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Domination and Inverse Domination Parameters Of *f* (N×R)

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ABSTRACT

The domination and inverse domination parameters have been already studied, in this paper our interest is to evaluate the domination and inverse domination parameters for the flower graph f_(n×r). Also we have proved the inverse domination parameter for the flower graph f_ n×r of order n r-1 is γ' $f_{n\times r}$ ={(k-1)n+γ' ($f_{n\times(3k+i)}$)k=1,2,3;i=0,1,2 and r≤11} where γ' (f_n×r be the inverse domination number of f_(n×r).

KEYWORDS Flower Graph, Dominating set, Inverse dominating set Domination and Inverse domination number.

1. Introduction:

Let G = (V, E) be a flower graph of order n(r − 1). A subset D of V is called dominating set if for every vertex $v \in V - D$, there exists a vertex u in such that u is adjacent to v. The smallest cardinality of a minimum dominating set in G is called the domination number of G and is denoted by $\gamma(G)$. Any dominating set with $\gamma(G)$ vertices is called γ −set of G.A dominating set D' contained in $V - D$ is called an inverse dominating set of G with respect to D.The smallest cardinality among all minimum dominating sets, in $V - D$ is called the inverse dominating set of G which has $\gamma'(G)$ vertices is called a γ' −set of G.

Definition: 1.1

If $e = \{u, v\}$ is an edge of G, written as $e = uv$, we say that e joins the vertices u and v. Also, we say that u and v are adjacent vertices, u and v are incident with e.

Definition: 1.2

A walk of a graph G is an alternating sequence of points and lines $v_0, x_1, v_1, x_2, v_2, \dots, v_{n-1}, x_n, v_n$ beginning and ending with points such that each line x_i is incident with v_{i-1} and v_i.

A walk in which all the vertices are distinct is called a Path. A path of n vertices is denoted by P_n. A closed path is called a Cycle. Generally a cycle with n vertices is denoted by C_n.

Definition: 1.3

Let $G = (V, E)$ be a graph. A subset D of V is called dominating set if every vertex in $V - D$ is adjacent to a vertex in D. The minimum cardinality of a dominating set in G is called the domination number of G and is denoted by $\gamma(G)$.

Definition: 1.4

Let $G = (V, E)$ be a graph. Let D be a minimum dominating set of G. If $V - D$ contains a dominating set D' is called an inverse dominating set with respect to D the minimum cardinality of all inverse dominating sets of a graph G is called the inverse domination number of G and it is denoted by $\gamma'(G)$.

Definition: 1.5

The degree of vertex v in a graph G is the number of edges of G incident with v and is denoted by $d_G(v)$ or deg v. (or) simply $d(v)$.

Definition: 1.6

A Graph G is called a $n \times r$ flower graph if it has m vertices which form a n-cycle and r sets of $n - 2$ vertices which form r-cycle around them n-cycle so that each r-cycle uniquely intersects with the n-cycle on a single edge. This graph is denoted by $f_{n\times r}$. It is clear that $f_{n\times r}$ has $n(r - 1)$ vertices and nr edges. The r cycles are called the petals and the n cycles is called the centre of $f_{n\times r}$. Then n vertices which form the centre are all of degree 4 and all the other vertices have degree 2.

Theorem: 1.7

Let $G = f_{n\times r}$ then,

$$\gamma'\,(f_{n\times r}) = \{(k-1)n + \gamma'(f_{n\times \overline{3k+i}}), \quad k = 1,2,3;\ i = 0,1,2 \ \text{ where, } r \le 11\}$$

Proof:

Let $G = f_{n\times r}$ be a flower graph of order $n(r - 1)$ and $r = 3k + i$

Case (i)

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The flower graph $G = f_{n\times 3}$ is given in figure 1.1

