ORIGINAL RESEARCH PAPER

Mathematics

Mathematical Model for Optimal Scheduling of Reservation system: A Study on APSRTC Vizianagaram Bus Depot.

KEY WORDS: Transportation problem, multi – criteria decision making, Goal programming, optimum scheduling, mathematical model and objective function.

Dr. G.A.Kalyani

Department of Mathematics & Statistics, M R (Autonomous College) Vizianagaram, Andhra Pradesh, India.

ABSTRACT

Decision making is the process of identifying and choosing alternatives based on the values and preferences of the decision maker. It is the process of sufficiently reducing uncertainty and doubt about alternatives to allow a reasonable choice to be made from among them. Decision making based solely on a single criterion appears insufficient as soon as the decision-making process deals with the complex organizational environment. The scheduling of crew, buses, depots and resources for effective running of the corporation require a scientific and rational decision making. Mathematical models provide the basic frame work for optimum utilization of resources. The APSRTC require an efficient decision making system for this optimal utilization of the resources. In this paper, we integrate mathematical modeling with transportation system and obtain the optimum scheduling for district transportation. The analysis is carried out with the optimal decision variables viewing the problem as a multi-criteria decision model. In this paper, a mathematical model for bus transportation system of Vizianagaram bus depot runs by APSRTC. The objective function of minimum transportation cost subject to the constraints is developed as a goal programming problem. Collecting the data from Vizianagaram bus depot on costs, waiting time of passengers, number of routes, number of buses, operation cost, maintenance cost, peak hour traffic, the model is developed and solved.

Introduction:

It is well known that the development of any nation is mainly linked with proper planning of urban areas. The major consideration of planning is to provide better facilities and opportunities to the persons there in. On par with the other facilities, providing better transportation is a must. The general transportation facilities are Rail transportation and Road transportation. In the transportation system, the phenomenon is to move persons or goods from one place to another is through bus transportation which involves huge funds, personal resources etc.

Decision making is the process of identifying and choosing alternatives based on the values and preferences of the decision maker. It is the process of sufficiently reducing uncertainty and doubt about alternatives to allow a reasonable choice to be made from among them. Decision making based solely on a single criterion appears insufficient as soon as the decision-making process deals with the complex organizational environment. So, one must acknowledge the presence of several criteria that lead to the development of multi-criteria decision making.

Optimization is a kind of the decision making, in which decisions have to be taken to optimize one or more objectives under some prescribed set of circumstances. These problems may be a single or multi-objective and are to be optimized (maximized or minimized) under a specified set of constraints. The constraints usually are in the form of inequalities or equalities. Such problems which often arise as a result of mathematical modeling of many real life situations are called optimization problems.

These optimization problems are quite common in transportation system like Road Transportation Corporation of Andhra Pradesh. Millions of people are utilizing the bus facility for their mode of transportation. The scheduling of crew, buses, depots and resources for effective running of the corporation require a scientific and rational decision making. Mathematical models provide the basic frame work for optimum utilization of resources. The APSRTC require an efficient decision making system for this optimal utilization of the resources.

Hence in this paper, we attempt to develop a mathematical model for optimal scheduling of reservation system applied to RTC bus station.

Single-objective optimization problem

In many real life situations optimization problems are modeled and solved as single-objective optimization problems in a deterministic and crisp environment. The general form of single-objective optimization problem is: Minimize (or Maximize) $f(X)$, $X = (x_1, x_2, \dots, x_n)$

Subject to

$g_j(X) \le 0, \quad j = 1, 2, \dots, k$

$l_j(X) \ge 0, \quad j = 1, 2, \dots, r$

$h_j(X) = 0, \quad j = 1, 2, \dots, m$

where $f, g_1, g_2, \dots, g_k, l_1, l_2, \dots, h_1, h_2, \dots, h_m$ are real valued functions defined on Rn) $X = (x_1, x_2, \dots, x_n) \in R_n$ is called decision vector and $x_1, x_2, \dots, x_n$ are called decision or unknown variables. In case all the functions (objective function and constraints) are linear then the above problem is called linear programming problem, otherwise it is called non-linear programming problem.

