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PRIME LABELLING OF SOME NEW STANDARD GRAPHS

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ABSTRACT

In this research paper we analyze the prime labelling of some new standard graphs. We need to prove that the graphs such as flower graph F l_n the splitting graph of Star $K_{1,n}$, the bistar $B_{n,n}$ the friendship graph F_n the graph SF(n,1) are prime graphs. A graph is said to be prime if it has a relatively prime labeling on its vertices which satisfies certain properties. The purpose of this paper is to give some new families of graphs that have a prime labeling and give some necessary and sufficient conditions for some families of prime graphs.

KEYWORDS Prime Labelling, Splitting graph, Star K_{1n} the bistar B_{nn} the friendship graph F_n, the graph SF (n, 1).

Introduction

All graphs in this paper are finite, simple and undirected. The symbols V(G) and E(G) will denote the vertex set and edge set of the graph G. For standard terminology and notations we follow Gross and Yellon[1]. We will give brief summary of definitions which are useful for the present investigation. It is well known that graph theory has applications in many other fields of study, including physics, chemistry, biology, communication, psychology, sociology, economics, engineering, operations research, and especially computer science. For all standard notation and terminology in graph theory we follow . Graph labelings where the vertices are assigned real values subject to certain conditions like as Graceful , Harmonious , Cordial , Prime and others, have often been motivated by practical problems such as coding theory, communication networks and astronomy, but they are also of logico- mathematical interest in their own right. An enormous body of literature has grown around the subject especially in the last thirty years or so, and is presented in a survey by Gallian .

Definition 1.1

Let G= G(V,E) be a graph. A bijection f: V→{1,2,3,....., |V| is called prime labelling if for each e={u,v} belong to E, we have GCD (f(u),f(v))=1. A graph that admits a prime labelling is called a prime graph.

Definition 1.2

The flower F l_n is the graph obtained from a helm H_n by joining each pendent vertex to the apex of the helm. It contains three types of vertices, an apex of degree 2n, n vertices of degree 4 and n vertices of degree 2.

Definition 1.3

For a graph G the splitting graph S′ of G is obtained by adding a new vertex v′ corresponding to each vertex v of G such that N(v)=N(v′).

Definition 1.5

The friendship graph Fn is one-point union of n copies of cycle C3.

Definition 1.6

An SF(n,m) is a graph consisting of a cycle C_n, n≥3 and n set of m independent vertices where each set joins each of the vertices of C_n.

Prime Labelling Of Some New Standard Graphs

Theorem 1:

Flower graph F l_n admits a prime labelling.

Proof:

Let V be the apex vertex, $v_1,v_2,\ldots\ldots.$vn be the vertices of degree 4 and $u_1,u_2,\ldots\ldots\ldots$un be the vertices of degree 2 of F l_n.
Then $|V(Fl_n)|$ = 2n+1 and $|E(Fl_n)|$ = 4n.

We define a prime labelling f: V→{1,2,3,….. $|V|$} given
by f(v) = 1
$f(v_i) = 1+2i$, 1≤i≤n
$f(u_i) = 2i$, 1≤i≤n.
There exists a bijection f: V→{1,2,3,…..,$|V|$ such that for each e={u,v} belongs to E, we have GCD (f(u),f(v))=1.
Hence the flower F l_n admits prime labelling.

Illustration 1: The prime labelling of the graph Fl_9 is shown in Figure1.

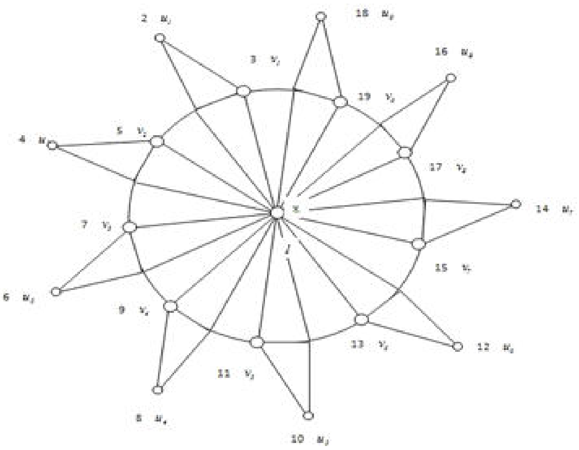

Figure 1

Theorem 2:

Splitting graph of star graph admits a prime labelling.

Proof:

Let v1,v2,…….vn be the vertices of star graph $K_{1,n}$ with v be the apex vertex. Let G be the splitting graph of $K_{1,n}$ and $v_1', v_2',\ldots\ldots, v_n'$

be the newly added vertices with K1,n to form G. We define f: V→{1,2,3,……$|V|$} by

f(v) = 1
f(v) = 2
f(v_i) = 1+ 2i , 1≤i≤n
f(v_i') = 2i+2 , 1≤i≤n.

In view of the above labelling pattern, G admits a prime labelling.

Illustration 2:
Figure 2 shows the prime labelling of splitting graph of $K_{1,8}$.