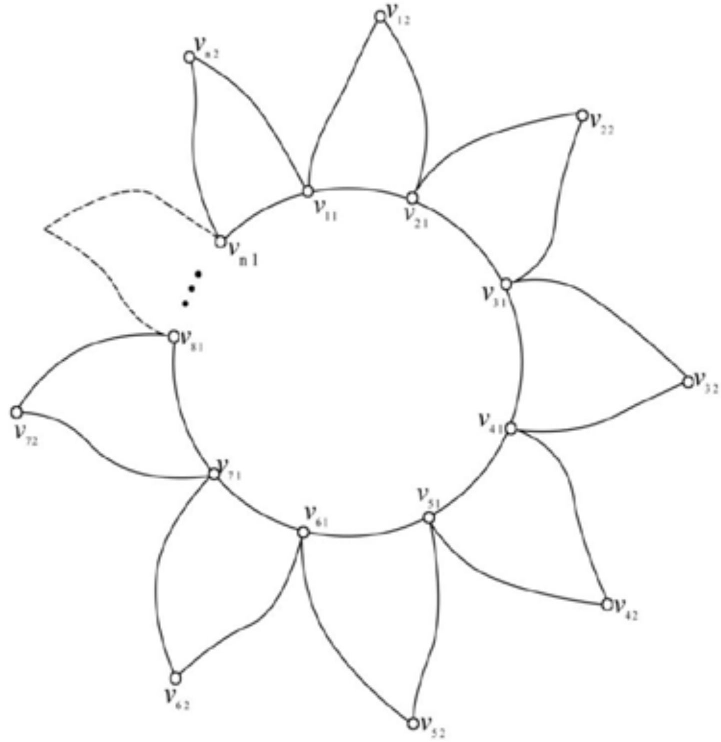

Figure 1.1

Now the vertices of G can be partitioned in to two sets S_1 and S_2 such that

$S_1 = \{v_{i\ 1} / \ i = 1,2,\dots,n\}$ and

$S_2 = \{v_{i\ 2} / \ i = 1,2,\dots,n\}$

Let $D = \{v_{\overline{2i+1}\ 1} / \ i = 0,1,2,\dots,\left\lceil\frac{n}{2}\right\rceil\}$ is the required minimum dominating set of G and

$D' = \{v_{\overline{2i+2}\ 1} / \ i = 0,1,2,\dots,\left\lceil\frac{n}{2}\right\rceil\}$ is the required inverse dominating set of G.

Thus the cardinality of D and D' is $\left\lceil\frac{n}{2}\right\rceil$.