The major objectives of the Research paper are:

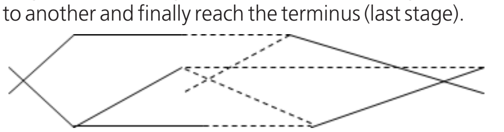
(i) to develop a suitable district transport model for Rural/Urban bus transportation system.
(ii) to obtain optimum scheduling to the model for different constraints.
(iii) to apply the developed model to Vizianagaram bus transportation system
(iv) to obtain the optimum operating strategies of district transportation system satisfying management and passengers requirement.
(v) to suggest the required modification for applying developed model for any district bus transportation system.

Urban / Rural bus transportation system

In this system buses starts at the initial stage with certain number of passengers, stops at each stage, transport some passengers from one stage to another at various stages and finally reaches the terminus (initial stage) and at the terminus it transports all the passengers remaining in the bus.

Urban / Rural bus transportation system:

In a urban / rural bus transportation system a bus starts initially from stage one (1st stage), transport the passengers from one stage to another and finally reach the terminus (last stage).



Starting Point
Terminus Stage 1 Stage 2 Stage (n-1)
Stage n

(vi) Diagram representing the urban / rural bus transportation system.

our prime motto is to develop a suitable mathematical model which assures comfortable journey to the passengers without any loss to the management. This can be achieved by controlling the number of passengers from one stage to the another subject to bus capacity, travelling cost and travelling line. Thus the decision variables here are for obtaining a feasible solution for this sort of models we first develop following constraints.

Logical Constraints

In circular Rural / Urban Transportation model, the transshipment from **i**[th] stage to the prior stages is not feasible.

$\therefore x_{ij} = 0 \quad i < j$ ……… 7.5

Formulation of the problem :

Each constraint can be viewed as an object for subject to the other. Specifically, if we consider the cost constraint as our objective function, than the objective is to minimize the loss to the management subject to time constraint and capacity constraint.

Let the priorities for cost time and capacity goals be P^1, P_2 and P_3.

Mathematical Model

$$\text{Minimize } Z = \left[p_1 d_l^+ + p_2 \sum_{l=2}^{n} d_l^- + p_3 \sum_{l=n+1}^{2n-1} d_l^- \right]$$

Subject to the constrain

$$\sum_{i=1}^{n-1} \sum_{j=1}^{n} c_{ij} x_{ij} - ct_{en} + d_l^- - d_l^+ = 0$$

$$t_{il+k} - \sum_{l=i}^{i+k-1} \sum_{j=i+1}^{n} t x_{lj} + d_m^- - d_m^+ = 0$$

$$i = 1,2,\dots,n-k-1$$

$$m = 2,3,\dots,n-k$$

$$k_0 - \sum_{l=i}^{i-1} x_{1i} - \sum_{l=j+1}^{n} x_{jl} + d_m^- - d_m^+ = 0$$

$$j = 1,2,\dots,n-1$$

$$m = n, n+1, \dots, 2n-1$$

$x_{ij}, d_l^-, d_e^+, d_m^-, d_m^+ \geq 0$, We set $p_1 = p_2 = p_3$ if all the goals are equally important.

Data and model for Vizianagaram bus Depot:

Table. 1 VIZIANAGARAM DEPOT AT A GLANCE

Total Product	RTC Depot	Schedule Hire	Total	Schedule Kms	Services Maximum
Super luxary	2	0	2	1182	2.13
Delux	8		8	3952	7.13
	22		22	8694	15.69
Express	21		37	18826	33.96
Palle Velugu	21		64	22581	40.72
Total	74	59	133	55442	
Spare	6	0	6		
				399	

Table. 2 SAMPLING DISTRIBUTION OF VIZIANAGARAM DEPOT ROUTES

Routes	C.P.B (In Rs.)	E.P.B (In Rs.)	Loss / Profit (In Rs.)
Anakapalli	8712	10374	+1662
Visakhapatnam	11401	9001	-2400
Srikakulam	11600	8666	-2934
Rajam	8976	7400	-1576
S. Kota	7700	6300	-1400
Garbham	7490	4200	-3290
Simhachalam	7896	4950	-2946
Andra	7200	8660	+1460

