Original Research Paper Statistics

Construction of Super-Saturated Designs Using Semi-Regular Group Divisible Designs

Dr. SK. Ameen Saheb	School of Mathematics and Statistics, University of Hyderabad, Hyderabad
Dr. N.Ch. Bhatra Charyulu	Department of Statistics, University College of Science, Osmania University, Hyderabad-7

ABSTRACT

A supersaturated design is a fractional factorial design in which the number of factors is more than the number of design points and the degrees of freedom for all its main effects and interaction terms exceed the number of design points. In this paper, we proposed a general method for constructing supersaturated designs using Cyclic Group Divisible designs. The construction is also illustrated with suitable example.

KEYWORDS Cyclic semi-regular group divisible design, Super saturated design

INTRODUCTION

Satterthwaite (1959) initially made an attempt to construct saturated designs randomly and suggested random balance designs. Booth and Cox (1962) proposed a systematic method for the construction of super-saturated designs, which are factorial designs in which the number of factors exceeds the number of design points.

Definition 1.1: A design X is said to be 'saturated' if the number of design points 'n' is equal to the number of factors 'v' plus one i.e. n = v+1.

Definition 1.2: A design X is said to be a super-saturated design, if the number of factors 'v' is more than the number of design points 'n' i.e. v > n.

The designs that are near orthogonal are preferable if it is not possible to conduct the experiment with orthogonal designs. The lack of orthogonality can be measured based on the dispersion matrix of the design.

Definition 1.3: A super-saturated design X is said to be E(s^2)-optimal super-saturated design, if, it has minimum value of $\sum_{1\le i<j\le m} s_{ij}^2 / {}^pC_2$ (i.e. the mean of s_{ij}^2 of all pairs (i, j) for (i≠j) is minimum) where s_{ij} is the sum of cross products between two different factors i and j in the design X.

After Booth and Cox (1962), no attempts were made till Lin (1993). Later Several researchers made attempts on the construction of super-saturated designs with their E(s^2) optimality. Some of them are Lin (1993), Nguyen (1996), Tang and Wu (1997), Deng, Lin and Wang (1999), Fang, Lin and Ma (2000), Liu and Zhang (2000), Lu and Sun(2001), Butler, Mead, Eskridge and Gilmour (2001), and Yamada and Lin (2002), Li and Lin (2003), Liu and Dean (2004), Fang, Gennian and Liu (2004), Aggarwal and Gupta (2004), Xu and Wu (2005), Koukouvinov, Mantos and Mylona (2007), Jones, Lin and Nachtsheim (2008) , Nguyen and Cheng (2008), Sun, Lin and Liu (2011,2012), Ameen saheb and Bhatracharyulu (2013, 2014) etc. In this paper, we proposed a new series of super-saturated design construction using cyclic semi-regular group divisible design.

Definition:1.4: A two associate class Partially Balanced Incomplete Block Design is said to be a Group Divisible design, if the 'v' treatments can be divided into 'm' groups of 'n' treatments each (m, n integers; m, n ≥ 2) such that any two treatments of the same group are first associates and two treatments from different groups are second associates. The association scheme of PBIBD for a group divisible type is

$$P^1=\begin{bmatrix} n-2 & 0 \\ 0 & n(m-1) \end{bmatrix} \qquad P^2=\begin{bmatrix} 0 & n-1 \\ n-1 & n(m-2) \end{bmatrix}$$

Definition 1.5: A group divisible partially balanced incomplete block design is said to be semi-regular if r > λ_1 and rk = λ_2v.

2. CONSTRUCTION OF NEW SERIES SUPERSATURATED DESIGNS :

Consider a Semi-Regular Group Divisible Partially Balanced Incomplete Block Design with parameters v = 4λ (m = 2, n = 2λ), b = 4λ², r = 2λ, k = 2, λ_1= 1 and λ_2=0 with λ = b/v (where λ =number of initial blocks) using initial blocks whose incidence matrix is N. Replace zeros in the incidence matrix with -1's. Transpose the transformed incidence matrix and augment a column with all +1's results to super saturated design is with 'v' design points in 'b' factors.

