ORIGINAL RESEARCH PAPER

Statistics

BAYESIAN ESTIMATION AND AN APPLICATION OF GIBBS SAMPLING TECHNIQUE AND RANDOM WALK METROPOLIS - HASTING ALGORITHM IN TWO PHASE LINEAR REGRESSION MODEL

KEY WORDS: Gibbs Sampling and RWM-H (Random Walk Metropolis - Hasting) Algorithm, Two Phase Linear Regression (TPLR) Model, Bayesian Estimation, Change Point.

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ABSTRACT

Here, in this research paper, we have applied the Gibbs Sampling Technique and RWM-H (Random Walk Metropolis - Hasting) Algorithm for the Bayesian Estimation of m, β1, β2 and 1/2. Also we have assumed that at some point of time say 'm', the co-efficient of regression changes from β1 to β2. Further, we have discussed about the effects of prior information on the Bayes estimates on the basis of the TPLR (Two Phase Linear Regression) Model with a Bayesian approach.

INTRODUCTION:

The regression coefficients are assumed constant as far as regression analysis is concerned, in general practice. A model can be suggested on the basis of the theoretical or empirical deliberations as far as real life situations are concerned. Here, we note that the model changes occasionally in either one or more of its parameters, most of the times. Generally, we all know that regression analysis is widely applicable in the fields of social, medical and engineering science. Here, we have emphasized on the point where the unknown change occurred and so we have focused on the shift point parameter. It is so because regression analysis indexes when or where the unknown change occurred in the system.

The Gibbs sampling technique for generating random variables from a marginal distribution is applied here in an indirect manner. Further, we note that we are not calculating the density. We have taken an assumption that we have a joint density as $\boldsymbol{f(x, y_1, \ldots, y_p)}$ and we want to obtain characteristic values of the marginal density

$$\boldsymbol{f(x) = \int \ldots \int f(x, y_1, \ldots, y_p)\, dy_1 \ldots dy_p} \qquad \textbf{(1)}$$

It may be mean or variance. In general, it may be considered as the most natural and straightforward approach to calculate $\boldsymbol{f(x)}$ and use it to obtain the desired characteristic values. There are many cases where the analytical and numerical integration in **(1)** are very much difficult to perform in practice. Therefore, we have applied Gibbs sampler in such cases as it provides an alternative method to obtain $\boldsymbol{f(x)}$.

On the basis of some research done in the past, we have assumed that we already have some technical knowledge about the parameter of the models which are already available. We have taken some notable references of researchers such as **Arnold Zellner (1971), A.F.M. Smith (1980) and A.K. Bansal and S. Chakravarty (1996)** who have studied the Bayesian estimation for the regression coefficient and change point of the TPLR model.

Now-a-days, simulation has become an increasingly important technique as an alternative to numerical as well as analytical approximation techniques. In this paper, we have shown how the routine **Bayesian Analysis of the Two Phase Linear Regression (TPLR) model** is made possible using simulation methods based on **Markov Chain Monte Carlo (MCMC) technique**. In the last decade, there has been a significant growth of interest in MCMC methods which include further refinements of standard Monte Carlo sampling techniques. We can say so very clearly from the developments which have taken place in the last decade.

It was the joint research work of **Gelfand and Smith (1990)** that gave the realization that Markov Chain could be used in wide variety of situations through their research studies and conclusions. Bayesian Analysis for the Block and Basu Bivariate Exponential distribution was done by **Jorge A. Achcar and Roseli A. Leandro (1998)** using **Metropolis Algorithm** with Gibbs steps and sampling technique. **S.K. Upadhyay and N. Vasistha (2000)** have worked jointly to overcome the computational difficulties using simulation approaches to Bayesian computation in reliability models, using MCMC methods. The understanding of how the development of Monte Carlo Methodology did not change our solution to the problems, but it changed the way of our thinking about the problems was given by **Christian Robert and George Casella (2011)**.

TWO PHASE LINEAR REGRESSION MODEL:

We shall consider the following Two Phase Linear Regression (TPLR) Model for our study.

$$\boldsymbol{Y_t = \begin{cases} \beta_1 X_t + \epsilon_t & t = 1, 2, \ldots, m \\ \beta_2 X_t + \epsilon_t & t = m+1, m+2, \ldots, n \end{cases}} \qquad \textbf{(2)}$$

Here, $\boldsymbol{X_t}$ is a non-stochastic explanatory variable. $\boldsymbol{\beta_1}$ and $\boldsymbol{\beta_2}$ are the regression parameters where $\boldsymbol{\beta_1 \neq \beta_2}$. The independently and identically distributed random errors are shown as $\boldsymbol{\epsilon_t}$ which follow the Normal distribution, i.e. $\boldsymbol{N(0, \sigma^2)}$. Further, we note that variance is strictly positive, i.e. $(\boldsymbol{\sigma^2} > 0)$.