Note :C.P.B = Cost per a bus per a day & E.P.B = Earning per a bus per a day

Table. 3 TOTAL NO. OF TRIPS IN 38 ROUTES OF VIZIANAGA RAM DEPOT

Services	No. of Trips
Super luxary	2
Delux	76
Metro Express	166
Express	118
Palle Velugu	184
Hire Express	94
Hire Palle velugu	386
Total	**1026**

Table. 4 TICKETS CHARGES FROM VARIOUS STAGES TO THE OTHER STAGES – PALLE VELUGU

K.ms	Sta-ge											
000	01	VZM											
004	02	4	Poolbaugh										
012	03	5	4	Nellimarla									
016	04	7	5	4	Gurla								
020	05	10	7	5	4	G. Valsa							
024	06	12	10	7	5	4	Atchi Pura						
028	07	14	12	10	7	5	4	Gari-vidi					
032	08	17	14	12	10	7	5	4	Ch palli				
038	09	19	17	14	12	10	7	5	4	Sambham			
042	10	21	19	17	14	12	10	7	5	4	B pali		
048	11	23	21	19	17	14	12	10	7	5	4	B odm	
052	12	26	23	21	19	17	14	12	10	7	5	4	Rajm

Table .5 TICKETS CHARGES FROM VARIOUS STAGES TO THE OTHER STAGES – EXPRESS

000	01	VZM								
012	02	10	Nelli-marla							
016	03	10	10	Gurla						
028	04	16	10	10	Gari Vidi					
032	05	18	11	10	10	Ch palli				
052	06	29	22	20	13	12	Rajam			

065	07	36	29	27	20	10	10	U. Metta		
069	08	38	31	29	23	20	10	10	Sankili	
075	09	41	35	32	26	241	13	10	10	Palkonda

Table. 6 AVERAGE RUNNING TIME BETWEEN SUCCESSIVE STAGES PER A BUS

Stage From – To	Time From – To
(1) – (2)	10
(2) – (3)	10
(3) – (4)	10
(4) – (5)	10
(5) – (6)	10
(6) – (7)	10
(7) – (8)	10
(8) – (9)	10
(9) – (10)	20
(10) – (11)	20
(11) – (12)	20

Table. 7 STAGES NUMBER OF PASSENGERS IN A BUS WITH SEVEN TRIPS A DAY

Stages S.No	Trips							Average
	↓ T_1	T_2	↓ T_3	T_4	↓ T_5	T_6	T_7	
1	47+60	16+30	45+50	38+66	45+70	23+20	5	31
2	42+50	16+30	44+30	22+60	44+70	22+20	5	28
3	30+50	24+10	40+30	24+60	32+40	20+20	6	25
4	32+50	24+20	35+30	22+40	30+40	20	6	24
5	32+50	25	32+30	18+40	36+40	18	1	22
6	16+20	28	34+20	20	28+30	28	2	22
7	12	18	45+10	7+10	12+25	18	5	18
8	12	20	42+10	7+10	10+15	20	14	18
9	14+10	20	38+10	5+20	10+15	32	20	20
10	14+10	16	5+20	5+20	5+20	20	5	10
		↑		↑		↑		

Note : ↑ : Up trips, ↓ Down trips.

In the above table 'Plus' number gives the number of students with bus pass. There are a number of engineering colleges, degree colleges, junior colleges near by- students go through busses which are overloaded at the college hours like 8.30 AM to 3.30 PM. In the above table it can be seen that, at a particular 2 or 3 stages there may not be much student rush but at that time it is preferable to have another bus as students are also paying for the student-pass. This amount is not counted on the Statistical and Ticket Account Record i.e., STAR sheet.

After obtaining the data as maintained above, to attain the optimal decision variable values, we incorporate the obtained numerical values in the modules discussed in earlier and solve them through weighted Goal Programming technique.