Example 3.1: Consider a cyclic semi-regular group divisible designwhose incidence matrix is N constructed with parameters v=4, b=4, r=2, k=3, m=2, n=2 and λ_1=0, λ_2=1 using initial block (1, 2). Obtain the supersaturated design X with 4 design points and 5 treatments by augmenting one column with all +1's. The resulting supersaturated design is

$$X=\begin{bmatrix} +1 & -1 & -1 & +1 & +1 \\ +1 & +1 & -1 & -1 & +1 \\ -1 & +1 & +1 & -1 & +1 \\ -1 & -1 & +1 & +1 & +1 \end{bmatrix}$$

Generating vectors for cyclic Semi Regular Group Divisible Designs

S No	Parameters of SRGD PBIBD									Intial blocks
	v	b	r	k	m	N	λ_1	λ_2	a	
1	4	4	2	2	2	2	0	1		(+, +, -, -)

2	4	8	4	2	2	2	0	2		(+, +, -, -), (+, +, -, -)
3	4	12	6	2	2	2	0	3		(+, +, -, -), (+, +, -, -), (+, +, -, -)
4	4	16	8	2	2	2	0	4		(+, +, -, -), (+, +, -, -), (+, +, -, -), (+, +, -, -)
5	4	20	10	2	2	2	0	5		(+, +, -, -), (+, +, -, -), (+, +, -, -), (+, +, -, -), (+, +, -, -)
6	6	18	6	2	2	3	0	2		(+, +, -, -, -, -, -), (+, -, +, -, -, -), (+, -, -, -, +, -)
7	8	16	4	2	2	4	0	1		(+, +, -, -, -, -, -), (+, -, +, -, -, -)
8	8	16	8	4	4	2	0	4		(+, +, +, +, -, -, -, -), (+, +, +, -, -, +, -, -)
9	8	32	8	2	2	4	0	2		(+,+,+,+,-,-,-,-),(+,+,+,-,-,+,-,-),(+,+,+,+,-,-,-,-), (+,+,+,-,-,+,-,-)
10	9	27	9	3	3	3	0	3		(+,+,+,-,-,-,-,-,-),(+,-,-,+,-,-,-,+,-),(+,-,-,-,+,-,+,--)
11	10	50	10	2	2	5	0	2		(+,+,-,-,-,-,-,-,-,-),(+,-,+,-,-,-,-,-,-,-), (+,-,-,-,+,-,-,-,-,-),(+,-,-,-,-,-,+,-,-,-), (+,-,-,-,-,-,-,-,+,-)
12	12	36	6	2	2	6	0	1		(+,+,-,-,-,-,-),(+,-,+,-,-,-),(+,-,-,-,+,-)
13	16	64	8	2	2	8	0	1		(+,+,-,-,-,-,-,-,-,-),(+,-,+,-,-,-,-,-,-,-), (+,-,-,-,+,-,-,-,-,-),(+,-,-,-,-,+,-,-,-)
14	20	100	10	2	2	10	0	1		(+,+,-,-,-,-,-,-,-,-),(+,-,+,-,-,-,-,-,-,-), (+,-,-,-,+,-,-,-,-,-),(+,-,-,-,-,-,+,-,-,-), (+,-,-,-,-,-,-,-,+,-)

References:

1. Ameen Saheb Sk. and Bhatra Charyulu N.Ch., Method for the construction of Super saturated design, International Journal of Mathematical Archive, Vol 4(12), 2013, pp 1-3.
2. Ameen Saheb: Study on Response Surface Model Reduction and Construction of Super-saturated Designs, Unpublished Ph.D Thesis submitted to Osmania University, Hyderabad, 2013.
3. Ameen Saheb Sk. and Bhatra Charyulu N.Ch., Construction of New Series of Super-Saturated Designs, Research Journal of Mathematical and Statistical Sciences, Vol. 2(3), 2014, pp 7-8.
4. Booth, K.H.V., Cox, D.R. Some systematic supersaturated designs, Technometrics, Vol. 4, 1962, 489-495.
5. Fang, K.T., Lin, D.K.J. and Ma, C.X. On the construction of multi-level supersaturated designs. Journal of Statistical Planning and Inference, 86, 2000, 239–252.
6. Gupta, S., and Chatterjee, K. Supersaturated designs: A review, Jour. Comb. Info. Sys. Sci., Vol. 23(4), 1998, 475-488.
7. Lin, D. K. J. A new class of Supersaturated designs, Technometrics, Vol. 35, 1993, 28-31.
8. Lin, D. K. J., Generating Systematic Supersaturated Designs, Technometrics, Vol. 37, 1995, 213-225.
9. Lu, X. and Zhang, Y. A new method in the construction of two-level supersaturated designs, Journal of Statistical Planning and Inference, 86, 2000, 229–238.
10. Nguyen, N. K., An algorithm approach to constructing supersaturated designs, Technometrics, 1996, Vol. 38, 69-73.
11. Plackett, R. L. and Burmann, J.P. The design of optimum multi-factorial experiments, Biometrica, Vol.33, 1946, 305-325.
12. Satterthwaite. F. Random Balance Experimentation, Technometrics, Vol. 1, 1959, 111-137.