ORIGINAL RESEARCH PAPER

Mathematics

ON THE HEPTIC DIOPHANTINE EQUATION WITH THREE UNKNOWNSWITH THREE UNKNOWNS $5(x^2+y^2)-9xy=35z^7$

KEY WORDS: Heptic equation with three unknowns, Integral solutions.

R.Anbuselvi Associate Professor of Mathematics, ADM College for women (Autonomous), Nagapattinam, Tamilnadu, India

K.Kannaki Lecturer of Mathematics, Valivalam Desikar Polytechnic College, Nagapattinam, Tamilnadu, India

ABSTRACT

We obtain three different patterns of non-zero integral solutions of the Heptic Diophantine equation with three unknowns $5(x^2+y^2)-9xy=35z^7$ by employing suitable transformations.

1. Introduction

Diophantine equations, homogeneous and non- homogeneous have aroused the interest of numerous mathematicians since antiquity as can be seen from [1-2]. The problem of finding all integral solutions of on Interminate equation with three or more variables in general presents a good deal of difficulties. Cubic equations in two variables fall into the theory of elliptic curves which is a very developed theory but still an important topic of current research [3-13]. For equations with degree at least three very little is known. In this communication a Heptic Polynomial equation with three variables represented by $5(x^2+y^2)-9xy=35z^7$ is considered and three different patterns of non-zero integral solutions have been presented.

2. Method of Analysis:

The equation under consideration is

$$5(x^2+y^2)-9xy=35z^7 \quad --- \quad (1)$$

Assigning the transformations

$$x=u+v,\ y=u-v \quad --- \quad (2)$$

in (1) leads to

$$u^2+19v^2=35z^7 \quad --- \quad (3)$$

Assume that $z=a^2+19b^2$ --- (4)

Where a and b are non-zero distinct integers in (3) we get

$$u^2+19v^2=35\,(a^2+19b^2)^7 \quad --- \quad (5)$$

Different patterns of solutions of (1) are presented below.

Pattern I:

Write 35 as

$$35=(4+i\sqrt{19})(4-i\sqrt{19}) \quad --- \quad (6)$$

use (6), (5) & (4) in (3) and applying the method of factorization, define

$$u+i\sqrt{19}v=\left(4+i\sqrt{19}\right)(a+i\sqrt{19}b)^7 \quad --- \quad (7)$$

Equating the real and imaginary parts, we have

$$u=u(a,b)=4a^7-1596a^5b^2+50540a^3b^4-192052ab^6-133a^6b$$
$$+12635a^4b^3-144039a^2b^5+130321b^7$$

$$v=v(a,b)=a^7-399a^5b^2+12635a^3b^4-48013ab^6+28a^6b$$
$$-2660a^4b^3+30324a^2b^5-27436b^7$$

Substituting the above values of u and v in equation (2), and hence the non-zero integral solutions of (1) are

$$\left.\begin{aligned} x &= 5a^7 - 1995a^5b^2 + 63175a^3b^4 - 240065ab^6 - 105a^6b \\ &\quad +9975a^4b^3 - 113715a^2b^5 + 102885b^7 \\ y &= 3a^7 - 1197a^5b^2 + 37905a^3b^4 - 144039ab^6 - 161a^6b \\ &\quad +15295a^4b^3 - 174363a^2b^5 + 157757b^7 \\ z &= a^2 + 19b^2 \end{aligned}\right\} \quad (8)$$

Pattern II:

Equ (3) can be written as

$$u^2 + 19v^2 = 35z^7 * 1 \quad \text{---} \quad (9)$$

Instead of (6), we write as

$$35 = \frac{(8+2\sqrt{19}i)(8-2\sqrt{19}i)}{4} \quad \text{---} \quad (10)$$

and also 1 as

$$1 = \frac{(9+i\sqrt{19})(9-i\sqrt{19})}{100} \quad \text{---} \quad (11)$$

use (5), (11), (10) in (9) and applying the method of factorization, define

$$u + i\sqrt{19} = \frac{1}{20}\left\{(8+2\sqrt{19}i)(9+i\sqrt{19})\left(a+i\sqrt{19}b\right)^7\right\} \quad \text{---} \quad (12)$$

Equating the real and imaginary part, we have

$$u = u(a,b) = \frac{1}{20}\{34a^7 - 13566a^5b^2 + 429590a^3b^4 - 1632442ab^6$$
$$-3458a^6b - 328510a^4b^3 - 3745014a^2b^5 + 3388346b^7\}$$

$$v = v(a,b) = \frac{1}{20}\{26a^7 - 10374a^5b^2 + 328510a^3b^4 - 1248338ab^6$$
$$+238a^6b - 22610a^4b^3 + 257754a^2b^5 - 233206b^7\}$$

Substituting the values of u and v in equ (2), then the values of x and y are given by

$$\left.\begin{aligned} x &= \frac{1}{20}\{60a^7 - 23940a^5b^2 + 758100a^3b^4 - 2880780ab^6 \\ &\quad -3220a^6b - 351120a^4b^3 - 3487260a^2b^5 - 3155140b^7\} \\ y &= \frac{1}{20}\{8a^7 - 3192a^5b^2 + 101080a^3b^4 - 384104ab^6 \\ &\quad -3696a^6b - 305900a^4b^3 - 4002768a^2b^5 + 3621552b^7\} \end{aligned}\right\} \quad (13)$$

As our interest is on finding integer solutions, we choose a and b suitably so that the values of x and y are in integers.

