ORIGINAL RESEARCH PAPER

Mathematics

EFFECTS OF AN INCLINED MAGNETIC FIELD ON THE MIXED CONVECTIVE FLOW OF A SECOND GRADE FLUID THROUGH POROUS MEDIUM IN A VERTICAL CHANNEL WITH PERMEABLE WALLS

KEY WORDS: MHD, Porous medium, Mixed Convective flow, Second grade fluid.

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ABSTRACT

In this paper, an investigation of the the effect of an inclined magnetic field on the fully developed mixed convection flow of a second grade fluid through a porous medium in a vertical channel with permeable walls is investigated. A closed form solutions of the equations governing the flow are obtained for the velocity and temperature distributions making use of regular perturbation technique. The velocity, temperature and concentration profiles are discussed through graphically as well as skin friction coefficient, Nusselt number presented in the form of tables and discussed for different values of governing flow parameter.

INTRODUCTION

Heat transfer in mixed convection in vertical channels occurs in many industrial processes and natural phenomena. It has therefore been the subject of many detailed, mostly numerical studies for different flow configurations. Most of the interest in this subject is due to its wide applications, for instance, in the design of cooling systems for electronic devices and in the field of solar energy collection. Cheng et al.[1] (1990) have studied the effects of heat transfer on fully developed mixed convection in vertical channels and flow reversal. Laminar mixed convection with viscous dissipation in a vertical channel was analyzed by Barletta [2] (1998). Boulama and Galanis [3] (2004) have investigated the fully developed mixed convection between parallel vertical plates with heat and mass transfer. Barletta et al.[4] (2005) have studied the Dual mixed convection flows in a vertical channel. Sajid et al. (2010) [5] have studied analytical solution for the problem of fully developed mixed convention flow of viscoelastic fluid between two permeable parallel vertical walls.

The use of electrically conducting fluids under the influence of magnetic fields in various industries has led to a renewed interest in investigating hydromagnetic flow and heat transfer in different geoemetries. Misra and Pal (1999) [6] have studied the hydromagnetic flow of a viscoelastic fluid in a parallel plate channel with stretching walls. The hydromagnetic fully developed laminar mixed convection flow in a vertical channel with symmetric and asymmetric wall heating conditions in the presence or absence of heat generation or absorption effects was studied by Chamkha [7] (2002). Bhargava et al. (2003) [8] have studied the effect of magnetic field on the free convection flow of micropolar fluid between two parallel porous vertical plates. Hayat et al. [9] (2004) have studied the Hall effects on the unsteady hydromagnetic oscillatory flow of a second grade fluid. The effect of an inclined magnetic field on unsteady free convection flow of a dusty viscous fluid between two infinite flat plates filled by a porous medium was investigated by Sandeep and Sugunamma [10] (2013). The unsteady MHD poiseuille flow between two infinite parallel plates in an inclined magnetic field with heat transfer has been studied by Idowu et al [11] (2014). Simon [12] (2014) have investigated the effect of heat of transfer on unsteady MHD couette flow between two infinite parallel porous plates in an inclined magnetic field. MHD oscillatory flow through a porous channel saturated with porous medium was investigated by Falade et al.[13] (2017).Recently Raghunath and Siva Prasad [14] (2018) have investigated Heat and mass transfer on MHD flow of Non-Newtonian fluid over an infinite vertical porous plate. Raghunath and Siva [15] Prasad Indigested Heat and mass transfer on Unsteady MHD flow of a second grade fluid through porous medium between two vertical plates.

In view of these, the effect of an inclined magnetic field on the fully developed mixed convection flow of a second grade fluid through porous medium in a vertical channel with permeable walls is investigated. The governing non – linear equations are solved for the velocity field and temperature field using the traditional perturbation technique. The effects of various emerging parameters on the velocity field and temperature field are discussed in detail through graphs.

