ORIGINAL RESEARCH PAPER

Mathematics

CONSTRAINTS FOR EQUIVALENCE OF HYDROSTATIC STRESS AND NORMAL PRINCIPAL STRESS

KEY WORDS: Characteristic equation, Eigen values, Hydrostatic stress, Principal stress, Strain.

Manjeel Kumar Assistant Professor in Mathematics, D.A.V. College, Hoshiarpur-146001

ABSTRACT

Hydrostatic stress be the one kind of average of the normal stress components of any stress tensor while Principal stress is one kind of maximum or minimum normal stress when the corresponding shear stresses are zero and the plane on which principal stress act is known as principal plane. As we know that there are infinite numbers of planes passing through the given point and the normal stress on each plane will be different from the other. Therefore If we know all the stress components $\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{zx}, \tau_{yx}, \tau_{zy}$ and τ_{xz} for a particular plane then the principal stresses are the roots of the characteristic equation whose coefficients are stress invaiants I_1, I_2 and I_3 Major aspect of this paper is to discuss the condition in terms of stress in-variants of equivalence of hydrostatic stress and principal stresses along the co-ordinate axes by taking principal axes as the axes of reference.

1. Introduction

There can be infinite number of planes passing through a point and normal stress on each plane will be different from one another. There will be only one plane where normal stress is maximum that plane is known as principal plane and the normal stress on this plane is known as principal stress. Also there is one more plane where normal stress value is minimum, this plane is also known as principal plane and normal stress is known as principal stress. By the maximum principal stress theory, a brittle material ruptures when the maximum principal stress in the specimen reaches some limiting value for the material. This critical value can be inferred as the tensile strength measured using a uni-axial tension test. In practice, this theory is simple, but can only be used for brittle materials. So Principal stresses at a given point in a stressed body is vitally used in design information. With the help of it we can predict whether the design is suitable to hold a given load at a suitable point or not. Principal stresses and principal planes form a backbone of material stress analysis. On the other hand in continuum mechanics, a *hydrostatic stress* is an isotropic *stress* that is given by the weight of water above a certain point. It is average pressure on the body when it under the water to a certain point.

2. **Basic Terminology**:

For the three- dimensional stress analysis, there are three components of normal stresses denoted as $\sigma_x, \sigma_y, \sigma_z$ while there are six shear stress components denoted as $\tau_{xy}, \tau_{yz}, \tau_{zx}, \tau_{yx}, \tau_{zy}$ *and* τ_{xz} as shown in the Fig.2.1. There are three planes of zero shear stress exist, that these planes are mutually perpendicular, and that on these planes the normal stresses have maximum or minimum values.

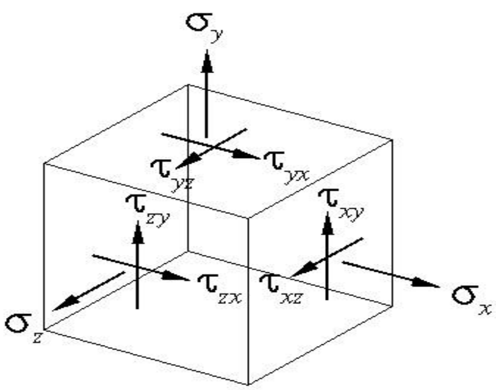

Fig.2.1

As the stress components are the roots of the characteristic equation

$$\begin{vmatrix} \sigma_x - \sigma & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y - \sigma & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z - \sigma \end{vmatrix} = 0.$$

Or $\sigma^3 - \sigma^2 I_1 + \sigma I_2 - I_3 = 0$ ……(i)

$$\text{Where } I_1 = \sigma_x + \sigma_y + \sigma_z$$

$$I_2 = \sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy} - \tau_{yz} - \tau_{zx}$$

$$I_3 = \sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{zx} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2$$

3. Derivation of Condition: By considering the planes of zero shear stress as the coordinate planes and normal to them as the coordinate axes which are the directions corresponding to optimal normal stress. Now, all the shear components become zero i.e. $\tau_{xy} = \tau_{yz} = \tau_{zx} = \tau_{yx} = \tau_{zy} = \tau_{xz} = 0.$

Therefore above expression reduced to

$$I_1 = \sigma_x + \sigma_y + \sigma_z \quad \ldots\ldots(ii)$$

$$I_2 = \sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x \quad \ldots\ldots(\boldsymbol{iii})$$

$$I_3 = \sigma_x\sigma_y\sigma_z \quad \ldots\ldots(iv)$$

Now by suitable transformation we can reduced the eq.(1) to another equation in which the second degree terms are not present. For that we make a substitution as

$$\sigma' = \sigma - \frac{I_1}{3} \quad \ldots\ldots(v)$$

Equation (1) reduced to the form

$$\sigma'^3 + 3A\sigma' + B = 0 \quad \ldots\ldots(vi)$$

$Where\ \ A = \frac{9I_1I_2 - 27I_3 - 2I_1^3}{27},\ B = \frac{3I_2 - I_1^2}{9}$

$$A = \frac{9(\sigma_x + \sigma_y + \sigma_z)(\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x) - 27(\sigma_x\sigma_y\sigma_z) - 2(\sigma_x + \sigma_y + \sigma_z)^3}{27}$$

$$= \frac{-\{(\sigma_x - \sigma_y)^3 + (\sigma_y - \sigma_z)^3 + (\sigma_z - \sigma_x)^3\}}{27} \quad \ldots..(vii)$$

$$B = \frac{3(\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x) - (\sigma_x + \sigma_y + \sigma_z)^2}{9}$$

$$= \frac{-\{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2\}}{18} \quad \ldots..(viii)$$

For equal principal stresses i.e. $\sigma_x = \sigma_y = \sigma_z$, we get from equation (vii) and (viii).

$$A = 0\ and\ B = 0$$

$$9I_1I_2 - 27I_3 - 2I_1^3 = 0\ and\ \ 3I_2 - I_1^2 = 0$$

$$\Rightarrow \quad 27I_3 = I_1^3$$

Under the condition $27I_3 = I_1^3$ we get hydrostatic stress as

$$\sigma_{hyd} = \frac{\sigma_x + \sigma_y + \sigma_z}{3} = \sigma_x$$

4. Conclusion:

When the three principal stresses defining a triaxial state of stress become equal i.e. $\sigma_x = \sigma_y = \sigma_z$ defining a hydrostatic state of stress and no shear stress developed. In amorphous metals, a very slight dependence of the yield stress on the hydrostatic stress is found experimentally also for this state of hydrostatic stress no plastic deformation occurs. Therefore above condition is very useful for the study of yield stress and plastic deformation.

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