Replace a by 20A and b by 20B in (4) and (13) we get

$$\left.\begin{aligned} x &= 20^6\{60A^7 - 23940A^5B^2 + 758100A^3B^4 - 2880780AB^6 \\ &\quad -3220A^6B - 351120A^4B^3 - 3487260A^2B^5 - 3155140B^7\} \\ y &= 20^6\{8A^7 - 3192A^5B^2 + 101080A^3B^4 - 384104AB^6 \\ &\quad -3696A^6B - 305900A^4B^3 - 4002768A^2B^5 + 3621552B^7\} \\ z &= 400A^2 + 7600B^2 \end{aligned}\right\} \quad (14)$$

Pattern III:

Let $35 = \frac{(11+i\sqrt{19})(11-i\sqrt{19})}{4}$ --- (15)

And also 1 a

$1 = \frac{(5+i3\sqrt{19})(5-i3\sqrt{19})}{196}$ --- (16)

Following the same procedure as in pattern II, the non-zero integral solutions of (1) are

$$\left.\begin{aligned} x &= 28^6\{36A^7 - 14364A^5B^2 + 454860A^3B^4 - 1728468AB^6 \\ &\quad -5068A^6B + 481460A^4B^3 - 5488644A^2B^5 + 4965916B^7\} \\ y &= 28^6\{-40A^7 + 15960A^5B^2 - 505400A^3B^4 + 1920520AB^6 \\ &\quad -5040A^6B + 478800A^4B^3 - 5458320A^2B^5 + 4938480B^7\} \\ z &= 784A^2 + 14896B^2 \end{aligned}\right\} \quad (17)$$

Note:

It is worth mentioning here that I can also be represented as follows

1. $35 = \frac{(12+3\sqrt{19}i)(12-3\sqrt{19}i)}{9}$
2. $35 = \frac{(16+4\sqrt{19}i)(16-4\sqrt{19}i)}{16}$
3. $35 = \frac{(20+5\sqrt{19}i)(20-5\sqrt{19}i)}{25}$
4. $35 = \frac{(24+6\sqrt{19}i)(24-6\sqrt{19}i)}{36}$ and so on.

3. Conclusion:

In this paper we have presented three different patterns of non-zero integral solutions of the Heptic Diophantine equation with three unknown (1). One may search for other patterns of solutions and their corresponding properties.

Reference:

1. L.E. Dickson, History of Theory of Numbers, Vol.2 Chelsea Publishing Company, New York (1952).
2. Mordell, L.J."Diophantine equations", Academic Press, New York 1969.
3. M.A. Gopalan, S. Vidhyalakshmi and S. Devibala, Integral solutions of $49x^2 + 50y^2 = 51z^2$ Acta Ciencia Indica, XXXIIM, No.2, 839(2006).
4. M.A. Gopalan, Note on the Diophantine Equation $x^2 + xy + y^2 = 3z^2$, Acta Ciencia Indica, XXVI M, No.3, 265, (2000).
5. M.A. Gopalan and Manju Somanath and N. Vanitha, Ternary Cubic Diophantine Equation $2^{2a-1}(x^2 + y^2) = z^3$, Acta Ciencia Indica, Vol.XXXIV M, No.3, 1135-1137 (2008).
6. M.A. Gopalan and G. Sangeetha, Integral solutions of Ternary non-homogeneous biquadratic equation $x^4 + x^2 + y^2 - y = z^2 + z$, Acta Ciencia Indica Vol.XXXVII M.No.4, 799-803 (2011).
7. M.A. Gopalan, Manju Somanath and G. Sangeetha, Integral solutions of non-homogeneous quadratic equation $x^4 - y^4 = (k^2 + 1)(z^2 - w^2)$, Archimedeas journal of mathematics 1(1), 51-57, 2011.
8. M.A. Gopalan and G. Janaki, Integral solutions of $(x^2 - y^2)(3x^2 + 3y^2 - 2xy) = 2(z^2 - w^2)p^3$, Impact. J. Sci. Tech., Vol.4, No.97-102, 2010.
9. M.A. Gopalan and G. Sangeetha, On the Sextic Diophantine equation with three unknowns $X^2 - XY + Y^2 = (K^2 + 3)^n z^6$, impact. J. Sci. Tech., Vol.4, No.4, 89-93, 2010.
10. Anbuselvi R, Kannaki K, On ternary Quadratic Equation $11x^2 + 3y^2 = 14z^2$ Volume 5, Issue 2, Feb 2016, Pg No. 65-68.
11. Anbuselvi R, Kannaki K, On ternary Quadratic Equation $x^2 + xy + y^2 = 12z^2$ IJAR 2016: 2 (3); 533-535.
12. Anbuselvi R, Kannaki K, On ternary Quadratic Equation $3(x^2 + y^2) - 5xy + x + y + 1 = 15z^3$ IJSR Sep 2016: 5(9); 42-48.
13. Anbuselvi R, Kannaki K, On ternary Quadratic Diophantine Equation $7(x^2 + y^2) - 13xy + x + y + 1 = 31z^2$ IERJ Feb 2017: 3(2); 52-57.