2. MATHEMATICAL FORMULATION

We consider the laminar mixed convection flow of a second grade fluid through a porous medium in a vertical permeable channel, the space between the plates being h, as illustrated in Fig. 1. It is assumed that the rate of injection at one wall is equal to the rate of suction at the other wall. A uniform magnetic field B_0 is applied in the transverse direction to the flow. A rectangular coordinate system (x,y) is chosen such that the x- axis is parallel to the gravitational acceleration vector , but with opposite direction and the - axis is transverse to the channel walls. The left wall (i.e, at y=0) is maintained at constant temperature T_1 and the right wall (i.e, at y=h) is maintained at constant temperature T_2, where . $T_1 > T_2$ The flow assumed steady and fully developed, i.e. the transverse velocity is zero. Then, the continuity equation drops to .$\partial u / \partial x = 0$

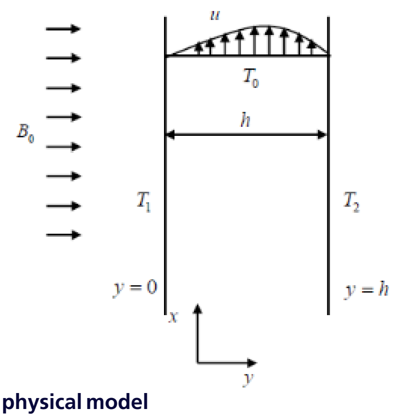

Fig. 1 The physical model

Viscoelastic fluids can be modeled by Rivlin – Ericksen constitutive equation

$$S = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 \quad (2.1)$$

where S is the Cauchy stress tensor, P is the scalar pressure,μ α_1 and α_2 are the material constants, customarily known as the coefficients of viscosity, elasticity and cross - viscosity, respectively. These material constants can be determined from viscometric flows for any real fluid. A_1 and A_2 are Rivlin-Ericksen tensors and they denote, respectively, the rate of strain and acceleration. A_1 and A_2 are defined by

$$A_1 = \nabla V + (\nabla V)^T \quad (2.2) \text{ and } A_2 = \frac{dA_1}{dt} + A_1(\nabla V) + (\nabla V)^T A_1 \quad (2.3)$$

where d/dt is the material time derivative and ∇ gradient operator and $(\,)^T$ transpose operator. The viscoelastic fluids when modeled by Rivlin-Ericksen constitutive equation are termed as second-grade fluids. A detailed account of the characteristics of second -

grade fluids is well documented by Dunn and Rajagopal (1995). Rajagopal and Gupta (1984) have studied the thermodynamics in the form of dissipative inequality (Clausius –Duhem) and commonly accepted the idea that the specific Helmholtz free energy should be a minimum in equilibrium. From the thermodynamics consideration they assumed

$$\mu \geq 0,\ \alpha_1 > 0,\ \alpha_1 + \alpha_2 = 0 \quad (2.4)$$

The basic equations of momentum and energy governing such a flow, subject to the Boussinesq approximation, are

$$\rho v_0 \frac{du}{dy} = -\frac{dp}{dx} + \mu \frac{d^2u}{dy^2} + \alpha_1 v_0 \frac{d^3u}{dy^3} - \left(\sigma B_0^2 \cos^2\alpha + \frac{\mu}{K}\right)u + \rho g \beta (T - T_0)$$

(2.5)

$$v_0 \frac{dT}{dy} = \alpha \frac{d^2T}{dy^2} \quad (2.6)$$

where p is the pressure, ρ is the density, μ is the dynamic viscosity of the fluid, g acceleration due to gravity , β coefficient of thermal expansion, α_1 is the viscoelastic parameter, σ is the electrical conductivity and $v0$is the transpiration cross flow velocity . Further, here dp/dx is a constant.