Hence $\gamma(G) = \gamma'(G) = \left\lceil\frac{n}{2}\right\rceil$

The flower graph $G = f_{n\times 6}$ is given in figure 1.2

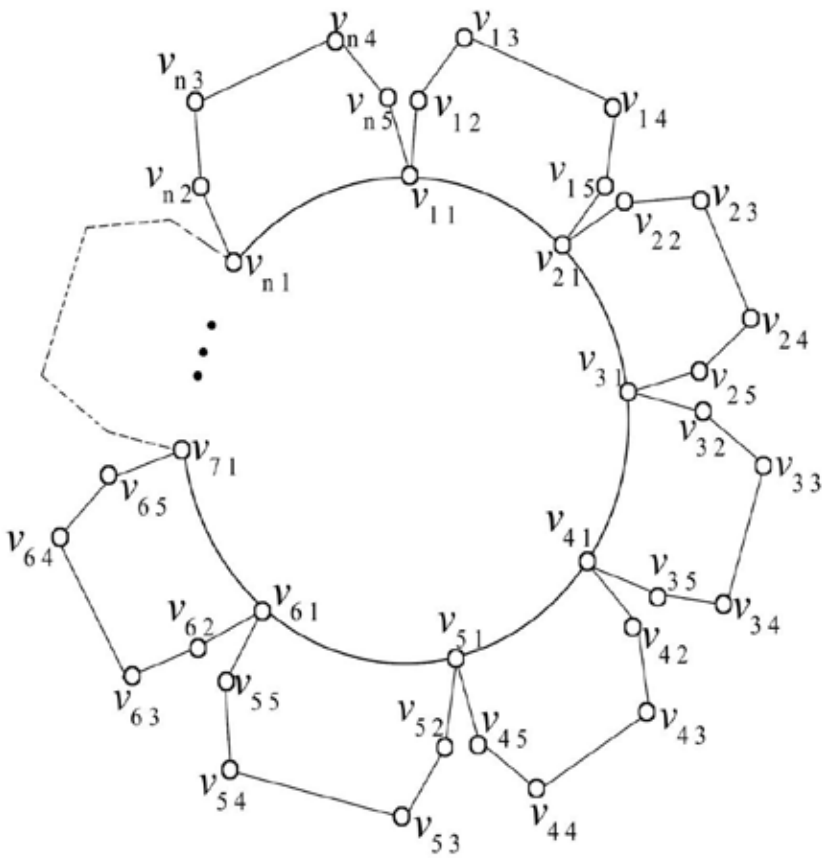

Figure 1.2

The vertices of G can be partitioned in to two sets S_1 and S_2 such that

$S_1 = \{v_{i\ 1} /\ \ i = 1,2,\dots,n\}$and

$S_2 = \{v_{i\ j} /\ \ i = 1,2,\dots,n; j = 2,3,\dots,5\}$

Let $= \{(v_{\overline{2i+1}\ 1}), (v_{\overline{2j+1}\ 4}), (v_{2k\ 3}) /\ \ i,j = 0,1,2,\dots,\left[\frac{n}{2}\right]; k = 1,2,\dots,\left[\frac{n}{2}\right] \}$ is the required minimum dominating set of G and

$D' = \{(v_{2i\ 1}), (v_{2j\ 4}), (v_{\overline{2k+1}\ 3}) /\ \ i,j = 1,2,\dots,\left[\frac{n}{2}\right]; k = 0,1,2,\dots,\left[\frac{n}{2}\right] \}$ is the required minimum inverse dominating set of G.

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Therefore, the cardinality of D and D' is $n + \left\lceil \frac{n}{2} \right\rceil$.

Hence, $\gamma(G) = \gamma'(G) = n + \left\lceil \frac{n}{2} \right\rceil$

The flower graph $G = f_{n\times 9}$ is given in figure 1.3

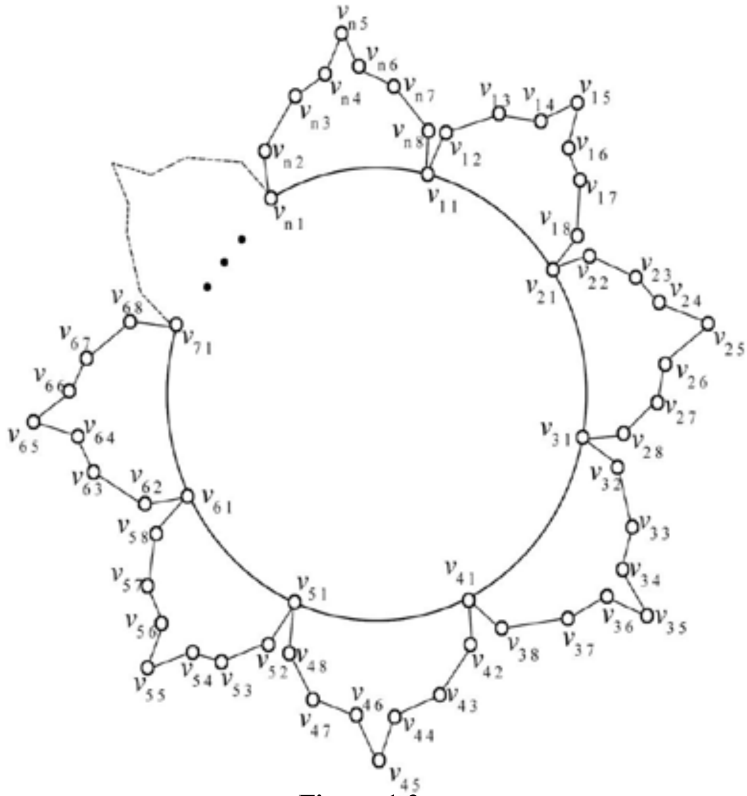


Figure 1.3

Now the vertex of G can be partitioned in to two sets S_1 and S_2 such that

$S_1 = \{v_{i\ 1} / \ i = 1,2,\dots,n\}$ and

$S_2 = \{v_{i\ j} / \ i = 1,2,\dots,n; j = 2,3,\dots,8\}$

Let $D = \left\{(v_{\overline{2i+1}\ 1}), \left(v_{\overline{2j+1}\ 4}\right), (v_{\overline{2k+1}\ 7})(v_{2d\ 3})(v_{2d\ 6}) / \ i,j,k = 0,1,2,\dots,\left\lceil\frac{n}{2}\right\rceil; d = 1,2,\dots,\left\lceil\frac{n}{2}\right\rceil \right\}$

is a required minimum dominating set of G and

$D' = \{(v_{2i\ 1}), \left(v_{2j\ 4}\right), (v_{2k\ 7}), (V_{\overline{2d+1}\ 3}), (V_{\overline{3d+1}\ 6}) / i,j,k = 1,2,\dots,\left\lceil\frac{n}{2}\right\rceil; d = 0,1,2,\dots,\left\lceil\frac{n}{2}\right\rceil \}$ is the required minimum inverse dominating set of G.