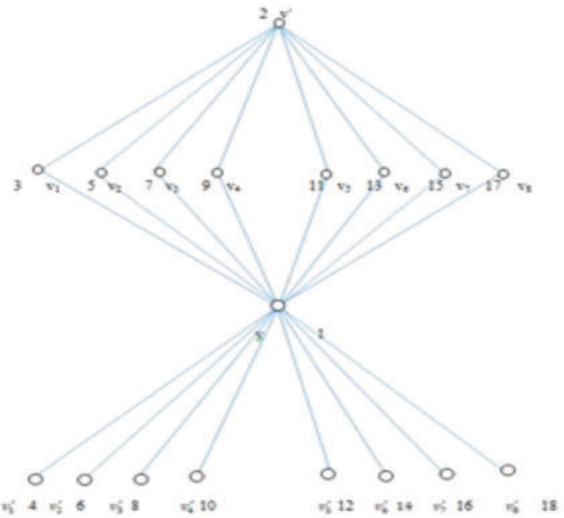

Figure 2

The bistar $B_{n,n}$ admits a prime labelling.

Proof:
Consider the two copies of $K_{1,n}$. Let $v_1,v_2,\ldots\ldots.v_n$ and $u_1,u_2,\ldots\ldots\ldots u_n$ be the corresponding vertices of each copy of $K_{1,n}$ with apex vertex v and u.

Let $e_i = vv_i$, $e_i' = uu_i$ and e=uv of bistar graph.
Note that then $|V(B_{n,n})| = 2n+2$ and $|V(B_{n,n})| = 2n+1$.
Define a prime labelling f: V→{1,2,3,…… $|V|$} as follows

f(u) = 1
f(v) = 2
f(u_i) = 2+ 2i, 1≤i≤n
f(v_i) = 2i+1, 1≤i≤n.

In view of above labelled pattern, Bn,n admits a prime labelling

Illustration 3:
Prime labelling of $B_{8,8}$ is shown in figure 3.

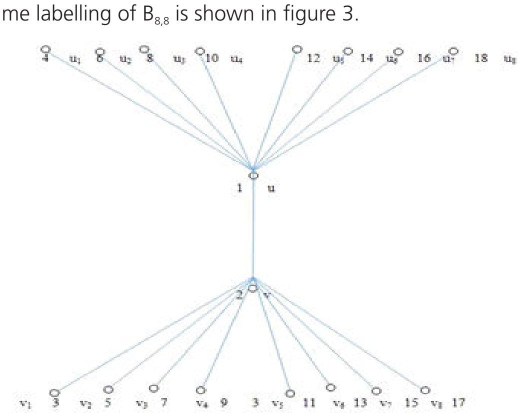

Figure 3

Theorem 4:
The friendship graph F_n admits a prime labelling.

Proof:
Let F_n be the friendship graph with n copies of cycle C_3. Let v' be the apex vertex, $v_1,v_2,\ldots\ldots.v_{2n}$ be the other vertices and $e_1,e_2,\ldots\ldots\ldots e_{3n}$ be the edges of F_n.

Define a prime labelling f: V→{1,2,3,…… $|V|$given

by f(v′) = 1
f(v_i) = i+1 for 1≤i≤n.

There exists a bijection f: V→{1,2,3,…$|V|$ such that for each e={u,v} belong to E, we have GCD (f(u),f(v))=1.
Hence the friendship graph F_n admits a prime labelling.

Illustration 4:
The prime labelling of F_6 is given by Figure 4.

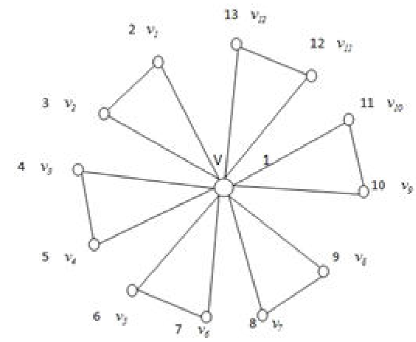

Figure 4

Theorem 5:
The graph SF(n,1) admits a prime labelling.

Proof:
Let G denote the graph SF(n,1).
Let $v_1,v_2,\ldots\ldots.v_n$ be the vertices of the cycle of SF(n,1) and v_j' for j = 1,2,3,…,n be the vertices joining the corresponding vertices vj.
Here p=2n and q=2n.
= 2j-1 for j = 1,2,3,….,n f (v_j')
= 2j for j = 1,2,3,….,n.

There exists a bijection f: V→{1,2,3,…..$|V|$ such that for each e={u,v} belong to E, we have GCD (f(u),f(v))=1.
The graph SF(n,1) admits prime labelling.

Illustration 5:
Figure 5 shows the prime labelling of SF(8,1).

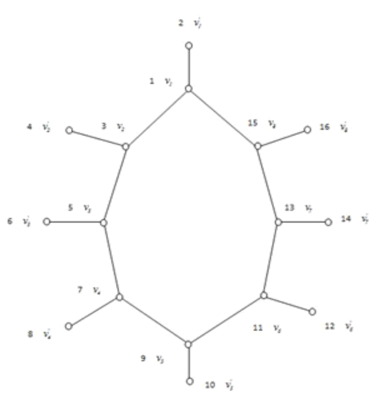

Figure 5

Conclusion

We have presented the prime labelling of certain classes of graphs such as flower F l_n the splitting graph of Star $K_{1,n}$the bistar $B_{n,n}$ the friendship graph F_n,the graph SF(n,1).In general, all the graphs are not prime, it is very interesting to investigate graph families which admit prime labelling.

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