The boundary conditions are given by
$u(0)(0)=,u(h)= 0,\ T(0) = T_1,\ T(h)= T_2$ (2.7)

Introducing the following non-dimensional variables

$$\bar{y} = \frac{y}{h},\ \bar{u} = \frac{u}{U},\ \theta = \frac{T - T_0}{T_2 - T_0}$$

into the equations (2.5) and (2.6), we obtain

$$kR\frac{d^3u}{dy^3} + \frac{d^2u}{dy^2} - R\frac{du}{dy} - N^2u + \frac{Gr}{\mathrm{Re}}\theta + A = 0 \quad (2.8)$$

$$\frac{d^2\theta}{dy^2} - R\,\mathrm{Pr}\frac{d\theta}{dy} = 0 \quad (2.9)$$

where $N = \sqrt{M^2\cos^2\alpha + \frac{1}{Da}}$ $Da = \frac{K}{h^2}$ is the Darcy number, $k = \frac{\alpha_1}{\rho h^2}$ is the viscoelastic parameter, $M = hB_0\sqrt{\frac{\sigma}{\mu}}$ is the Hartmann number, $R = \frac{\rho v_0 h}{\mu}$ is the cross flow Reynolds number, $Gr = \frac{g\beta(T_2 - T_1)h^3}{\nu^2}$ is the Grashof number, $\mathrm{Re} = \frac{\rho U_0 h}{\mu}$ is the Reynolds number, $\mathrm{Pr} = \frac{\nu}{\alpha}$ is the Prandtl number,

$r_T = \frac{T_1 - T_0}{T_2 - T_0}$ is the wall temperature parameter and

$A = -\left(\frac{dp}{dx}\right)\frac{U_0\nu}{h^2}$ is the constant pressure gradient.

The corresponding dimensionless boundary conditions are given by

$$u(0) = u(1) = 0,\ \theta(0) = r_T,\ \theta(1) = 1 \quad (2.10)$$

3. PERTURBATION SOLUTION

We consider the first - order perturbation solution of the BVP (2.8) - (2.10) for small k. Since the constitute equation (2.1) has been derived up to only the first - order of smallness of k, therefore, the perturbation solution obtained by retaining the terms up to the same order of smallness of must be quite logical and reasonable. We write and

$$u = u_0 + ku_1 \quad (3.1)$$

$$\theta = \theta_0 + k\theta_1 \quad (3.2)$$

Substituting Eqs. (3.1) and (3.2) into the Eqs. (2.8) and (2.9) and boundary conditions (2.10) and then equating the like powers of k, we obtain

3.1 Zeroth-order system (k^0)

$$\frac{d^2u_0}{dy^2} - R\frac{du_0}{dy} - N^2u_0 = -\frac{Gr}{\mathrm{Re}}\theta_0 - A \quad (3.3)$$

$$\frac{d^2\theta_0}{dy^2} - R\,\mathrm{Pr}\frac{d\theta_0}{dy} = 0 \quad (3.4)$$

Together with boundary conditions

$$u_0(0) = u_0(1) = 0,\ \theta_0(0) = r_T,\ \theta_0(1) = 1 \quad (3.5)$$

3.2 First-order system(k^1)

$$\frac{d^2u_1}{dy^2} - R\frac{du_1}{dy} - N^2u_1 = -R\frac{d^3u_0}{dy^3} - \frac{Gr}{\mathrm{Re}}\theta_1 \quad (3.6)$$

$$\frac{d^2\theta_1}{dy^2} - R\,\mathrm{Pr}\frac{d\theta_1}{dy} = 0 \quad (3.7)$$

Together with boundary conditions

$$u_1(0) = u_1(1) = 0,\ \theta_1(0) = 0,\ \theta_1(1) = 0 \quad (3.8)$$

3.3 Zeroth-order solution

Solving equations (3.3) and (3.4) using the boundary conditions (3.5), we get

$$\theta_0 = \frac{(1 - r_T e^{R\,\mathrm{Pr}}) + (r_T - 1)e^{R\,\mathrm{Pr}\,y}}{(1 - e^{R\,\mathrm{Pr}})} \quad (3.9)$$

$$u_0 = c_1e^{ay} + c_2e^{by} + \frac{Gr}{\mathrm{Re}}\left(c_3 - c_4e^{R\,\mathrm{Pr}\,y}\right) + \frac{A}{N^2} \quad (3.10)$$