Thus the cardinality of D and D' is $2n + \left\lceil \frac{n}{2} \right\rceil$.

Hence $\gamma(G) = \gamma'(G) = 2n + \left\lceil \frac{n}{2} \right\rceil$

Thus, $\gamma(f_{n\times r}) = \left\{(k-1)n + \left\lceil \frac{n}{2} \right\rceil\right\}$

$\gamma'(f_{n\times r}) = \left\{(k-1)n + \left\lceil \frac{n}{2} \right\rceil, \text{ where } r = 3k\right\}$ ....

Put $k = 1$ in eqn (1) we get, $\gamma(G) = \gamma'(G) = \left\lceil \frac{n}{2} \right\rceil$

Put $k = 2$ in eqn (1) we get, $\gamma(G) = \gamma'(G) = n + \left\lceil \frac{n}{2} \right\rceil$

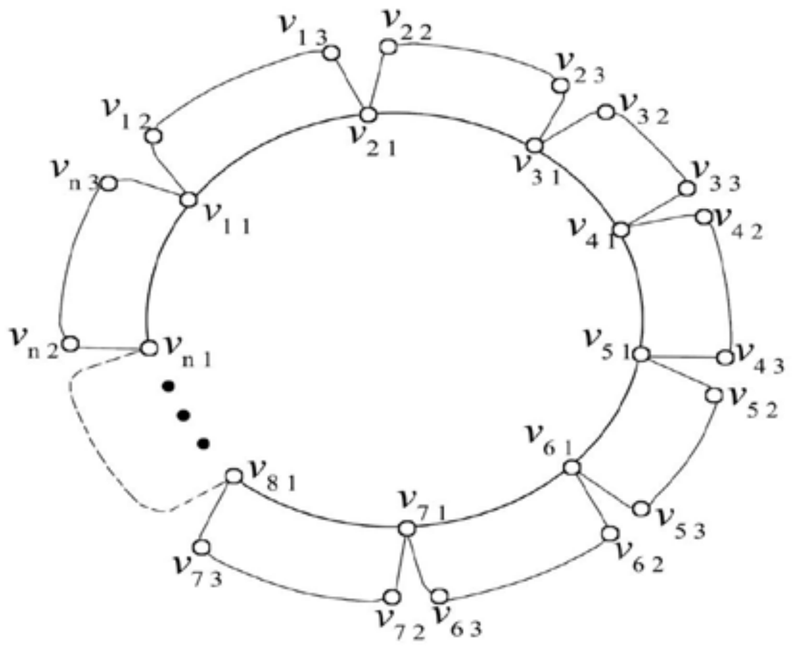
Put $k = 3$ in eqn (1) we get, $\gamma(G) = \gamma'(G) = 2n + \left\lceil \frac{n}{2} \right\rceil$

Therefore, $\gamma(f_{n\times r}) = \left\{(k-1)n + \left\lceil \frac{n}{2} \right\rceil\right\}$

$\Longrightarrow \gamma'(f_{n\times r}) = \left\{(k-1)n + \left\lceil \frac{n}{2} \right\rceil \text{ where } r = 3k, r \leq 11\right\}$

Case : (ii)

The flower graph $G = f_{n\times 4}$ is given in figure 1.4



$S_1 = \{v_{i\ 1} / \ i = 1,2,\dots,n\}$and

$S_2 = \{v_{i\ j} / \ i = 1,2,\dots,n; j = 2,3,\dots,6\}$

Let $D = \{(v_{i\ 2}), (v_{j\ 5}) / \ i = 1,2,\dots,n\}$is the required minimum dominating set of G and $D' = \{(v_{i\,1}), (v_{j\,4}) / \ i = 1,2,\dots,n; j = 1,2,\dots,n\}$is the required minimum inverse dominating set of G.

Thus the cardinality of D and D' is $2n$.

