ORIGINAL RESEARCH PAPER

Statistics

# CONSTRUCTION OF A NEW SERIES OF PBTD

**KEY WORDS:** Balanced Ternary Design; Partially Balanced Ternary Design.

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**ABSTRACT**

Partially Balanced Ternary Design's were introduced by Mehata, Agarwal and Nigam (1975) as generalization of Balanced Ternary Design's (BTD). This paper provides a new series for the construction of Partially Balanced Ternary Design using Balanced Ternary Design. The method is illustrated with a suitable example.

## 1. INTRODUCTION

The arrangement of 'v' treatments in 'b' blocks, each of sizes $k_1$, $k_2$, … , $k_b$, each of the treatment appears $r_1$, $r_2$, … , rv blocks such that some pairs of treatments occur in $l_1$ blocks, some pairs of treatments occur in $l_2$blocks, soon some rest of pairs of treatments occur in lm blocks then the design is said to be a 'General Incomplete Block Design'. In any incomplete block design if each treatment occurs at most once in blocks then the design is said to be 'binary' block design and if it occurs at most (n-1) times the design is said to be 'n- ary' block design.

Balanced n-ary Block Designs were introduced by Tocher (1952) as generalization of Balanced Incomplete Block Designs (binary) by allowing a treatment to occur more than once in a block.

**DEFINITION 1.1:** A Balanced n-ary Block Design (BnBD) is one whose incidence matrix $N_{BxV}$ has $n_{ij}$ (j = 1,2, … , B, i= 1,2,…,V), as elements where nij takes any one of the n-distinct values 0, 1, … , n-1 and the variance of the comparison between any two treatment is the same. For such a design, 'V' treatments are arranged in 'B' blocks each of size K such that every treatment is replicated 'R' times and S $n_{ij}n_{ij}'$ = p is constant.

**DEFINITION 1.2:** A block design with 'V' treatments, 'B' blocks is said to be Partially Balanced n-ary Block Design with p- associate classes if

(i) The incidence matrix $N_{BxV}$ has n entries 0,1,2, ..n-1
(ii) The row sum $N_{BxV}$ is K
(iii) The column sum of $N_{BxV}$ is R and the column sum of squares is d
(iv) The inner product of any two columns of NBxV is pa, if q and f are mutually ath associates a=1, 2, …, p
(v) There exists a relationship between the treatments defined as
(a) Any two treatments are either $1^{st}$ , $2^{nd}$, or pth associate being symmetrical,
(b) Each treatment q has na- a associates. If q and f are $a^{th}$ associates the number of treatments that are $j^{th}$associates of q and kth associates of f is pjk

In particular, in the incidence matrix NBxV, elements nij takes three values 0,1,2 the design corresponding to the incidence matrix is called 'Partially Balanced Ternary Design (PBTD)' and. nij takes four values 0, 1, 2, 3 the corresponding design is called 'Partially Balaced Quarternary Design (PBQD)'. This paper provides new series of methods for the constructions of Partially Balanced Ternary Design (PBTD).

## 2. METHOD OF CONSTRUCTION OF PARTIALLY BALANCED TERNARY DESIGN

**METHOD 2.1:** A new series of Partially Balanced Ternary Design can be obtained from the combinatorial arrangement of the incidence matrix of a Balanced Ternary Design is presented below.

**Method of Construction:**

**Step 1:** Let N be the incidence matrix of a Balanced Ternary Design with parameters V, B, R, K and p. Let J be the dual design of N.

**Step 2:** Arrange the incidence matrix and its dual in the form

$$N' = \begin{bmatrix} N & J \\ J & N \end{bmatrix}$$

**Step 3:** The resulting design is the incidence matrix of three associate class Partially Balanced Incomplete Block Design with parameters V = 2V, B = 2B, R = B+R, K = V+K. $\pi_1$ =+πB, π2 = 2($r_1$ + 2$r_2$).

**Theorem 2.1:** A Partially Balanced Ternary Design with parameters V = 2V, B = 2B, R = B+r, K = V+K can be constructed using the combinatorial arrangement of N and J in N, where N is the incidence matrix of Balanced Ternary Design.

$$N' = \begin{bmatrix} N & J \\ J & N \end{bmatrix}$$

**Proof:** let NVxB be the incidence matrix of a Balanced Ternary Design with parameters V, B, R, K,π. Let the incidence matrix contains r1 number of 1's and $r_2$ number of 2's in each row, then R = $r_1$+ 2$r_2$. Let the $k_1$ be the number of 1's and $k_2$ be the number of 2's in the each column K = $k_1$+ 2$k_2$. Let the number of pairs (0, 0), (0, 1), (1, 0), (1, 1) (0, 2), (2, 0), (2, 2), (2, 1), (1, 2) are occurring in any two rows constant number of times in N.

The arrangement incidence matrix N and J in N' in the form

$$N' = \begin{bmatrix} N & J \\ J & N \end{bmatrix}$$

As a results it will contains 2V treatments, 2B blocks, each block size is V+K and each treatment replicated B+V times due to augmenting J of order VxB. The pairs of occurrences within first V rows and within last V rows will be same as one associate ($\pi_1$= π+B) and First V rows with second V rows will form different associate ($\pi_2$ = $r_1$ +2$r_2$) forms a PBTD.

The method is illustrated in the example 2.1

**EXAMPLE 2.1:** Let N be the incidence matrix of a Balanced Ternary Design with parameters V = 4, B = 12, K = 4, R = 12, π = 10.

$$N = \begin{bmatrix} 1 & 1 & 0 & 2 & 1 & 1 & 2 & 0 & 1 & 1 & 2 & 0 \\ 1 & 1 & 2 & 0 & 2 & 0 & 1 & 1 & 2 & 0 & 1 & 1 \\ 2 & 0 & 1 & 1 & 1 & 1 & 0 & 2 & 0 & 2 & 1 & 1 \\ 0 & 2 & 1 & 1 & 0 & 2 & 1 & 1 & 1 & 1 & 0 & 2 \end{bmatrix}$$

The combinatorial arrangement of N and J in N provides

$$N' = \begin{bmatrix} N & J \\ J & N \end{bmatrix}$$

$$N' = \begin{bmatrix} 1 & 1 & 0 & 2 & 1 & 1 & 2 & 0 & 1 & 1 & 2 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 0 & 2 & 0 & 1 & 1 & 2 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 & 1 & 1 & 0 & 2 & 0 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 1 & 1 & 0 & 2 & 1 & 1 & 1 & 1 & 0 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 2 & 1 & 1 & 2 & 0 & 1 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 0 & 2 & 0 & 1 & 1 & 2 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 0 & 1 & 1 & 1 & 1 & 0 & 2 & 0 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 2 & 1 & 1 & 0 & 2 & 1 & 1 & 1 & 1 & 0 & 2 \end{bmatrix}$$

The resulting design is a PBTD with parameters $V = 8$, $B = 24$, $K = 8$, $R = 24$, $\pi_1 = +B$, $\pi_2 = 2(r_1 + 2r_2)$

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