Where $a = \frac{R + \sqrt{R^2 + 4N^2}}{2}$, $b = \frac{R - \sqrt{R^2 + 4N^2}}{2}$, $c_3 = \frac{(1 - r_T e^{R\,\mathrm{Pr}})}{(1 - e^{R\,\mathrm{Pr}})N^2}$,

$c_4 = \frac{(r_T - 1)}{(1 - e^{R\,\mathrm{Pr}})(R^2\mathrm{Pr}^2 - R^2\mathrm{Pr} - N^2)}$, $c_5 = \frac{Gr}{\mathrm{Re}}(f_1 - f_2) + \frac{A}{N^2}$,

$c_6 = \frac{Gr}{\mathrm{Re}}(f_1 - f_2e^{R\,\mathrm{Pr}}) + \frac{A}{N^2}$, $c_1 = \frac{c_6 - c_5e^b}{e^b - e^a}$ and $c_2 = \frac{c_5e^a - c_6}{e^b - e^a}$.

3.4 First-order solution (or Solution for a second - grade fluid)

Solving Eq. (3.7) with corresponding boundary conditions, we obtain

$$\theta_1 = 0 \quad (3.11)$$

Substituting the equations (3.10) and (3.11) into the Eq. (3.6) and then solving the resulting equation with the corresponding conditions, we get

$$u_1 = c_7e^{ay} + c_8e^{by} - c_{10}ye^{ay} - c_{11}ye^{by} + c_9e^{R\,\mathrm{Pr}\,y} \quad (3.12)$$

where $c_9 = \frac{Gr}{\mathrm{Re}}c_4\frac{R^4\mathrm{Pr}^3}{(R^2\mathrm{Pr}^2 - R^2\mathrm{Pr} - N^2)}$, $c_{10} = \frac{Rc_1a^3}{2a - R}$, $c_{11} = \frac{Rc_2b^3}{2b - R}$ $c_{12} = c_9e^{R\mathrm{Pr}} - c_{10}e^a - c_{11}e^b$,

$c_7 = \frac{c_{12} - c_9e^b}{e^b - e^a}$, $c_8 = \frac{e^ac_9 - c_{12}}{e^b - e^a}$.

Finally, the perturbation solutions up to first order for θ and u are given by

$$\theta = \frac{(1 - r_Te^{R\,\mathrm{Pr}}) + (r_T - 1)e^{R\,\mathrm{Pr}\,y}}{(1 - e^{R\,\mathrm{Pr}})} \quad (3.13)$$

and

$$u = (c_1 + kc_7 - kc_{10}y)e^{ay} + (c_2 + kc_8 - kc_{11}y)e^{by} + \frac{Gr}{\mathrm{Re}}\left(c_3 - c_4e^{R\mathrm{Pr}\,y}\right) + kc_9e^{R\mathrm{Pr}\,y} + \frac{A}{N^2}$$

The rate of heat transfer coefficient in terms of Nusselt number uN at the plate y=0 of the channel is given by

$$Nu = \left.\frac{\partial\theta}{\partial y}\right|_{y=0} = \frac{(r_T - 1)R\,\mathrm{Pr}}{(1 - e^{R\,\mathrm{Pr}})}$$

Note that when k=0, $Da\to\infty$, R=0 and $M\to0$ our results reduces to those given by Aung and Worku (1986).

4. RESULTS AND DISCUSSIONS

In order to see the effects of $M, k, Da, \alpha, R, Pr, Gr$, Re and r_T on the velocity u, we have plotted Figs. 2-11.Fig. 2 illustrates the effect of viscoelastic parameter k on u for M=1, ,Tr=0.5 ,R=5 Da=0.5, α=π/6, A=1,Gr=1, Pr=2and Re=1. It is found that, the velocity u decreases with an increase in k. The point where the maximum velocity occurs is shifted away from the upper wall as the value of the viscoelastic parameter is increased. Further it is observed that, the velocity is more for Newtonian fluid (0→k) than that of second grade fluid. The effect of Hartmann number M on u for $k = 0.02, r_T = 0.5, R = 5, A = 1$ $Da = 0.5$ $\alpha = \pi/6$ Gr=1, Pr=2 and Re=1 is depicted in Fig. 3. It is observed that, the velocity u is decreases with increasing M. Further, it is found that, the velocity is more for non-conducting (magnetic) (i.e.,0→M) second grade fluid than that of conducting second grade fluid. Fig. 4 depicts the effect of Darcy number Da on u for r_T=0.5,