Hence , $\gamma(G) = \gamma'(G) = 2n$

The flower graph $G = f_{n\times 10}$ is given in figure 1.6

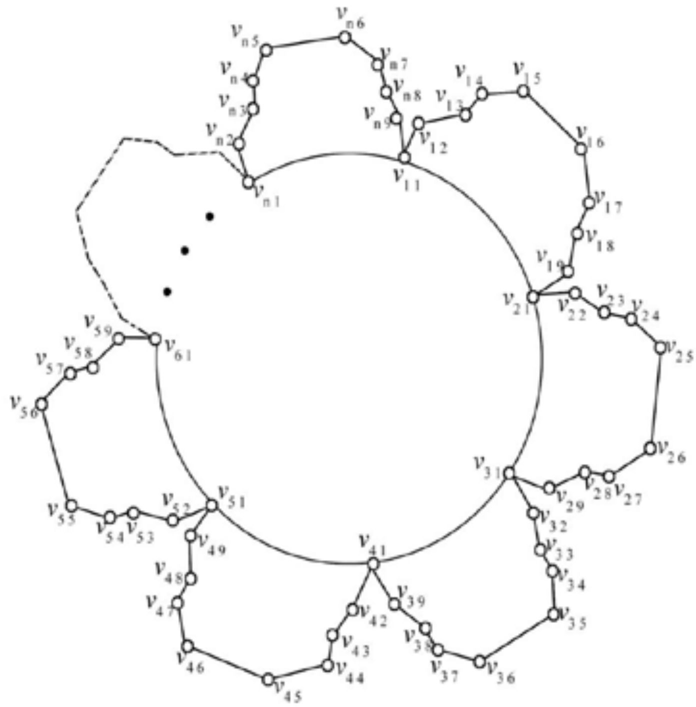


Figure 1.6

Now the vertices of G can be partitioned in to two sets S_1 and S_2 such that

$S_1 = \{v_{i\ 1} / \ i = 1,2,\dots,n\}$and

$S_2 = \{v_{i\ j} / \ i = 1,2,\dots,n; j = 2,3,\dots,9\}$

Let $D = \{v_{i\,j} /\ i = 1,2,\dots,n;\ j = 1,4,7\}$ is the required minimum dominating set of G and $D' = \{v_{i\,j} /\ i = 1,2,\dots,n; j = 2,5,8\}$ is a required minimum inverse dominating set of G.

Thus the cardinality of D and D' is $3n$.

Hence $\gamma(G) = \gamma'(G) = 3n$

Thus, $\gamma(f_{n\times r}) = \{(k-1)n + n\}$

$\gamma'(f_{n\times r}) = \{(k-1)n + n$ where $r = 3k+1, r \le 11\}$ …… (2)

Put $k = 1$ in eqn (2), we get, $\gamma(G) = \gamma'(G) = n$

Put $k = 2$ in eqn (2), we get, $\gamma(G) = \gamma'(G) = 2n$

Put $k = 3$ in eqn (2), we get, $\gamma(G) = \gamma'(G) = 3n$

Therefore, $\gamma(f_{n\times r}) = \{(k-1)n + n$

$\gamma'(f_{n\times r}) = \{(k-1)n + n$ where $r = 3k+1, r \le 11\}$

Case : (iii)

