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Technique Solution of Linear Equations, Homogeneous and Non-Homogeneous Equations

Alok Kumar	Assistant Professor, Deshbandhu College, Delhi University. New Delhi-110019.
Rahul Singh	Assistant Professor, Ramanujan College, Delhi University. New Delhi-110019.
Dheeraj Tiger	Assistant Professor, Rajdhani College, Delhi University. New Delhi-110015.
Ajay Kumar Agrawal	Assistant Professor, Motilal Nehru College, Delhi University. New Delhi-110021.

ABSTRACT

In the present paper gives an algorithm to solution of systems of linear equations by finding the rank of coefficient matrix and rank of augmented matrix. Algorithm gives solutions of both types of equations Homogeneous system and Non Homogeneous system.

KEYWORDS Homogeneous System, Non-homogeneous System, Rank

A system of linear equations in variables:

x1,x2,……………xn:

$$a_{11}x_1+a_{12}x_2+\cdots\ldots\ldots\ldots\ldots a_{1n}x_n=b_1$$
$$a_{21}x_1+a_{22}x_2+\cdots\ldots\ldots\ldots\ldots a_{2n}x_n=b_2$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$a_{m1}x_1+a_{m2}x_2+\cdots\ldots\ldots a_{mn}x_n=b_m$$

System written in matrix form as

$$\begin{pmatrix} a_{11} & a_{12} & a_{1n} \\ a_{21} & a_{22} & a_{2n} \\ \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{mn} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ \ldots \\ x_n \end{pmatrix}=\begin{pmatrix} b_1 \\ b_2 \\ \ldots \\ b_m \end{pmatrix}$$

Matrix $[a_{ij}]$ of order called the coefficient matrix of the given system of equations. This matrix is denoting by.AX=B
If is a zero matrix that is b1=b2.........bm = 0 then the system is called homogeneous system of linear equations. If that is are not all zero, then this system is called linear non-homogeneous system, The matrix obtained by adding to the coefficient matrix an additional column is called augmented matrix that is the matrix

$$\begin{pmatrix} a_{11} & a_{12} & a_{1n} & b_1 \\ a_{21} & a_{22} & a_{2n} & b_2 \\ \vdots & \vdots & \vdots & \\ a_{m1} & a_{m2} & a_{mn} & b_m \end{pmatrix}$$

If the given system of linear equations is such that the equations are all satisfied simultaneously by at least one set of values of the variables, then it is said to be consistent. Such a set of values of the variables is called a solution. The system is said to be inconsistent if the equations are not satisfied simultaneously by any set of values of the variables, that is if the system has no solution.

Rank:If be a matrix of order .The order of the largest square sub matrix of whose determinant has a non-zero value is called the rank of the matrix

Rank of Non-singular matrix is equal to its order.

Theorem: A homogeneous system is always consistent. This is because a homogeneous system has at least one solution, trivial solution or zero solution given by

Theorem: A necessary and sufficient condition for the system of linear equations to be consistent is that the coefficient matrix and the augmented matrix of the system are of the same rank.If the coefficient matrix and the augmented matrix are differ by rank then system is inconsistent.

If the system having equations and unknowns,the system is consistent with that is the number of equations is same as numbers of variables.then the system has a unique solution.

If the system having equations and unknowns,the system is consistent with that is the numbers of variables is more than the numbers of variables, If the Rank of augmented matrix the rank of coefficient matrix then solution is unique,

If the Rank of augmented matrix the rank of coefficient matrix then the system has infinitely many solution.

Consider the following system of equations

(i) x+y-z=2
2x-y+z=1
3x-y+z=0

The augmented matrix of the given system is

$$[A:B]=\begin{bmatrix} 1 & 1 & -1 & : & 2 \\ 2 & -1 & 1 & : & 1 \\ 3 & -1 & 1 & : & 0 \end{bmatrix}$$

By elementary row operations,the matrix is equivalent to the matrix

$$\begin{bmatrix} 1 & 1 & -1 & : & 2 \\ 0 & 1 & -1 & : & 1 \\ 0 & 0 & 0 & : & 1 \end{bmatrix}$$

Rank of augmented matrix is 3, and rank of coefficient matrix is 2

So the given system of equations is inconsistent.

(ii) $x + 3y + 2z = 0$
$2x - y + 3z = 0$
$3x - 5y + 4z = 0$
$x + 17y + 4z = 0$

This is a homogeneous equations the coefficient matrix is

$$[A] = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix}$$

By elementary row operations this matrix is reduce

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Rank less the numbers of unknowns ,hence system has infinitely many solutions.

Conclusion: We would like to state here that we can solve system of equations in n-variables and m-equations where n is greater than m and n is less than m. It is very fast technique and rank gives a very important role for solution of system of equations. By rank we can find out linearly independent vectors and linearly dependent vectors. System of equations play a role for solves the physics model, network analysis. Linear equations we can also solve algebra problems and curve fitting. System of linear equations use in chemistry and it is also used in business model and our daily life problem.

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