M=1, k=0.01,α=π/6 ,R=5,A=1, Gr=1, Pr=2 and Re=1 .It is found that, the velocity u is increases with increasing Da. Effect of inclination angle α on u for r_T=0.5, M=1, k=0.01, α=π/6,R=5,A=1, Gr=1, Pr=2 and Re=1 is shown in Fig. 5. It is noted that, the velocity is u increases with increasing α. Fig. 6 shows the influence of R on for M=1, r_T=0.5, k=0.01 A=1, Da=0.5, α=π/6, Pr=2, Gr=1 and Re=1 It is observed that, the velocity u decreases with increasing R . The effect of Prandtl number Pr on u for M=1, r_T=0.5,R=5, A=1,Da=0.5, α=π/6, Pr=2, Gr=1 and Re=1 is shown in Fig. 7. It is found that, the velocity u decreases on increasing Prandtl number . Fig. 8 depicts the effect of Grashof number Gr on u for Da=0.5, α=π/6, M=1, r_T=0.5, R=5, A=1, k=0.01,Pr=2, and Re=1 . It is observed that, the velocity increases with increasing Grashof number . The Gr on u of Reynolds number Re on u for$M$=1, r_T=0.5, R=5, Da=0.5, α=π/6, A=1, Pr=2, Gr=1and Re=1 is shown in Fig. 9. It is found that, the velocity u decreases with increasing Reynolds number. Fig. 10 illustrates the effect of wall temperature parameter r_T on u for M=1, k=0.01,R=5, Da=0.5, α=π/6, A=1, Pr=2, Gr=1and Re=1 . It is observed that, the velocity u increases with increasing r_T.Fig. 11 shows the effect of R on the temperature θ for r_T=0.5,and Pr=2. It is found that, the temperature θ decreases on increasing R.The effect of Prandtl number Pr on the temperature θ for r_T=0.5 and R=5, is presented in Fig. 12. It is observed that, the temperature θ is decreases with increasing Prandtl number Pr . Fig. 13 depicts the effect of r_T on temperature θ for R=5, and Pr=2. It is found that, the temperature θ increases with an increase in r_T.Table-1 shows the effect of R on Nusselt number Nu for and . It is found that, the decreases with increasing. Table-2 depicts the effect of Pr on Nusselt number Nu for r_T=0.5 and R=5. It is observed that, the decreases with increasing Pr. Table-3 illustrates the effect of r_T on Nusselt number Nu for R=5 and Pr=2. It is noted that, the Nu decreases with increasing r_T

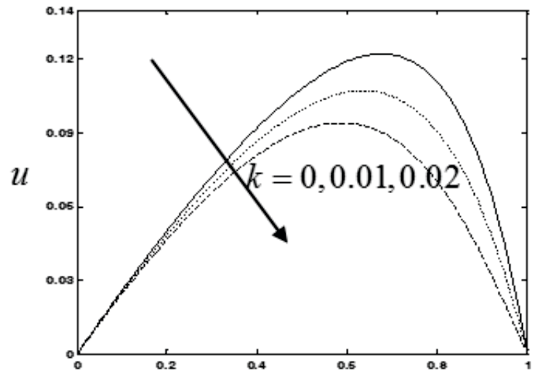

Fig. 2. Effect of viscoelastic parameter k on u for $r_T = 0.5, M = 1, Da = 0.5$ $\alpha = \pi/6$, $R = 5$, $A = 1$, $Gr = 1, \Pr = 2$ and $\mathrm{Re} = 1$

