ORIGINAL RESEARCH PAPER

Mathematics

AN INTERVAL VALUED LINEAR PROGRAMMING PROBLEM WITH TRAPEZOIDAL Z FUZZY NUMBER

KEY WORDS: Z number, interval valued Z fuzzy number (IVZFN), Z fuzzy linear programming problem (ZFLPP), interval valued Z fuzzy linear programming problem (IVZFLPP)

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ABSTRACT

This paper gives the modified interval valued Z fuzzy numbers (IVZFN) for solving interval valued Z fuzzy linear programming problem (IVZFLPP) with trapezoidal Z fuzzy numbers by assuming different cut values. An illustrative numerical example is presented in order to clarify the proposed approach.

1.INTRODUCTION:

The belief of fuzzy sets was introduced by Zadeh [9] and it was indiscriminate to trapezoidal fuzzy sets by Atanassov [2,3]. Zadeh have also anticipated a belief, namely Z-number, which is an order pair of fuzzy numbers$(\tilde{A},\tilde{R})$ The first component $\tilde{A}$, plays the role of a fuzzy restriction. And the second component $\tilde{R}$ is a reliability of the first component[10]. This manuscript focuses on trapezoidal Z fuzzy numbers (TZFNS) and interval valued Z fuzzy numbers by presumptuous various cut values from them. When we judge the interval valued Z fuzzy numbers(IVZFNS), the arithmetic operations defined on them are of great authority. In the literature, Interval Arithmetic was first suggested by Dwyer [5] in 1951. The same was developed by Moore [6], Ganesan.K. and Veeramani.P[6] and Nagoor Gani. A and Irene Hepzibah. R[8]. Here in this work, we used the same operations to interval valued Z fuzzy numbers(IVIFNS) to get the preferred conclusion. Many researchers have applied the fuzzy set theory to the field of decision making. Bellman and Zadeh [4] proposed the concept of decision making In fuzzy envoironment. Zimmermann[11] proposed the first formation of fuzzy linear programming problem. The paper is organized as follows: Section 2 introduces the preliminaries of fuzzy set, trapezoidal fuzzy number, Z fuzzy n number, interval valued Z fuzzy numbers. Section 3 deals with the formulation of Z fuzzy linear programming problem(ZFLPP) , interval valued ZFLPP and ranking function. Section 4 discusses the algorithm for solving IVZFLPP. In section 5, an application of these are discussed by a numerical illustration and some concluding remarks are given in Section 6.

II.PRELIMINARIES:

A. DEFINITION 1: [9]

Let X be a nonempty set. A fuzzy set $\tilde{A}$ of X is defined as $\tilde{A}=\{(x,\mu_{\tilde{A}}(x))/\ x\in X\}$ where $\mu_{\tilde{A}}(x)$ is called the membership function which maps each element of X to a value between 0 and 1

B. DEFINITION 2: [9]

A fuzzy number is a generalization of a regular real number and which does not refer to a single value but rather to a connected set of possible values, where each possible value has its weight between 0 and 1. This weight is called the membership function.

A fuzzy number $\tilde{A}$ is a convex normalized fuzzy set on the real line R such that:

- There exist at least one x ∈ R with $\mu_{\tilde{A}}(x) = 1$
- $\mu_{\tilde{A}}(x)$ is piecewise continuous

C. DEFINITION 3: [1]

A trapezoidal fuzzy number A can be expressed as [a1,a2,a3,a4] and its membership function is defined as

$$\tilde{a} = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, & for\ x\in[a_1,a_2] \\ 1 & for\ x\in[a_2,a_3] \\ \dfrac{a_4-x}{a_4-a_3}, & for\ x\in[a_3,a_4] \\ 0 & otherwise \end{cases}$$

D. DEFINITION 4: [1]

If $\tilde{A}$= [a1,a2,a3,a4] is a trapezoidal fuzzy number, we will let $\tilde{A}_\alpha = [A_\alpha^-, A_\alpha^+]$ where $[A_\alpha^-, A_\alpha^+] = (\alpha(a_2-a_1)+a_1, a_4-$ be the closed interval which is α–cut for $\tilde{A}$ in $0 \le \alpha \le 1$.

E. DEFINITION 5: [10]

A Z-number is an ordered pair of fuzzy numbers denoted as $Z=(\tilde{A},\tilde{R})$ The first component $\tilde{A}$, a restriction on the values, is a real-valued uncertain variable X. The second component $\tilde{R}$ is a measure of reliability for the first component.