The flower graph $G = f_{n\times 5}$ is given in figure 1.7

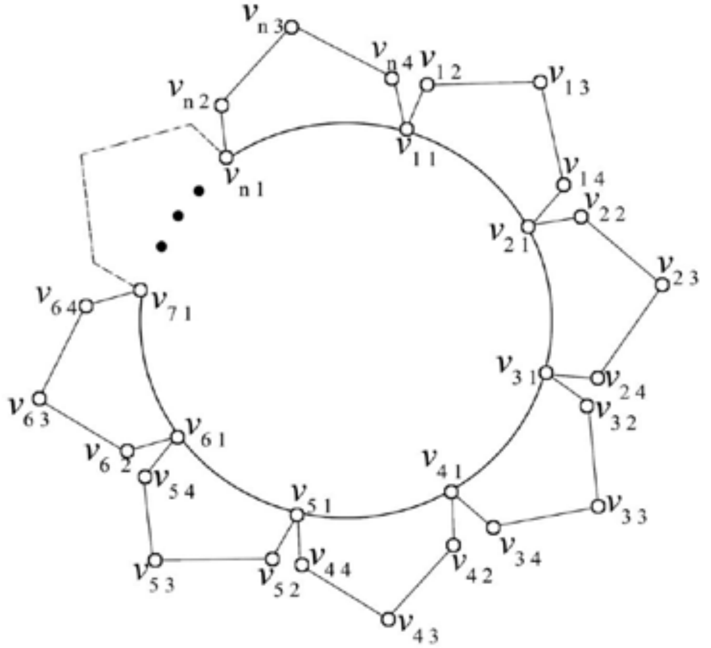


Figure 1.7

Now the vertices of G can be partitioned in to two sets S_1 and S_2 such that

$S_1 = \{v_{i\ 1} / \ i = 1,2,\dots,n\}$ and

$S_2 = \{v_{i\ j} / \ i = 1,2,\dots,n; j = 2,4\}$

Let $D = \{(v_{\overline{3\iota+2}\ 1}), (v_{\overline{3\iota+2}\ 4}), (v_{3j\ 3}) / \ i = 0,1,2,\dots,n;\ j = 1,2,\dots,n\}$ is the required minimum dominating set of G and

$D' = \{(v_{\overline{3\iota+1}\ 1}), (v_{\overline{3k+2}\ 3}), (v_{3d\ 2}) / \ i = 0,1,2,\dots,n; d = 1,2,\dots,n;$ and $j = 1,4\}$ is a required minimum inverse dominating set of G.

Thus the cardinality of D and D' is $n + \left\lceil \frac{n+1}{3} \right\rceil$

Hence $\gamma(G) = \gamma'(G) = n + \left\lceil \frac{n+1}{3} \right\rceil$

The flower graph $G = f_{n\times 8}$ is given in figure 1.8

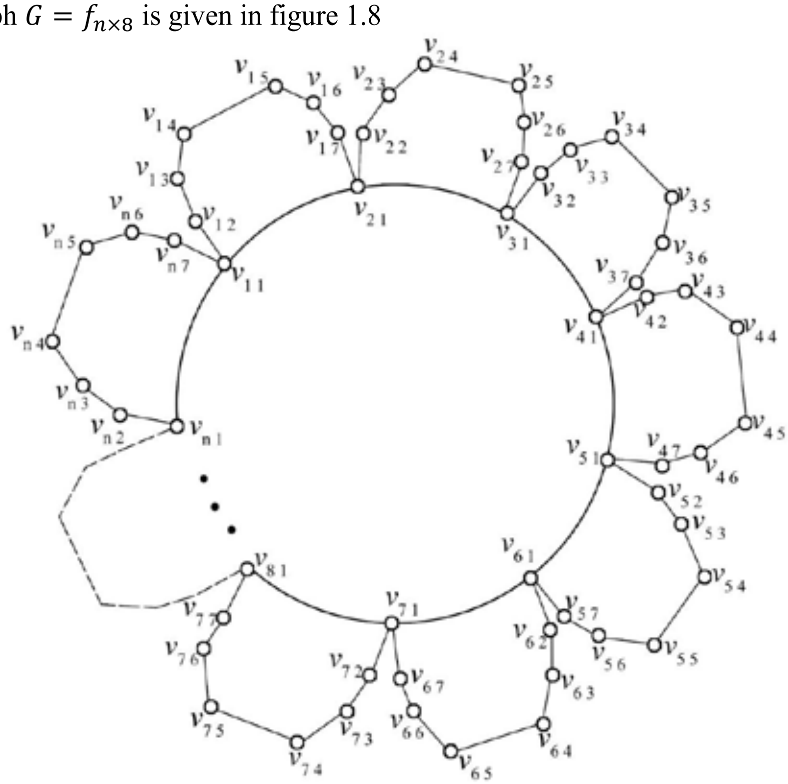


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Figure 1.8

Now the vertices of G can be partitioned in to two sets S_1 and S_2 such that

$S_1 = \{v_{i\ 1} /\ \ i = 1,2,\dots,n\}$ and

$S_2 = \{v_{i\ j} /\ \ i = 1,2,\dots,n;\ j = 2,3,\dots,7\}$

Let $D = \{(v_{\overline{3\iota+2}\ j}), (v_{\overline{3\iota+1}\ k}), (v_{3d\ P}) / i = 0,1,2,\dots,n;\ j = 1,4,7; k = 2,5; p = 3,6; d = 1,2,\dots,n\}$ is the required minimum dominating set of G and

$D' = \{(v_{\overline{3\iota+1}\ j}), (v_{\overline{3\iota+2}\ k}), (v_{3\ \overline{3\iota+2}}) /\ i = 0,1,2,\dots,n; j = 1,4,7; k = 3,6\}$ is a required minimum inverse dominating set of G.

