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RADIO LABELING OF N-SUN AND JAHANGIR GRAPH

KEY WORDS: radio, n-sun, Jahangir

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ABSTRACT

For a graph G, let diam(G) denote the diameter of G. For any two vertices u and v in G, let $d_G(u,v)$ denote the distance between u and v. A multilevel distance labeling (or radio labeling) of G is a function f that assigns to each vertex a non-negative integer such that for any pair of vertices u, v, it is satisfied that $f(u)–f(v)\geq diam(G)–d_G(u,v)+1$. The span of f is max f(v) (That is, largest number in f(v)). The radio number of G denoted by rn(G), is the maximum span of a distance labeling for G. This paper determines the radio number for uniform caterpillar.

INTRODUCTION

Multi-level distance labeling can be regarded as an extension of distance two labeling which is motivated by the channel assignment problem introduced by Hale[3]. Suppose we are given a set of stations or transmitters, the task is to assign to each station (or transmitter) with a channel (non-negative integer) such that the interference is avoided. The interference is closely related to the stations-the closer are the stations the stronger the interference that might occur. To avoid the interference, the separation of the channels assigned to nearby stations must be large enough. To model this problem, we construct a graph so that each station is represented by a vertex, and two vertices are adjacent when their corresponding stations are close. The ultimate goal is to find a valid labeling such that the span(range) of the channels used is minimized.

Let G be a connected graph. We denote the distance between any two vertices u and v by $d_G(u,v)$ The diameter of G, denoted by diam(G), is the maximum distance between any two vertices. A distance two labeling (orλ – labeling) with span k is a function $f:V(G)\to\{0,1,2,...,k\}$, such that the following are satisfied

$$|f(u)-f(v)|\geq\begin{cases}2 & \text{if} \quad d(u,v)=1\\ 1 & \text{if} \quad d(u,v)=2\end{cases}$$

The λ-number of G is the smallest k such that G admits a distance –two labeling with span k. Since introduced by Griggs and Yeh[2] in 1992, distance-two labeling has been studied extensively (cf.[1, 2, 5, 6, 7, 8, 10, 11, 14, 15, 17, 18]). A multi-level distance labeling (or radio-labeling) [16,12] with span k for a graph G is a function $f:V(G)\to\{0,1,2,...,k\}$, such that the following holds for any u and V: $|f(u)-f(v)|\geq diam(G)-d_G(u,v)+1$.

The radio number (as suggested by the AM/FM radio channel assignment [5]) for a graph G, denoted by rn(G), is the maximum span of a distance labeling for G. Note that when diam(G) = 2, distance two labeling coincides with multi-level distance labeling, and in this case $\lambda(G)$=rn(G). Besides its motivation by the channel assignment, radio labeling itself is an interesting graph labeling problem and has been studied by several authors. The radio numbers for paths and cycle were investigated by Chartrand et al. , Chartrand Erwin and Zhang [1] and Zhang[6] and were completely solved by Liu and Zhu [7]. In this paper, we completely settle the radio numbers for uniform caterpillar.

Definition1.1: The distance between two vertices u and v of a graph $d_G(u, v)$, is the length of the shortest path between u and v.

Definition1.2: The diameter of a graph G, diam(G), is the maximum distance in a graph, taken over all pair of vertices.

Definition1.3: A radio labeling is a function f that assigns positive integer values to vertices so as to satisfy the radio condition $|f(u)–f(v)|\geq diam(G)–d_G(u,v)+1$.

Remark1.4: For any two vertices u and v, $|f(u)–f(v)| \geq diam(G)$ $d_G(u,v)+1$. Also, $diam(G)–d_G(u,v)\geq 0$. Therefore, $|f(u)–f(v)| \geq 1$Hence, $f(u)\neq f(v)$. Hence, f is a one to one function.

Definition1.5: The span of a radio labeling f, span(f), is the maximum integer assigned by f to a vertex in G.

Definition1.6: The radio number of G, rn(G), is the minimum achievable span.

Note1.7: As we are seeking for the minimum for the minimum span of a distance labeling for G, without loss of generality, we assume that the label 1 is used by any distance labeling.

Definition1.8: A n-sun graph U_n as a graph on 2n nodes consisting of a central complete graph K_n with an outer ring of n vertices, each of which is joined to both endpoints of the closet outer edge of the central core.

