ORIGINAL RESEARCH PAPER

Mathematics

GENERALIZED αb*-CLOSED SETS IN TOPOLOGICAL SPACES

KEY WORDS: gαb*-closed, gαb*-open, gαb*-nbhd.

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ABSTRACT

In this paper a new class of generalized closed sets in topological spaces, namely generalized αb*-closed(briefly, gαb*-closed) set is introduced. We give some basic properties and various characterizations of gαb*-open sets. Also we introduce gαb*-neighbourhood in a topological spaces and investigate some basic properties.

1.INTRODUCTION

In 1970, Levine introduced the class of generalized closed sets. The notion of generalized closed sets has been extended and studied exclusively in recent years by many topologists. In 1996, Andrjivic gave a new type of generalized closed sets in topological spaces called b-closed sets. Later in 2012 S.Muthuvel and P.Parimelazhagan introduced b*-closed sets and investigated some of their properties.

In this paper, a new class of closed set called generalized αb*-closed set is introduced. The notion of generalized αb*-closed set and its different characterizations are given in this paper. It has been proved that the class of generalized αb*-closed set lies between the class of b-closed set and gb-closed set.

2.Preliminaries

Throughout this paper (X, τ) represents a topological space on which no separation axiom is assumed unless otherwise mentioned. For a subset A of a topological space X, cl(A) and int(A) denote the closure of A and the interior of A respectively. (X, τ) will be replaced by X if there is no changes of confusion. We recall the following definitions and results.

Definition 2.1. Let (X, τ) be a topological space. A subset A of the space X is said to be semi-open [9] if A cl(int(A)) and semi-closed [3] if int(cl(A)) A.

α-open [13] if A int(cl(int(A))) and α-closed ifcl(int(cl(A))) A.

pre-open [14] if A int(cl(A)) and pre-closed if cl(int(A)) A.

b-open [16] if A⊆int(cl(A)) ∪ cl(int(A)) and b-closed if int(cl(A))∩cl(int(A)) ⊆A.

regular open if int(cl(A))=A and regular closed if cl(int(A))=A.

π-open [4] if A is the union of regular open sets and π-closed if A is the intersection of regular closed sets Definition 2.2. Let (X, τ) be a topological space and A X. The b-closure(resp.pre-closure, semi-closure, α-closure) of A, denoted by bcl(A) (resp .pcl(A), scl(A), αcl(A)) and is defined by the intersection of all b-closed (resp. pre-closed, semi-closed, α-closed) sets containing A.

Definition 2.3. Let (X, τ) be a topological space. A subset Aof X is said to be generalized closed [8](briefly g-closed) if cl(A) U whenever A U and U is open in (X, τ).

generalized b-closed [2] (briefly gb-closed) if bcl(A) U whenever A U and U is open in (X, τ).

regular generalized closed [7] (briefly rg-closed) if cl(A) U whenever A U and U is regular open in (X, τ).

regular generalized b-closed [17](briefly rgb-closed) if bcl(A) U whenever A U and U is regular open in (X, τ).

generalized αb-closed [15](briefly gαb-closed) if bcl(A) U whenever A U and U is α-open in (X, τ)

generalized pre-regular closed [20] (briefly gpr-closed) if pcl(A) U whenever A U and U is rg-open in (X, τ).

generalized p-closed (briefly gp-closed) if pcl(A) U whenever A U and U is open in (X, τ).

α-generalized closed [10] (briefly αg-closed) if αcl(A) U whenever A U and U is an open in (X, τ).

π-generalized b-closed [6](briefly πgb-closed) if bcl(A) U whenever A U and U is π-open in (X, τ).

π-generalized pre-closed [6](briefly πgb-closed) if pcl(A) U whenever A U and U is π-open in (X, τ).

π-generalized semi-closed [6](briefly πgb-closed) if scl(A) U whenever A U and U is π-open in (X, τ).

weakly closed [19](briefly w-closed) if cl(A) ⊆ U whenever A U and U is a semi-open in (X, τ).

weakly generalized closed [18] (briefly wg-closed) [2] if cl(int(A)) U whenever A U and U is an open in (X, τ).

semi weakly generalized closed (briefly swg-closed) [] if scl(A) U whenever A U and U is an wg-open in (X, τ).

w-closed [19] if cl(A)⊆U whenever A U and U is a semi-open in (X, τ).

w α-closed [3] if αcl(A)⊆U whenever A⊆U and U is a w-open in (X, τ).