The cardinality of D and D' is $2n + \left\lceil \frac{n+1}{3} \right\rceil$

Therefore, $\gamma(G) = \gamma'(G) = 2n + \left\lceil \frac{n+1}{3} \right\rceil$

The flower graph $G = f_{n\times 11}$ is given in figure 1.9

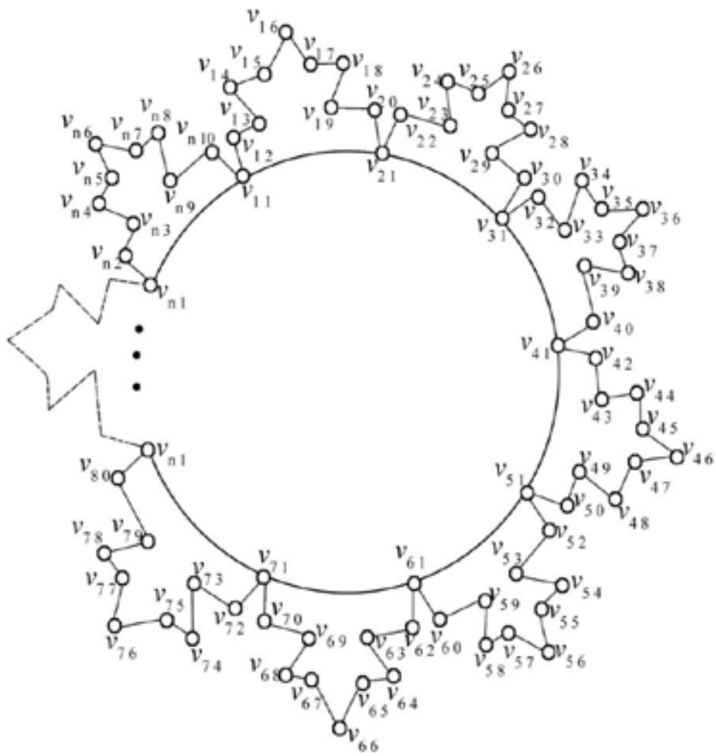

Figure 1.8

Now the vertices of G can be partitioned in to two sets S_1 and S_2 such that

$S_1 = \{v_{i\ 1} / \ i = 1,2,\dots,n\}$ and

$S_2 = \{v_{i\ j} / \ i = 1,2,\dots,n;\ j = 2,3,\dots,7\}$

Let $D = \{(v_{\overline{3\iota+2}\ j}), (v_{\overline{3\iota+1}\ k}), (v_{3d\ P}) / i = 0,1,2,\dots,n;\ j = 1,4,7; k = 2,5; p = 3,6; d = 1,2,\dots,n\}$ is the required minimum dominating set of G and

$D' = \{(v_{\overline{3\iota+1}\ j}), (v_{\overline{3\iota+2}\ k}), (v_{3\ \overline{3\iota+2}}) / \ i = 0,1,2,\dots,n; j = 1,4,7; k = 3,6\}$ is a required minimum inverse dominating set of G.

The cardinality of D and D' is $2n + \left\lceil \frac{n+1}{3} \right\rceil$

Therefore, $\gamma(G) = \gamma'(G) = 2n + \left\lceil \frac{n+1}{3} \right\rceil$