For solving the cost objective function, we modify the cost variables

C_i = Cost of traveling from initial stage to i +1th stage i = 1,2,….,10
As the decision values are non-negligible and are increasing order, we have

$C_1 > C_2, \ldots, > C_{10}$

These constraints can be mathematically shown as :
$C_1 > 0$

$C_i+_1 - C_i > 2$, for all I.

Therefore the decision variation here are cost variables i.e., C_i's for i = 1,2,…,10.

The known constants here are x_{ij} 's and t_{ij}'s.

Cost per a bus = $\frac{\text{Total cost for all trips}}{\text{Total no. of trips}}$

Average running time from 1st Stage to terminus = 140 minutes.

Average earning per a bus = $\frac{\text{Total Revenue}}{\text{Total no. of trips}}$

$$= \frac{4033}{7} = 576 \simeq 580\ Rs.$$

Our problem is $z = \left(2d_1^-, \sum_{i=2}^{10} d_i^-\right)$

Subject to the constraints

$34C_1+42C_2+35C_3+37C_4+26C_5+38C_6+42C_7+6C8+21C_9+2C_{10}+ d_1^- - d_1^+ = 580$

Subject to the constraints

$$C_1+d_2^- - d_2^+ = 0$$

$$C_2-C_1 + d_3^- - d_3^+ = 4$$

$$C_3-C_2 + d_4^- - d_4^+ = 2$$

$$C_4-C_3 + d_5^- - d_5^+ = 2$$

$$C_5-C_4 + d_6^- - d_6^+ = 3$$

$$C_6-C_5 + d_7^- - d_7^+ = 2$$

$$C_7-C_6 + d_8^- - d_8^+ = 3$$

$$C_8-C_7 + d_9^- - d_9^+ = 2$$

$$C_9-C_8 + d_{10}^- - d_{10}^+ = 2$$

$$C_{10}-C_9 + d_{11}^- - d_{11}^+ = 2 \quad \text{and}$$

$C_1,C_2,C_3,C_4,C_5,C_6,C_7,C_8,C_9,C_{10} \geq 0\ \forall$ i=1,2,…,10

This problem is solved through weighted goal programming technique using LINGO Software.

Optimum decision variables are

$$C_1^* = 3.5, C_2^* = 2.5, C_3^* = 2.5,$$

$$C_4^* = 3.5, C_5^* = 2.0, C_6^* = 2.5,$$

$$C_7^* = 3.5, C_8^* = 2.0, C_9^* = 2.5, C_{10}^* = 2.0$$

Optimal Solution:
Stages Average number of passengers

1	42
2	39
3	41
4	35
5	25
6	43
7	47
8	06
9	20
10	04

Actually if there is no loss the RTC has to run the buses with this optimal rates but the present charges are :

$C_1=4, C2=5, C3=7, C4=10, C5=12, C6=14, C7=17$, C8=19,C9=21,C10=23

The differences between the existing charges and optimal charges are :

$$C_1 - C_1^* = 0$$

$$C_2 - C_2^* = 0.5$$

$$C_3 - C_3^* = 0.5$$

$$C_4 - C_4^* = 0.5$$

$$C_5 - C_5^* = 0.5$$

$$C_6 - C_6^* = 0$$

$$C_7 - C_7^* = 0.5$$

$$C_8 - C_8^* = 0$$

$$C_9 - C_9^* = 0.5$$

$$C_{10} - C_{10}^* = 0$$

This prevailing situation is optimal with respect to the management.

But passengers convenience is also a major consideration for public sector undertaking.

Summary and conclusions:

In many rural routes in Vizianagaram depot occupancy ratio is much less and CPB is less than EPB which mean that there is a loss to the depot. The reasons are many. People of rural areas particularly nearby villages are opting for autos as they don't want to wait for the RTC bus to come. At peak load hours like 8.00 to 10.30 and 3.00 to 6.00, we can see the autos are jam packed with passengers of many sectors like labourers, teachers, students, small traders etc. They want to go to their work places as well as their homes without much waiting time, so that they can have enough of time to spend with their families. Also every day many patients are visiting the Vizianagaram city for their medical checkup and diagnostic tests. In Vizianagaram district we see that every day there are road accidents due to rash driving of autos and many are injured or died, which is a great loss to their families. Also complaints of blocking main roads and holding traffic up stopping abruptly in the middle of the road are common.