α-generalized closed [10](briefly α g-closed [2] if α cl(A) U whenever A U and U is open in (X, τ).

α- generalized regular closed (briefly α gr-closed [2] if αcl(A) U whenever A U and U is regular open in (X, τ).

strongly b*-closed [21](briefly sb*-closed) if cl(int(A))) U whenever A) U and U is b-open in (X, τ).

The complements of the above mentioned closed sets are their respective open sets.

Theorem 2.4.[21]For a topological space (X,τ),
Every open set is b*-open.
Every α-open set is b*-open.
Every semi-open set is b*-open.

Theorem 2.5.[22] For any subset A of a topological space (X,τ),
sint (A)= A∩cl(int(A))
pin(A)= A∩int(cl(A))
scl (A)= A ∪ int(cl(A))
pcl(A)= A ∪ cl(int(A)).

Remark 2.6. Jankovic and Reilly pointed out that every singleton {x} of a space X is either nowhere dense or pre-open. This provides another decomposition X=X1 ∪ X2, where X1={x∈X/ {x} is nowhere dense} and X2={x∈X/ {x} is pre-open}.

Definition 2.7. The intersection of all gb-open sets containing A is called the gb-kernel of A and it is denoted by gb-ker(A).

Lemma 2.8. For any subset A of X, X2∩cl(A) gb-ker(A).
Remark 2.9. cl(X\A) = X\int(A)

3. Generalized αb*-closed set

Definition 3.1. A subset A of a topological space (X, τ) is called a generalized αb*-closed set (briefly, gαb*-closed) if αcl(A) U whenever A U and U is b*-open in (X, τ).

Theorem 3.2. For a topological space (X, τ),
Every closed set is g αb*- closed.
Every α-closed set is gαb*-closed.
Every regular closed set is g αb* -closed.
Every π-closed set is g αb*- closed.

Proof:
Let A be a closed set. Let A U, U is b*-open in X. Since A is closed, then cl(A)=A U. But αcl (A) cl(A). Thus we have αcl (A) U whenever A U and U is b*-open. Therefore, A is a g αb*- closed set.

Let A be a α-closed set. Let A U, U is b*-open. Since A α-closed, αcl (A)=A U whenever A U and U is b*-open. Therefore, A is g αb*- closed set.

Let A be a regular closed set. Since every regular closed set is closed. Then by (I), A is gαb*-closed set.

Let A be a π-closed set. Since every π-closed set is closed. Then by (I), A is gαb*-closed set.

Theorem 3.3. For a topological space (X, τ),
Every g αb*- closed set is gb-closed.
Every g αb*- closed set is gp-closed.
Every g αb*- closed set is gs-closed.
Every g αb*- closed set is sg-closed.
Every gαb* - closed set is rgb-closed.

Proof:
Let A be a gαb*-closed set. Let A U, U is open. Since open set is b*-open, then U is b*-open. Since A is gαb*-closed, αcl(A) U. But bcl(A)⊆αcl(A). Thus, we have bcl(A) U whenever A U and U is open. Therefore, A is gb-closed set.

Let A be a g αb*- closed set. Let A U, U is open. Since open set is b*-open, then U is b*-open. Since A is g αb*- closed, αcl (A) U. But pcl(A)⊆αcl(A). Thus, we have pcl(A) U whenever A U and U is open. Therefore, A is gb-closed set.

Let A be a gαb*-closed set. Let A U, U is open. Since open set is b*-open, then U is b*-open. Since A is g αb*- closed, αcl(A) U. But scl(A)⊆αcl(A). Thus, we have scl(A) U whenever A U and U is open. Therefore, A is gs-closed set.

Let A be a gαb*-closed set. Let A U, U is semi-open. Since semi-open set is b*-open, then U is b*-open. Since A is gαb*-closed, αcl(A) U. But scl(A)⊆αcl(A). Thus, we have scl(A) U whenever A U and U is semi-open. Therefore, A is sg-closed set.