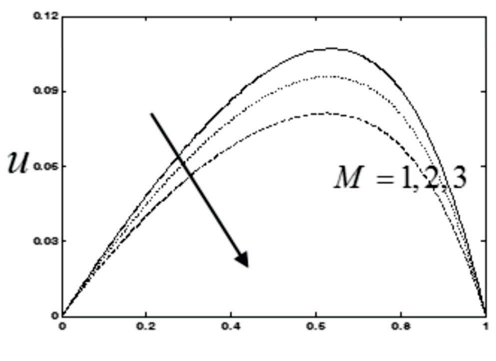

Fig. 3. Effect of Hartmann number M on u for $k = 0.01, R = 5, r_T = 0.5$, $Da = 0.5$, $\alpha = \pi/6, A = 1$, $Gr = 1, \Pr = 2$ and $\mathrm{Re} = 1$.

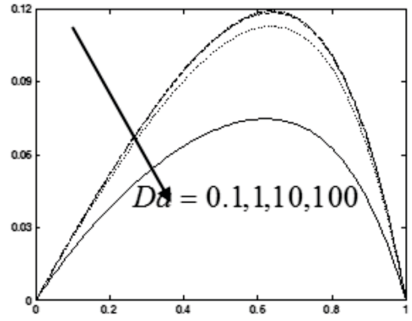

Fig. 4. Effect of Darcy number Da on u for $r_T = 0.5, M = 1, k = 0.01$, $\alpha = \pi/6$, $R = 5$, $A = 1$, $Gr = 1, \Pr = 2$ and $\mathrm{Re} = 1$.

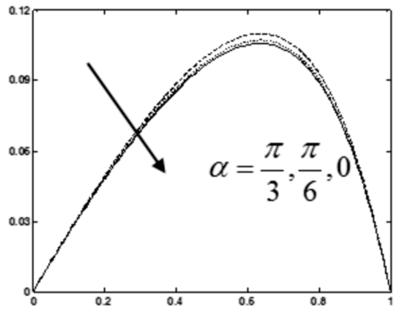

Fig. 5. Effect of inclination angle α on u for $r_T = 0.5, M = 1, k = 0.01, \alpha = \pi/6$, $R = 5$, $A = 1$, $Gr = 1, \Pr = 2$ and $\mathrm{Re} = 1$.

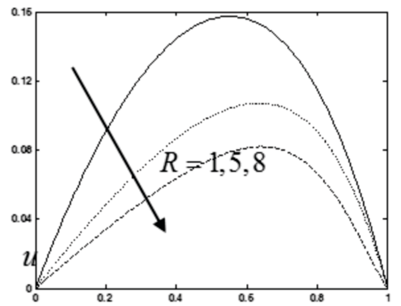

Fig. 6. Effect of R on u for $r_T = 0.5, M = 1, k = 0.01, A = 1$, $Da = 0.5, \alpha = \pi/6$, $Gr = 1, \Pr = 2$ and $\mathrm{Re} = 1$.

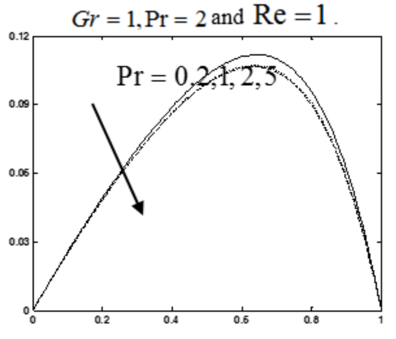

Fig. 7. Effect of Prandtl number Pr on u for $M = 1, r_T = 0.5, R = 5$, $Da = 0.5$, $\alpha = \pi/6$, $A = 1$, $Gr = 1, k = 0.02$ and $\mathrm{Re} = 1$.

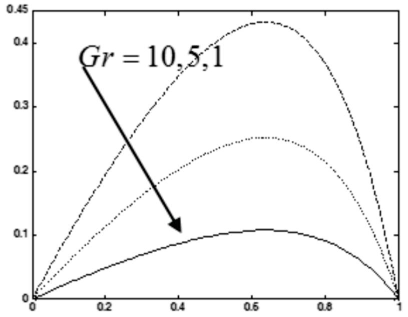

Fig. 8. Effect of Grashof number Gr on u for $r_T = 0.5, R = 5, M = 1, Da = 0.5$, $\alpha = \pi/6$, $k = 0.01, A = 1, \Pr = 2$ and $\mathrm{Re} = 1$.