III FORMULATION OF PROBLEM:

A. FORMULATION OF Z FUZZY LINEAR PROGRAMMING PROBLEM (ZFLPP):

The general form of optimization problem with Z- fuzzy objective function $\tilde{z}$ and m Z- fuzzy constraints is given by

$$\max\ z_k(\tilde{A},\tilde{R}) = \sum_{j=1}^{n} \tilde{c}_j^k \tilde{R}_j^k \tilde{x}_j\ ,$$

Where $k = 1,2,3,....., K$

Subject to
$$\sum_{i=1}^{n} \tilde{a}_{ij}\tilde{R}_{ij}\tilde{x}_j \le \tilde{k}_i\tilde{R}_{Ij}\ ,$$
$$i = 1,2,3,.....,m;\quad j = 1,2,3,.....,n$$
$$\tilde{x}_j \ge 0,\ j = 1,2,3,....,n.$$

B. Formulation of Interval Valued Z Fuzzy Linear Programming Problem (IVZFLPP):

By assuming the prescribed value of α the problem can be restated as

$$\max\ z_k(\tilde{A},\tilde{R})_\alpha = \sum_{j=1}^{n} (\tilde{c}_j^k)_\alpha(\tilde{R}_j^k)_\alpha(\tilde{x}_j)_\alpha$$

Where $k = 1,2,3,....., K$

Subject to
$$\sum_{i=1}^{n} (\tilde{a}_{ij})_\alpha(\tilde{R}_{ij})_\alpha(\tilde{x}_j)_\alpha \le (\tilde{k}_i)_\alpha(\tilde{R}_{Ij})_\alpha\ ,$$
$$i = 1,2,3,.....,m;\quad j = 1,2,3,.....,n$$
$$(\tilde{x}_j)_\alpha \ge 0,\ j = 1,2,3,....,n.$$

Volume-8 | Issue-1 | January-2019 | PRINT ISSN - 2250-1991

C. RANKING FUNCTION:

Let $\tilde{A}=\{(a,b)(c,d)\}$, be the interval valued Z fuzzy numbers. The average representation of this interval valued Z fuzzy numbers is given by R $(\tilde{A})$, where $R(\tilde{A})=\frac{(a+b+c+d)}{4}$

Accordingly for any two IVZFN $\tilde{A}$ and $\tilde{B}$ we have $\tilde{A} \geq \tilde{B}$ iff $R(\tilde{A}) \geq R(\tilde{B})$

$\tilde{A} \leq \tilde{B}$ iff $R(\tilde{A}) \leq R(\tilde{B})$ and $\tilde{A} = \tilde{B}$ iff $R(\tilde{A}) = R(\tilde{B})$

IV. ALGORITHM FOR SOLVING INTERVAL VALUED Z FUZZY LINEAR PROGRAMMING PROBLEM:

By using the concepts mentioned above, the algorithm is proposed for solving Z fuzzy linear programming problem.

Step 1: Formulate the Z fuzzy linear programming problem

$$\max z_k(\tilde{A},\tilde{R})=\sum_{j=1}^{n}\tilde{c}_j^k \tilde{R}_j^k \tilde{x}_j$$

Subject to

$$\sum_{i=1}^{n}\tilde{a}_{ij}\tilde{R}_{ij}\tilde{x}_j \leq \tilde{k}_i\tilde{R}_{IJ}, i=1,2,3,.....,m;$$

$$j=1,2,3,.....,n$$

Step 2: Set α cut level values, to formulate the above problem into interval valued Z fuzzy linear programming problem (IVZFLPP).

Step 3: Convert the interval valued Z fuzzy linear programming problem into crisp linear programming problem by using III.C for the intervals of uncertainty and reliablity values.

Step 4: Compute the net Evaluations by using the relation $\tilde{z}_j - \tilde{c}_j = \tilde{c}_B\tilde{a}_j - \tilde{c}_j$ and examine the sign of $\tilde{z}_j - \tilde{c}_j$

(a) if all $\tilde{z}_j - \tilde{c}_j \geq 0$ then the solution is optimal solution.
(b) If atleast one $\tilde{z}_j - \tilde{c}_j < 0$ then the solution is not optimal.

Step 5: Go To Step 2 Until You Get Optimal Solution.

V. NUMERICAL EXAMPLE:

A cultivator is to cultivate banana, turmeric, cotton, potato, onion and watermelon in a season in areas be x1, x2, x3, x4, x5, and x6 respectively. The farmer has a total land of 10 acres, maximum irrigation of 40, number of labour during flood will not exceed 42000 and during drip will not exceed 10500 and a maximum labour work time available to him is 300 hours. The price for the crops, water requirement, irrigation, net savings, yield, labour and work time for the crops are given in the Table-1. Find by using which crop the labour gets maximum net savings, the cultivator gets maximum profit and maximum yield also the water requirement during both flood and drip

Crop	Dura-tion	No of Irrig-ation	Water Require-ment (F)	Water Require-ment (D)	Yield (F)	Labo-ur (F)	Lab-our (D)	Pri-ce	Net Savings On Labour (in thousands)
Banana	300	35	8000	4100	45000	42000	10500	28	1218
Turme-ric	260	25	9500	7000	25000	30000	7500	20	470
Cotton	165	11	3200	2000	1400	13200	3300	60	70.8
Potato	140	12	2500	1400	1500	14400	3600	20	15.6
Onion	90	9	2100	1300	4000	10800	2700	20	69.2
Water melon	60	5	3000	2000	3500	6000	1500	15	46.5

TABLE I

Solution:

Step 1: Formulating the above problem into linear programming problem

Max NS = $1281 x_1 + 470 x_2 + 70.8 x_3 + 15.6 x_4 + 69.2 x_5 + 46.5 x_6$
Max P = $28 x_1 + 20 x_2 + 60 x_3 + 20 x_4 + 20 x_5 + 15 x_6$
Max Y = $45000 x_1 + 25000 x_2 + 1400 x_3 + 1500 x_4 + 4000 x_5 + 3500 x_6$
Max WF = $8000 x_1 + 9500 x_2 + 3200 x_3 + 2500 x_4 + 2100 x_5 + 3000 x_6$
Max WD = $4100 x_1 + 7000 x_2 + 2000 x_3 + 1400 x_4 + 1300 x_5 + 2000 x_6$

Subject to ,
$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \leq 10$
$300 x_1 + 260 x_2 + 165 x_3 + 140 x_4 + 90 x_5 + 60 x_6 \leq 300$
$35 x_1 + 25 x_2 + 11 x_3 + 12 x_4 + 9 x_5 + 5 x_6 \leq 40$
$42000 x_1 + 30000 x_2 + 13200 x_3 + 14400 x_4 + 10800 x_5 + 6000 x_6 \leq 42000$
$10500 x_1 + 7500 x_2 + 3300 x_3 + 3600 x_4 + 2700 x_5 + 1500 x_6 \leq 10500$
$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$

This formulation is converted into Z fuzzy linear programming problem.

Step 2: Setting α cut level values, to formulate the above formulation into interval valued Z fuzzy linear programming problem (IVZFLPP).

Step 3: By using III.C the interval valued Z fuzzy linear programming problem is converted into crisp linear programming problem

Step 4: Computing the value of $\tilde{z}_j - \tilde{c}_j$

Step 5: Hence we got the optimum solution as shown in Table 2.

	α = 0	α = 0.5	α = 1
Max NS =	1277	1277.5	1278
x1 =	1	1	1
x2 =	0	0	0
x3 =	0	0	0
x4 =	0	0	0
x5 =	0	0	0
x6 =	0	0	0
Max P =	102.8	107.4	112.29
x1 =	0	0	0
x2 =	0	0	0
x3 =	1.74	1.82	1.9
x4 =	0	0	0
x5 =	0	0	0
x6 =	0	0	0
Max Y =	46250	45500	44750
x1 =	1	1	1
x2 =	0	0	0
x3 =	0	0	0
x4 =	0	0	0
x5 =	0	0	0
x6 =	0	0	0
Max WF =	10278.99	10654.77	11690.74
x1 =	0	0	0
x2 =	1.13	1.17	0
x3 =	0	0	0
x4 =	0	0	0
x5 =	0	0	0
x6 =	0	0	4.37
Max WD =	7976.72	8210.62	9287.04
x1 =	0	0	0
x2 =	1.13	1.17	0
x3 =	0	0	0
x4 =	0	0	0
x5 =	0	0	0
x6 =	0	0	4.37

TABLE II

VI. CONCLUSION:

A cultivator is cultivating banana, turmeric, cotton, potato, onion and watermelon in his land. The Labours in the farm will get maximum net savings and the cultivator will get maximum yield while cultivating banana in the farm, whereas water requirement is more during both flood and drip is happen when he cultivates turmeric and hence the cultivator got maximum profit only when he cultivate cotton in his farm.

REFERENCES:

1. Adrian I. Ban and Delia A. Tuse ,Trapezoidal/triangular intuitionistic fuzzy numbers versus interval-valued trapezoidal/triangular fuzzy numbers and applications to multicriteria decision making methods, Notes on Intuitionistic Fuzzy Sets, ISSN 1310–4926 Vol. 20, 2014, No. 2, 43–51, 2014.
2. Atanassov.K.T , Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20, 87-96, 1986.
3. Atanassov.K.T , More on intuitionistic fuzzy sets, Fuzzy sets and systems, 33, 37-46, 1989.
4. Bellman,R.E. and Zadeh, L.A., "Decision making in a fuzzy environment", Management Science, 17, 141-164,1970.
5. Dwyer, P.S.,Linear Computation, New York, 1951.
6. Ganesan.K. and Veeramani.P., "On Arithmetic operations of Interval Numbers", Uncertainty,Fuzziness and Knowledge based Systems, 13,619-631,2005.
7. Ireme Hepzibah R and Vidhya R, " Modified New Operations for Interval Valued Intutionistic Fuzzy Numbers(IVIFNS): Linear Programming Problem with Hexagonal Intutionistic Fuzzy Numbers, International Journal of Pure and Applied

Mathematics, Volume 106 No. 8 2016, 97-106
8. Nagoor Gani. A and Irene Hepzibah. R, Multi-Objective Fuzzy Linear Programming Problems with interval numbers, Bulletin of Pure and Applied Sciences, Vol.26E(No.2): P.51-56,2007.
9. Zadeh,L.A., "The concept of a Linguistic variable and its applications to approximate Reasoningparts I, II and III", Inform.Sci.8,199-249, 1975.
10. Zadeh L. A., A Note on Z-numbers, Information Sciences, 181, 2923-2932, 2011.
11. Zimmerman H. J., Fuzzy Programming and Linear Programming with Several Objective Functions, Fuzzy Sets and Systems, Vol. 1, 45-55, 1978.