Let A be a gαb*-closed set. Let A U, U is regular-open. Since every regular open set is b*-open, then U is b*-open. Since A is gαb*-closed, αcl(A) U. But bcl(A)⊆αcl(A). Thus, we have bcl(A) U whenever A U and U is regular-open. Therefore, A is rgb-closed.

Theorem 3.4. For a topological space (X, τ),
1. Every g αb*- closed set is g αb- closed set.
2. Every g αb*- closed set is πgb- closed set.
3. Every g αb*- closed set is πgp-closed set.
4. Every g αb* -closed set is πgs-closed set.
5. Every g αb*- closed set is sgb-closed set.
6. Every g αb* -closed set is gpr-closed set.

Proof.
1. Let A be a g αb*- closed set. Let A U, U is α- open. Since every α-open set is b*-open, then U is b*-open. Since A is gαb*-closed, αcl(A) U. But bcl(A)⊆αcl(A). Thus, we have bcl(A) U whenever A U and U is α-open. Therefore, A is g αb- closed.
2. Let A be a gαb*-closed set. Let A U, U is π-open. Since every π-open set is b*-open, then U is b*-open. Since A is g αb*-closed, α cl(A) U. But bcl(A)⊆αcl(A). Thus, we have bcl(A) U whenever A U and U is π-open. Therefore, A is πgb-closed.
3. Let A be a g αb*- closed set. Let A U, U is π-open. Since every π-open set is b*-open, then U is b*-open. Since A is gαb*-closed, αcl(A) U. But pcl(A)⊆αcl(A). Thus, we have pcl(A) U whenever A U and U is π-open. Therefore, A is πgp-closed.
4. Let A be a gαb*-closed set. Let A U, U is π-open. Since every π-open set is b*-open, then U is b*-open. Since A is g αb*-closed, αcl(A) U. But scl(A)⊆αcl(A). Thus, we have scl(A) U whenever A U and U is π-open. Therefore, A is πgs-closed.
5. Let A be a gαb*-closed set. Let A U, U is semi-open. Since every semi- open set is b*-open, then U is b*-open. Since A is gαb*-closed, αcl(A) U. But bcl(A)⊆αcl(A). Thus, we have bcl(A) U whenever A U and U is semi-open. Therefore, A is sgb-closed.
6. Let A be a g αb*- closed set. Let A U, U is regular-open. Since every regular open set is b*-open, then U is b*-open. Since A is gαb*-closed, αcl(A) U. But pcl(A)⊆αcl(A). Thus, we have pcl(A) U whenever A U and U is regular-open. Therefore, A is gpr-closed.

Remark 3.5. The reverse implications of the above theorems need not be true which is shown in the following examples.

Example 3.6. Let X= {a,b,c,d} with τ= {ϕ, X, {a}, {c},{a,c},{c,d},{a,c,d},{a,b,c}}.
1. gαb*-closed sets in (X, τ) are ϕ, X, {b}, {d}, {a,b}, {b,d}, {a,b,d}, {b,c,d}.
2. regular-closed sets in (X, τ) are ϕ, X, {a,b}, {b,c,d}.
3. π-closed sets in (X, τ) are ϕ, X, {b}, {a,b}, {b,c,d}.
4. sg-closed sets in (X, τ) are ϕ, X, {b}, {d}, {a,b}, {b,d}, {a,b,d}, {b,c,d}.
5. gb-closed sets in (X, τ) are ϕ, X,{a},{b},{d},{a,b}, {a,d}, {b,d},{c,d},{a,b,d},{b,c,d}.
6. gs-closed sets in (X, τ) are ϕ, X,{a},{b},{d}, {a,b}, {a,d}, {b,d},{c,d},{a,b,d},{b,c,d}.

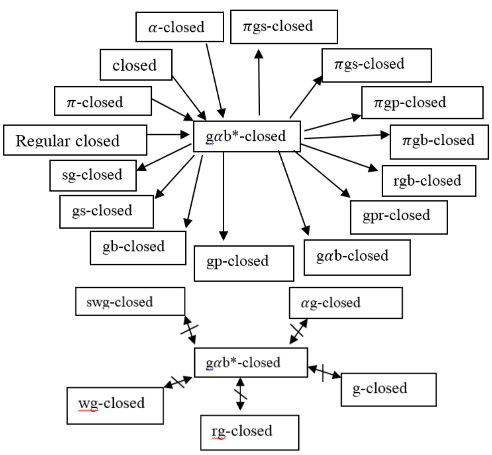
7. gpr-closed sets in (X, τ) are ϕ, X, {b}, {d}, {a,b}, {a,c}, {a,d}, {b,c}, {b,d}, {a,b,c}, {a,c,d}, {a,b,d}, {b,c,d}.
8. πgp-closed sets in (X, τ) are ϕ, X, {b}, {d}, {a,b}, {a,c}, {a,d}, {b,c}, {b,d}, {a,b,c}, {a,c,d}, {a,b,d}, {b,c,d}.