Definition1.9: Jahangir graphs J_{mn}, for $m\geq 3$ is a graph on (nm + 1) vertices. That is a graph consisting of a cycle nmC with one additional vertex which is adjacent to m vertices of nmC at distance n to each other on .C_{nm}

Theorem2.1: $rn(U_n)=4n–2$ for all $n\geq 5$ where U_n is the sun graph with 2n vertices.

Proof: Let G = (V, E) be a sun graph U_n with $V = \{u_1, u_2, u_3, ..., u_{2n}\}$ is the vertex set and $E = \{u_iu_{i+1} / 1\leq i \leq 2n-1\} \cup \{u_iu_j / 1\leq i, j \leq 2n; i\neq j\}$ $\cup \{u_{2n}u_1\}$ is the edge set. We first show that $rn(U_n)\geq 4n–2$. Let f be a radio labeling of the sun graph U_nThe diameter of U_n is three as the maximum distance between any two vertices is three. Therefore, by the definition of radio labeling we get that,

$$|f(u)-f(v)|\geq 4-d_G(u,v).$$

We have the distance between any two of the odd vertices $u_1, u_2, ..., u_{2n-1}$ is one and hence their corresponding labeling should differ by three. Also we have the distance between any two of the even vertices u_2, u_6, u_{10} ,...,is three and hence their corresponding labeling should differ by one. Similarly, we have the distance between any two of the even vertices u_4, u_8, u_{12} ,..., is three and hence their corresponding labeling also should differ by one. Without loss of generality we assume that .$f(u_1)=1$

As there are 2n vertices with $|f(u_i)-f(u_j)|\geq 3$ for $1\leq Ii, j\leq 2n$ with $i\neq j$, the radio number of the sun graph can not be less than (4n – 2). We prove that the radio number of sun graph U_n is exactly (4n – 2) by giving a suitable labeling. We define the radio labeling for the vertices of the sun graph U_n as follows.

Case(I): Let us first consider the case when n is odd. Let n = (2m + 1). Define

$$f(u_i)=\begin{cases} 1 & i=1 \\ \dfrac{n+i+3}{2} & i=2 \\ \dfrac{i+8}{4} & i=0 \bmod 4 \\ \dfrac{2n+i+8}{4} & i=2 \bmod 4 \\ \dfrac{2n+3i-1}{2} & i=1 \bmod 2 \end{cases}$$

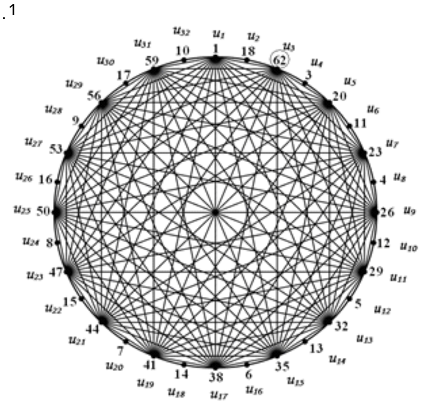
Here, the labels assigned to the vertices are $u_1, u_4, u_8, u_{12}, \ldots, v_{4n}$, $u_2, u_6, u_{10}, \ldots, u_{4n-2}$ are respectively 1, 3, 4, 5,…, (n + 2) and the vertices $u_3, u_5, u_7, \ldots, u_n$ are respectively (n + 4), (n + 7), (n + 10), …, (4n + 2). Therefore, in this case, the radio number is (4n – 2).

Case(ii): Now let us consider the case when n is even. Let n = 2m. Define

$$f(u_i)=\begin{cases} 1 & i=1 \\ n+2 & i=2 \\ \dfrac{i+8}{4} & i=0 \bmod 4 \\ \dfrac{2n+i+6}{4} & i=2 \bmod 4 \\ \dfrac{2n+3i-7}{2} & i=1 \bmod 2,\ i\neq 3 \\ 4n-2 & i=3 \end{cases}$$

Here, the labels assigned to the vertices are
$u_1, u_4, u_8, u_{12}, \ldots, v_{4n}, u_2, u_6, u_{10}, \ldots, u_{4n-2}$
are respectively 1, 3, 4, 5,…, (n + 2) and the vertices
$u_5, u_7, \ldots, u_n, u_3$
are respectively (n + 4), (n + 7), (n + 10), …, (4n – 5), (4n – 2). Therefore, in this case, the radio number is (4n – 2). Thus the above labeling pattern shows that
$rn(U_n)=4n-2,\ n\geq 5.$

Example3.1: Radio labeling for the n-sun graph U16 is given in figure 3.1



U16 (Fig.3.1)

Theorem2.2: $rn(J_{2,m})=2(m+1),\ m\geq 4,$ where mJ,2 is the jahangir graph with 2m vertices.