In today's fast world, everything has become very fast and speed has become the other name for life. This theory is applicable for all the different aspects of life. And so it even includes fast driving and riding as well. To add to this the auto manufacturers are coming up with faster cars and bikes everyday. This has lead to an increase in the number of accidents every year and it seems to increase year after year. Although different governmental and non governmental organizations do carry out workshops and other training programs to make people aware of careless driving, yet this whole process has not been very successful till date In this paper, a mathematical model for bus transportation system of Vizianagaram bus depot run by APSRTC is carried utilizing, the multi criteria decision modeling. After conducting brain storming sessions, with the personal of bus depot and interacting with the passengers, these important goals are observed. They are time constraint, cost constraint and logical constraint. Collecting the data from Vizianagarm bus depot on costs, waiting time of passengers, number of routes, number of buses, operation cost, maintenance cost, peak hour traffic, the model is developed and solved. The optimum number of passengers to be transported from one stage to another stage is obtained. These optimum values are useful for scheduling the transportation system, most effectively and efficiently.

This study is very helpful for understanding the dynamics of bus transportation system in a urban/rural area and useful optimal decision making. It is possible to develop this model for overall scheduling of bus transportation system in Andhra Pradesh with several other constraints. This paper has provided an opportunity to interact with the officers and employees of APSRTC bus depot at Vizianagaram and public using the transportation system.

References:

[1] Ammar, E E. and Youness, E A., Study on multi-objective transportation problem with fuzzy numbers. Applied Mathematics and Computation, Vol. 166, 2005, pp.241-253.

[2] B.G. Raghavendra and M. Muthiraja; Optimal allocation of buses to Depots – a case study – opsearch Vol.24, No.4 (1987).

[3] Charnes, A and W.W. Cooper ; Management models and industrial applications of linear programming. Vol. I and II, New york; Wiley 1961

[4] Charnes and Cooper; Goal Programming and multiple objective optimizations, European Journal of operational research (1977) pp39-54.

[5] Jaiprakash and Sinha ; Bus transportation Crews planning by goal programming ; Socio Economic Planning sciences, Vol.18, (1984) pp 207-210

[6] Kasana, H. S. and Kumar, K. D., Introductory Operations Research: Theory and Applications, Springer, 4th ed., 2008.

[7] Kautmann, A. and Gupta, M. M., Fuzzy Mathematical Models in Engineering and Management Science, Amsterdam, The Netherland Elsevier, 1988.

[8] Kikuchi, S., A method to defuzzify the number: transportation problem application. Fuzzy Sets and Systems, Vol. 116,2000, pp. 3-9.

[9] Klir, G. J. and Yuan, B., Fuzzy sets and Fuzzy Logic: Theory and Applications,5th ed., (Prentice Hall of India, 1995).

[10] K.V. Ramani, G. Radhuram ; Utilization of transportation units – an optimal alloction study; Opsearch Vol. 17 No.1 (1980).

[11] Li, L. and Lai, K. K., A frizzy approach to the multi-objective transportation problem. Computers and Operational Research, Vol. 28,2000, pp. 43-57.

[12] Lohgaonkar, M. H. and Bajaj V. H., Fuzzy approach to solve multi-objective capacitated transportation problem. International Journal of Bioinformatics Research, Vol. 2,2010, pp. 10-14.

[13] Taha, H.A., Operation Research: An Introduction, 5th ed. (Macmillan, NewYork. 1992).

[14] Wahed, W. F. and Sinna, M. A., A hybrid fuzzy-goal programming approach to multiple objective decision making problems. Fuzzy Sets and Systems, Vol. 119. 2001,pp.71-85.