partial z} \sigma_z + \alpha_t (T - T_0)$$

$$\epsilon_\theta = -\partial \frac{\alpha_1}{r} + \frac{1}{r} \sigma_\theta - \frac{\partial}{\partial z} \sigma_z + \alpha_t (T - T_0)$$

$$\epsilon_z = -\partial \frac{\alpha_1}{r} - \frac{\partial}{\partial z} \sigma_\theta - \frac{1}{r} \sigma_z + \alpha_t (T - T_0), \gamma_{rz} = \frac{2(1+\nu)}{E} \sigma_{rz}$$

Thus the stress-strain relationship can be represented in matrix form as,

$$\begin{Bmatrix} 1 & -\nu & -\nu \\ -\nu & 1 & -\nu \\ -\nu & -\nu & 1 \end{Bmatrix} \begin{Bmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_z \end{Bmatrix} = E \begin{Bmatrix} \epsilon_r \\ \epsilon_\theta \\ \epsilon_z \end{Bmatrix} - \alpha_t (T - T_0) \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}$$

$$[P]\{\sigma\} = E[\{\epsilon\} - \{\epsilon_0\}], \quad \{\sigma\} = E[\{\epsilon\} - \{\epsilon_0\}] [P]^{-1}$$

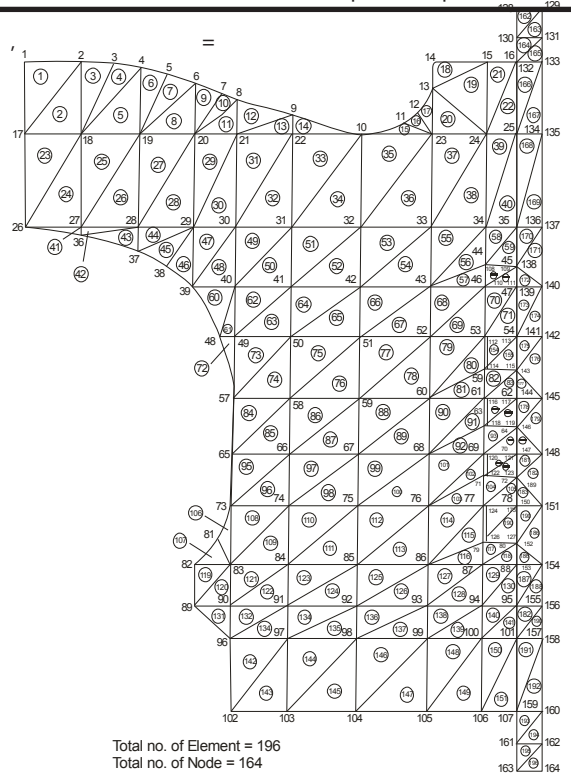
Let the determinant of the matrix $[P] = \Delta$

$$\Delta = (1 - \nu^2) + \nu(1 - \nu - \nu^2) - \nu(\nu^2 + \nu)$$

$$\Delta = (1 - \nu^2) - 2\nu^2(1 + \nu) = (1 + \nu)(1 - \nu - 2\nu^2)$$

$$\Delta = (1 + \nu)[(1 + \nu) - 2\nu(1 + \nu)]$$

$$\Delta = (1 - \nu)^2(1 - 2\nu)$$



Total no. of Element = 196
Total no. of Node = 164

Fig. 1.3 Piston showing the Node Number and Element Number

1.4 CONCLUSION

From the isothermic plots the temperature decreases in the radial direction at any cross section away from the piston center. Also temperature decreases from top face to the bottom face of the piston. This is to be expected as heat must flow from hottest side i.e. piston crown towards water and air sides, which are relatively at lower temperature. From the isotherms, it is clear that the temperature is highest at center point of the top of the piston. So it is necessary to provide some cooling arrangement either by insulating the piston or by circulating coolant in side cylinder.

1.5 SCOPE FOR FUTURE WORK

1. Now a day the designers are interested to make the engine adiabatic by insulating various parts like cylinder wall, valve and piston surface etc. Thus this analysis can be further extended to study heat flow pattern and thermal stress behavior in adiabatic engine.
2. Problems can be considered for radiative heat exchange in combination with convective heat exchange in case of air-cooled engine.
3. The program developed in this thesis work can be used to analyze transient case also.
4. The friction occurring between surface of piston ring and cylinder wall is being neglected here but its effect can be incorporated to solve the problem more realistically.

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