Proof: Let G = (V, E) be a Jahangir graph $J_{2,m}$ with $V=\{u_1, u_2, \ldots, u_{2m}\}$ is the vertex set and $E=\{u_i u_{i+1} / 1\leq i\leq 2n-1\} \cup \{u_{2m} u_1\} \cup \{u_0 u_i / i=2, 4, 6, \ldots\}$ is the edge set. We first show that $rn(J_{2,m})\}2(m+1)$. Let f be a radio labelingof the Jahangir graph $J_{2,m}$ The diameter of $J_{2,m}$ is four as the maximum distance between any two vertices is four. Therefore, by the definition of radio labeling we get that, $|(f(u_i)-f(u_j)|\geq 3-d_G(u,v)$ We have the distance between any two of the even vertices $u2, u4, u6, \ldots, u_{2n}$ is two and hence their corresponding labeling should differ by three. As $d_G(u0, u_{2i-1})=2$ for $1\leq\iota\leq m$, $|f(u_0)-f(u_{2i-1})|\geq 3$ for $1\leq\iota\leq m$. That is, $f(u_{2i-1})\geq f(u_0)+3$ or $f(u_{2i-1})\leq f(u_0)-3$ for all $1\leq\iota\leq m$. Let us first assume that $f(u_{2i-1})\geq f(u_0)+3$ for $1\leq\iota\leq m$. Hence $f(u_0)$ has to be 1. Let us have that $f(u_1)=4$, As there are 2m vertices with $|f(u_0)-f(u_{2i-1})|\geq 3$ for $1\leq\iota\leq 2m$, the radio number of the Jahangir graph can not be less than (4m + 2). We prove that the radio number of sun graph $J_{2,m}$ is exactly (4m + 2) by giving a suitable labeling. We define the radio labeling for the vertices of the jahangir graph $J_{2,m}$ as follows.

Case(I): Let us first consider the case when n = (3m + 1). Define

$$f(u_i)=\begin{cases} 1 & i=0 \\ 4 & i=1 \\ \dfrac{4m+2i+10}{3} & i=2 \\ \dfrac{8m+2i+10}{3} & i=0 \bmod 3 \\ \dfrac{2i+10}{3} & i=1 \bmod 3 \\ \dfrac{4m+2i+10}{3} & i=2 \bmod 3 \end{cases}$$

Therefore, the above definition we conclude that in this case, the radio number is (4m + 2).

Case(ii): Let us first consider the case when n = (3m + 2). Define

$$f(u_i)=\begin{cases} 1 & i=0 \\ 4 & i=1 \\ \dfrac{8m+2i+10}{3} & i=2 \\ \dfrac{2m+2i+10}{3} & i=0 \bmod 3 \\ \dfrac{2i+10}{3} & i=1 \bmod 3 \\ \dfrac{8m+2i+10}{3} & i=2 \bmod 3 \end{cases}$$

Therefore, the above definition we conclude that in this case, the radio number is (4m + 2).

Case:(iii) Let us first consider the case when n = 3m. Define

$$f(u_i)=\begin{cases} 1 & i=0 \\ 4 & i=1 \\ 4m+2 & i=2 \\ \dfrac{4m+2i+6}{3} & i=0 \bmod 3 \\ \dfrac{2i+10}{3} & i=1 \bmod 3 \\ \dfrac{8m+2i+2}{3} & i=2 \bmod 3 \end{cases}$$

Therefore, the above definition we conclude that in this case, the radio number is (4m + 2). Thus the above labeling pattern shows that $rn(j_{2,m})=4m+2,\ m\geq 4$.

Example3.1: Radio labeling for the Jahangir graph $J_{2,12}$ is given in figure 3.2.

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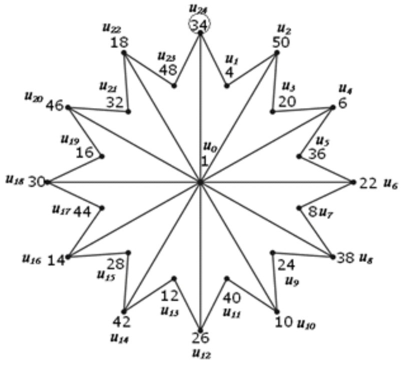

$J_{2,12}$ (Fig.3.2)

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