The flower graph $G = f_{n\times 11}$ is given in figure 1.9

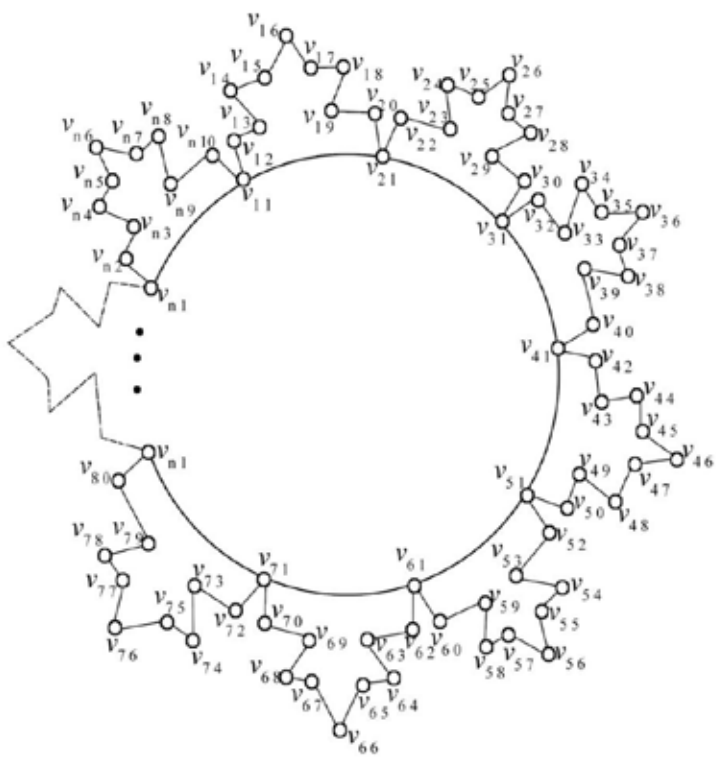

Figure 1.9

Now the vertices of G can be partitioned in to two sets S_1 and S_2 such that

$S_1 = \{v_{i\ 1} / \ i = 1,2,\dots,n\}$ and

$S_2 = \{v_{i\ j} / \ i = 1,2,\dots,n;\ j = 2,3,\dots,10\}$

Let $D = \{(v_{\overline{3\iota+1}\ j}), (v_{\overline{3\iota+2}\ k}), (v_{3d\ P}) / i = 0,1,2,\dots,n;\ j = 1,4,7,10; k = 3,6,9; d = 1,2,\dots,n;$

$p = 2,5,8\}$ is the required minimum dominating set of G and

$D' = \{(v_{\overline{3\iota+1}\ j}), (v_{\overline{3\iota+2}\ k}), (v_{3d\ p}) / \ i = 0,1,2,\dots,n; j = 2,5,8; k = 1,4,7; d = 1,2,\dots,n; p = 3,6,9\}$ is

a required minimum inverse dominating set of G.

The cardinality of D and D' is $3n + \left\lceil \frac{n+1}{3} \right\rceil$

Hence $\gamma(G) = \gamma'(G) = 3n + \left\lceil \frac{n+1}{3} \right\rceil$

Thus $\gamma(f_{n\times r}) = \left\{(k-1)n + n + \left\lceil \frac{n+1}{3} \right\rceil\right\}$

$\gamma'(f_{n\times r}) = \left\{(k-1)n + n + \left\lceil \frac{n+1}{3} \right\rceil\right\}$ where $r = 3k + 2, r \le 11\}$ ……(3)

Put $k = 1$ in eqn (3),we get, $\gamma(G) = \gamma'(G) = n + \left\lceil \frac{n+1}{3} \right\rceil$

Put $k = 2$ in eqn (3),we get, $\gamma(G) = \gamma'(G) = 2n + \left\lceil \frac{n+1}{3} \right\rceil$

Put $k = 3$ in eqn (3),we get, $\gamma(G) = \gamma'(G) = 3n + \left\lceil \frac{n+1}{3} \right\rceil$

Therefore, $\gamma(f_{n\times r}) = \left\{(k-1)n + n + \left\lceil \frac{n+1}{3} \right\rceil\right\}$

$\gamma'(f_{n\times r}) = \left\{(k-1)n + n + \left\lceil \frac{n+1}{3} \right\rceil\right\}$ where $r = 3k + 2, r \le 11\}$

compaining eqn (1),(2) and (3) we get,

$$\gamma(f_{nxr}) = \{(k-1)n + \gamma(f_{nx\overline{3k+i}})\}$$

$$\gamma'(f_{nxr}) = \{(k-1)n + \gamma'(f_{nx\overline{3k+i}})\}$$

where $r = 3k + i, i = 0,1,2$ for case (i), (ii)and (iii)respectively and $k = 1,2,3$

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