Example 3.7. Let X= {a, b, c, d} with τ= {ϕ, X, {a}, {b}, {a,b}, {b,c}, {b,c,d}, {a,b,c}}.

1. g αb*- closed sets in (X, τ) are ϕ, X, {a}, {c}, {d}, {a,c}, {a,d}, {c,d}, {a,c,d}, {b,c,d}.
2. closed sets in (X, τ) are ϕ, X, {a},{d}, {a,d}, {c,d}, {a,c,d},{b,c,d}.
3. sg b-closed sets in (X, τ) are ϕ, X, {a}, {c}, {d}, {a,c}, {a,d}, {b,c}, {c,d}, {a,c,d}, {b,c,d}.
4. rgb-closed sets in (X, τ) are ϕ, X, {a},{b}, {c}, {d}, {a,b}, {a,c}, {a,d}, {b,c}, {b,d}, {c,d}, {a,b,c}, {a,b,d}, {a,c,d}, {b,c,d}.
5. g αb -closed sets in (X, τ) are ϕ, X, {a}, {c}, {d}, {a,c}, {a,d}, {b,c}, {c,d}, {a,c,d}, {b,c,d}.
6. gp-closed sets in (X, τ) are ϕ, X, {a}, {c}, {d}, {a,c}, {a,d}, {b,d}, {c,d}, {a,b,d}, {a,c,d}, {b,c,d}.
7. g-closed sets in (X, τ) are ϕ, X, {a}, {d}, {a,d}, {b,d}, {c,d}, {a,b,d}, {a,c,d}, {b,c,d}.
8. rg-closed sets in (X, τ) are ϕ, X, {a}, {d},{a,b} {a,c}, {a,d}, {b,d}, {c,d}, {a,b,c}, {a,b,d}, {a,c,d}, {b,c,d}.
9. πgs-closed sets in (X,τ) are ϕ, X, {a},{b}, {c}, {d}, {a,c}, {a,d}, {b,c}, {b,d}, {c,d}, {a,b,d}, {a,c,d}, {b,c,d}.
10. πgp-closed sets in (X, τ) are ϕ, X, {a}, {c}, {d}, {a,c}, {a,d}, {b,c}, {b,d}, {c,d}, {a,b,d}, {a,c,d}, {b,c,d}.
11. πgb-closed sets in (X, τ) are ϕ, X, {a},{b}, {c}, {d}, {a,c}, {a,d}, {b,c}, {b,d}, {c,d}, {a,b,d}, {a,c,d}, {b,c,d}.

Remark 3.8. The gαb*-closed sets are independent from αg-closed set, g-closed set,rg-closed set, wg-closed set, swg-closed set`

Remark 3.9. From the above results, we have the following implications diagrams.



4.Characterization

Theorem 4.1. If a set A is gαb*-closed in (X,τ), then αcl(A)\A contains no non empty b*-closed sets in (X,τ).

Proof:
Let F be a b*-closed subset of X such that F αcl(A)\A. Then F αcl(A) (X\A).That implies, F αcl(A) and F (X\A). Then A X\F and X\F is b*-open in (X, τ). Since A is gαb*-closed in X, αcl(A) X\F, F X\αcl(A). Thus F αcl(A)∩(X\αcl(A))= ϕ. Hence αcl(A)\A does not contain any non-empty b*-closed sets.

Theorem 4.2. If a subset A is gαb*-closed set in (X, τ) and A B αcl(A), then B is also a gαb*-closed set.

Proof: Let A be a g αb*- closed set and B be any subset of X such that A B α cl(A). Let U be b*-open in (X, τ) such that B U. Then A U. Also since A is g αb*- closed, αcl(A) U. Since B αcl(A), αcl(B) α cl (αcl(A))=αcl(A) U. This implies, αcl(B) U. Thus B is a gαb*-closed set.