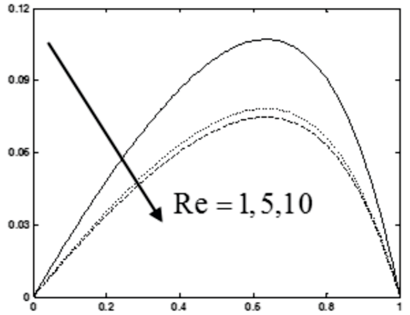

Fig. 9. Effect of Reynolds number Re on u for $M = 1, r_T = 0.5, R = 5, Da = 0.5$, $\alpha = \pi/6$, $Gr = 1, \Pr = 2, A = 1, k = 0.01$ and $\mathrm{Re} = 1$.

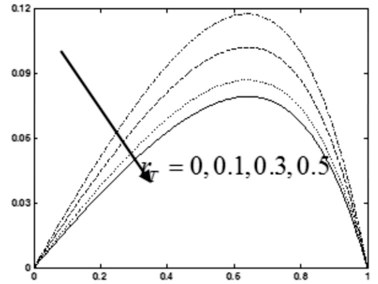

Fig. 10. Effect of wall temperature parameter k on u for $M = 1, k = 0.02$, $Da = 0.5$, $\alpha = \pi/6$, $R = 5$, $A = 1$, $Gr = 1, \Pr = 2$ and $\mathrm{Re} = 1$

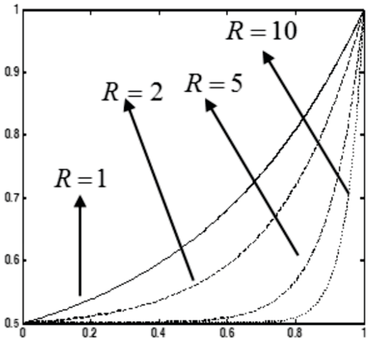

Fig. 11. Effect of R on θ for $r_T = 0.5$ and $\Pr = 2$.

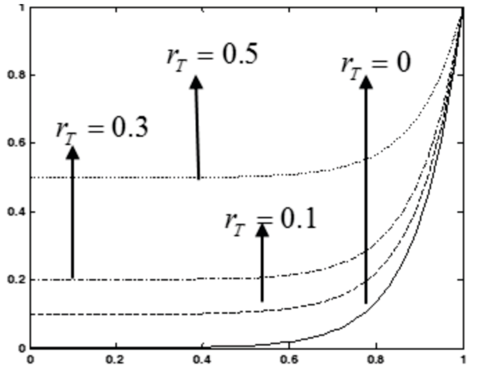

Fig. 13. Effect of r_T on θ for $R = 5$ and $\Pr = 2$.

TABLE-1: Effect of *R* on Nusselt number *Nu* for r_T=0.5, and Pr=2.

R	*Nu*
1	0.1565
2	0.0373
5	0.0002

TABLE-2: Effect of Pr on Nusselt number *Nu* for r_T=0.5, and *R*=5.

Pr	*Nu*
0.2	0.2910
1	0.0170
2	0.0002

TABLE-3: Effect of r_T on Nusselt number *Nu* for *R*=5, and Pr=5.

r_T	*Nu*
0	0.0005
0.1	0.0004
0.3	0.0003
0.5	0.0002

5. CONCLUSION

In this chapter, the effect of an inclined magnetic field on the fully developed mixed convection flow of a second grade fluid through a porous medium in a vertical channel with permeable walls is investigated. The governing non – linear equations are solved for the velocity field and temperature field using the traditional perturbation technique. It is found that, the velocity u decreases with increasing k, M, R, Re and Pr, while it increases with increasing ,Da, α, Gr and r_T. Also, it is observed that, the temperature θ decreases with increasing R and Pr, while it increases with increasing r_T.

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