Definition 4.3. Let (X, τ) be a topological space and Y be a subspace of X. Then the subset A of Y is b*-open in Y if A=G Y, where G is b*-open in X.

Theorem 4.4. Let A⊆Y⊆X and suppose that A is gαb*-closed in X then A is gαb*-closed relative to Y.

Proof: Given that A ⊆ Y ⊆ X and A is a gαb*-closed set in X. To prove that A is gαb*-closed set relative to Y. Let us assume that A⊆Y∩U, where U is b*-open in X. Since A is gαb*-closed set in X, then αcl(A)⊆U. That implies Y∩αcl(A)⊆Y∩U, where Y∩αcl(A) is the α-closure of A in Y and Y∩U is b*-open in Y. Therefore αcl(A)⊆Y∩U in Y. Hence, A is gαb*-closed set relative to Y.

Theorem 4.5. Let A be any g αb*- closed set in (X, τ). Then A is α-closed in (X, τ) iff αcl(A)\A is b*-closed.

Proof: Necessity: Since A is α-closed set in (X, τ), αcl(A)=A. Then αcl(A)\A= ϕ, which is a b*-closed set in (X, τ). Sufficiency: Since A is gαb*-closed set in (X, τ), by Theorem 4.1, αcl(A)\A does not contains any non-empty b*-closed set. Therefore, αcl(A)\A= ϕ. Hence αcl(A)=A. Thus A is α-closed set in (X, τ).

Theorem 4.6. For every element x in a space X, X–{x} is gαb*-closed or b*-open.

Proof: Case (I): Suppose X–{x} is not b*-open. Then X is the only b*-open set containing X–{x}. This implies αcl(X–{x})⊆X. Hence X–{x} is gαb*-closed.

case (ii): Suppose X–{x} is not gαb*-closed. Then there exists a b*-open set U containing X–{x} such that αcl(X–{x}) does not contained in U. Now αcl(X–{x}) is either X–{x} or X. If αcl(X–{x})=X–{x}, then X–{x} is α-closed. Since every α-closed set is gαb*-closed, X–{x} is gαb*-closed, which is a contradiction. Therefore αcl(X–{x})=X. To prove that, X–{x} is b*-open. Suppose not. Then by case (i), X–{x} is g αb*-closed. There is a contradiction to our assumption. Hence X–{x} is b*-open.

Theorem 4.7. If A is both b*-open and gαb*-closed set in X, then A is α-closed set

Proof: Since A is b*-open and gαb*-closed in X, αcl(A)⊆A. But always A⊆αcl(A). Therefore, A=αcl(A). Hence A is a α-closed set.

Definition 4.8. The intersection of all b*-open sets containing A is called the b*-kernel of A and it is denoted by b*-ker(A).

Theorem 4.9. A subset A of X is gαb*-closed iff αcl(A)⊆b*-ker(A).

Proof: Necessity: Let A be a g αb*- closed subset of X and x∈αcl(A). Suppose x∉b*-ker(A). Then there exists a b*-open set U containing A such that x∉U. Since A is gαb*-closed set, then αcl(A)⊆U. This implies that, x∉αcl(A), which is a contradiction to x∈αcl(A). Therefore αcl(A)⊆b*-ker(A).

Sufficiency: Suppose αcl(A)⊆b*-ker(A). If U is any b*-open set containing A, then b*-ker(A)⊆U. That implies, αcl(A)⊆U. Hence A is gαb*-closed in X.

Remark 4.10. For any subset A of X, gb-ker(A)⊆b*-ker(A).

Theorem 4.11. For any subset A of X, X2∩αcl(A)⊆b*-ker(A).

Proof: Since αcl(A)⊆cl(A), then X2∩αcl(A)⊆X2 cl(A). Then by Lemma 2.8 and Remark 4.10, X2∩αcl(A)⊆b*-ker(A).

Theorem 4.12. A subset A of X is gαb*-closed if and only if X1∩αcl(A)⊆A.

Proof: Necessity: Suppose that A is gαb*-closed and x∈X1∩αcl(A). Then x∈X1 and x∈αcl(A). Since x∈X1, then int(cl({x}))=∅. That implies, cl(int(cl({x})))=∅. Therefore {x} is α-closed. Then {x} is b*-closed. If x does not belongs to A, then U=X–{x} is a b*-open set containing A and so αcl(A)⊆U. Since x∈αcl(A), x∈U. This is a contradiction to x not in U. Hence X1∩αcl(A)⊆A.

Sufficiency: Let X1∩αcl(A)⊆A. Then X1∩αcl(A)⊆b*-ker(A). Now, αcl(A)= X∩αcl(A)=(X1∪X2) ∩αcl(A) = (X1∩αcl(A)) ∪ (X2∩αcl(A)) ⊆b*-ker(A). Then by Theorem 4.9, A is gαb*-closed.

Remark 4.13. Union of any two gαb*-closed sets in (X, τ) is also a gαb*-closed set`

Proof. Let A and B be two gαb*-closed sets in a topological space (X, τ). Let U be any b*-open set containing A∪B. Then A⊆U and B⊆U. Since A and B are gαb*-closed sets, αcl(A)⊆U and αcl(B)⊆U. Now, αcl(A∪B)= αcl(A)∪ αcl(B)⊆U and hence A∪B is gαb*-closed set.

Theorem 4.14. Arbitrary intersection of gαb*-closed sets is gαb*-closed.

Proof. Let {Ai} be the collection of gαb*-closed sets of X. Let A=∩Ai. Since A⊆Ai, for each i, then αcl(A)⊆αcl(Ai). That implies, X1∩αcl(A)⊆X1∩αcl(Ai). Since each Ai is gαb*-closed, then by Theorem 4.12, X1∩αcl(Ai)⊆Ai, for each i. Now, X1∩αcl(A)= X1∩αcl(∩Ai)⊆∩(X1∩αcl(Ai))⊆∩Ai=A. Again by Theorem 4.12, A is gαb*-closed.

Remark 4.15. The set of all gαb*-closed sets in a topological space X, form a topology on X.

Theorem 4.16. Let A be a gαb*-closed in X. Then
1. sint(A) is gαb*-closed.
2. If A is regular open, then pint(A) and scl(A) are also gαb*-closed.
3. If A is regular closed, then pcl(A) is also gαb*-closed.

Proof: Let A be a gαb*-closed set of X.
1. Since cl(int(A)) is closed, then by Theorem 3.2, cl(int(A)) is gαb*-closed, sint(A) is closed.
2. Suppose A is regular open, then int(cl(A))=A. By Lemma 2.5, scl(A)=A. Since A is gαb*-closed, then scl(A) is gαb*-closed. Similarly pint(A) is gαb*-closed.
3. Suppose A is regular closed, cl(int(A))=A. Then by Lemma 2.5, pcl(A)=A, and hence gαb*-closed.

5. Generalized αb*-open

Definition 5.1. A subset A of (X, τ) is said be generalized αb*-open (briefly gαb*-open) set if its complement X\A is gαb*-closed in X. The family of all gαb*-open sets in X is denoted by gαb*-O(X).

Theorem 5.2. Let (X, τ) be a topological space and A⊆X. Then A is a gαb*-open if and only if F⊆αint(A), whenever F⊆A and F is b*-closed.

Proof: Necessity: Let A be a gαb*-open set in (X, τ). Let F⊆A and F is b*-closed. Then X\A is gαb*-closed and it is contained in the b*-open set X\F. Therefore αcl(X\A)⊆X\F. This implies that X\αint(A) ⊆ X\F. Hence F⊆αint(A).

Sufficiency: If F is b*-closed set such that F⊆αint(A) whenever F⊆A. It follows that X\A⊆X\ F and X\αint(A)⊆X\F. Therefore αcl(X\A) ⊆X\ F. Hence X\A is gαb*-closed and hence A is gαb*-open.

Theorem 5.3. If a set A is gαb*-open and B⊆X such that αint(A)⊆B⊆A, then B is gαb*-open.

Proof: If α int(A)⊆B⊆A then, X\A⊆X\B⊆X\αint(A). That is, X\A⊆X\B⊆αcl(X\A). Since X\A is gαb*-closed, then by

Theorem 2.2, X\B is gαb*-closed and hence B is gαb*-open.

Theorem 5.4. If a subset A is gαb*-open in X and G is b*-open in X with αint(A)∪(X\G)⊆ G then X= G.

Proof: Suppose that G is b*-open and αint(A)∪(X\G)⊆ G. This implies, X\G⊆(X\αint(A)) A=αcl(X\A)\(X\A). Since X\A is gαb*-closed and X\G is b*-closed, then by Theorem 4.1, X\G=ϕ. Hence X=G.

Remark 5.5. Union of gαb*-open sets is gαb*-open in a topological space X.

Remark 5.6. Intersection of g αb*-open sets is also a g αb*-open in X.

Theorem 5.7. If B is g αb*-open and αint(B) ⊆A, then A∩B is gαb*-open.

Proof: Suppose B is gαb*-open and αint(B)⊆A. Then αint(A∩B) ⊆A∩B⊆B. By Theorem 5.3, A∩B is gαb*-open.

6. g αb*-neighbourhood

Definition 6.1. Let X be a topological space and let x∈X. A subset N of X is said to be a gαb*-neighbourhood (shortly, gαb*-nbhd) of x if there exsits a gαb*-open set U such that x∈U⊆N.

Definition 6.2. A subset N of a space X, is called a gαb*-nbhd of A⊆X if there exists an gαb*-open set U such that A⊆U⊆N.

Theorem 6.3. Every nbhd N of x∈X is a gαb*-nbhd of x.

Proof: Let N be a nbhd of point x∈ X. Then there exists an open set U such that x∈U⊆N. Since every open set is g αb*-open, U is a gαb*-open set such that x∈U⊆N. This implies, N is a gαb*-nbhd of x.

Remark 6.4. The converse of the above theorem need not be true which is shown in the following example.

Example 6.6. Let X = {a, b, c, d} with topology τ = {ϕ, {a},{b}, {a,b},{b,c},{a,b,c},{b,c,d} X}. In this topological space (X, τ), gαb*-O(X) = {ϕ, {a}, {b}, {a, b}, {b, c}, {b, d}, {a, b,c}, {a, b, d}, {b, c d}}. The set {b,d} is the gαb*-nbhd of d, since {b,d} is gαb*-open set such that d ∈ {b,d}⊆ {b, d}. However, the set {b, d} is not a nbhd of the point d.

Remark 6.7. Every gαb*-open set is a gαb*-nbhd of each of its points.

Theorem 6.8. If F is a gαb*-closed subset of X and x∈X\F, then there exists a g αb*-nbhd N of x such that N∩F=ϕ

Proof: Let F be gαb*-closed subset of X and x∈ X\F. Then X\F is gαb*-open set of X. By Theorem, X\F contains a gαb*-nbhd of each of its points. Hence there exists a gαb*-nbhd N of x such that N⊆ X\F. Hence N∩F=ϕ.

Definition 6.9. The collection of all gαb*-neighborhoods of x∈X is called the gαb*-neighborhood system of x and is denoted by gαb*-N(x).

Theorem 6.10. Let (X, τ) be a topological space and x ∈ X. Then

(I) g αb*-N(x)≠ϕ and x∈each member of gαb*-N(x)

(ii) If N ∈ gαb*-N(x) and N ⊆ M, then M ∈ gαb*-N(x).

(iii) Each member N ∈gαb*-N(x) is a superset of a member G∈gαb*-N(x) where G is a gαb*-open set.

Proof:

(i) Since X is gαb*-open set containing x, it is a gαb*-nbhd of every x∈X. Thus for each x∈X, there exists atleast one gαb*-nbhd, namely X. Therefore, gαb*-N(x)≠ϕ. Let N∈gαb*-N(x). Then N is a g αb*- nbhd of x. Hence there exists a g αb*-open set G such that x∈G ⊆ N, so x ∈ N. Therefore x∈every member N of gαb*-N(x).

(ii) If N ∈gαb*-N(x), then there is a gαb*-open set G such that x∈G⊆N. Since N⊆M, M is gαb*-nbhd of x. Hence M∈gαb*-N(x).

(iii) Let N∈gαb*-N(x). Then there is a gαb*-open set G, such that x ∈ G⊆N. Since G is gαb*-open and x∈G, G is gαb*-nbhd of x. Therefore G∈gαb*-N(x) and also G⊆N.

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