BAYES ESTIMATION:

The likelihood function of $\boldsymbol{\beta_1, \beta_2, \sigma^{-2}}$ and *'m'* on the basis of the sample information $\boldsymbol{Z = (x_t, y_t)}$, where $\boldsymbol{t = 1, 2, \ldots, m, m+1, \ldots, n}$ is taken as under:

$$L(\beta_1, \beta_2, \sigma^{-2}, m|Z) = \frac{1}{(2\pi)^{\frac{n}{2}}}\left(e^{-\frac{A}{2\sigma^2}}\right).(\sigma^{-n}).\ exp\left[-\frac{1}{2}\beta_1^2\left(\frac{S_{m1}}{\sigma^2}\right) + \beta_1\left(\frac{S_{m3}}{\sigma^2}\right)\right].\exp\left[\frac{-1}{2}\beta_2^2\left(\frac{S_{n1}-S_{m1}}{\sigma^2}\right) + \beta_2\left(\frac{S_{m4}}{\sigma^2}\right)\right] \quad \textbf{(3)}$$

where,

$$S_{k1} = \sum_{i=1}^{k} x_i^2, \qquad S_{k2} = \sum_{i=1}^{k} x_i y_i,$$

$$S_{m3} = S_{m2}, \qquad S_{m4} = S_{n2} - S_{m2}$$

$$A = \sum_{i=1}^{n} y_i^2 \qquad \textbf{(4)}$$

APPLICATION OF GAMMA PRIOR ON $1/\sigma^2$ AND INFORMATIVE PRIORS ON β_1 AND β_2 WITH UNKNOWN σ^{-2}:

Here, the **TPLR model (2)** is taken into consideration with unknown $\boldsymbol{\sigma^{-2}}$ just as it was used by **Broemeling et.al. (1987)**. Further, uniform prior for change point *'m'* is assumed as $\boldsymbol{g(m) = \frac{1}{n-1}}$

Moreover, we have normal prior density on β_1 and β_2 as,

$$g(\beta_1) = \frac{1}{\sqrt{2\pi}\sigma_1} e^{-\frac{1}{2}\left(\frac{\beta_1-\mu_1}{\sigma_1}\right)^2}$$

$$g(\beta_2) = \frac{1}{\sqrt{2\pi}\sigma_2} e^{-\frac{1}{2}\left(\frac{\beta_2-\mu_2}{\sigma_2}\right)^2}$$

Further, we assume the ***Gamma (c, d)*** prior for $\frac{1}{\sigma^2}$ as under:

$$g\left(\frac{1}{\sigma^2}\right) = \frac{c^d}{\Gamma d}\left(\frac{1}{\sigma^2}\right)^{d-1} e^{-\frac{c}{\sigma^2}} \qquad \sigma^2 > 0$$

Also we note that ***c*** and ***d*** are the hyper parameter which can be obtained by using following relations:

$d = \frac{1}{\emptyset^2}$, $\qquad c = \frac{1}{\mu_3 \emptyset^2}$.

We note that μ_3 and $\emptyset$ are prior mean and prior coefficient of variation of $\frac{1}{\sigma^2}$ respectively.

Hence, joint prior *p.d.f.* of $\boldsymbol{\beta_1}$, $\boldsymbol{\beta_2}$, $\boldsymbol{\sigma^{-2}}$ and ***'m'*** will be:

$$g(\beta_1, \beta_2, \sigma^{-2}, m) = \frac{c^d}{2\pi\sigma_1\sigma_2\sigma^{2(d-1)}\Gamma d\,(n-1)}\, e^{-\frac{1}{2}\left(\frac{\beta_1-\mu_1}{\sigma_1}\right)^2} e^{-\frac{1}{2}\left(\frac{\beta_2-\mu_2}{\sigma_2}\right)^2} \qquad \textbf{(5)}$$

Using Likelihood function **(3)** with the joint prior density, the joint posterior density of $\beta_1, \beta_2, \sigma^{-2}, m$ say $g\,(\beta_1, \beta_2, \sigma^{-2}, m|Z)$ will be:

$$g(\beta_1, \beta_2, \sigma^{-2}, m|Z) = K_1\left[e^{\left[-\frac{1}{2}\beta_1^2 B_1 + \beta_1 A_1\right]} e^{\left[-\frac{1}{2}\beta_2^2 B_2 + \beta_2 A_2\right]}\, e^{-\frac{1}{\sigma^2}\left(\frac{A+B}{2}+c\right)} \sigma^{-2\left(\frac{n}{2}+d-1\right)}\right] \Big/ h_1(z) \qquad \textbf{(6)}$$

where,

$$K_1 = \frac{c^d \exp\left[-\frac{1}{2}\left(\left(\frac{\mu_1}{\sigma_1}\right)^2 + \left(\frac{\mu_2}{\sigma_2}\right)^2\right)\right]}{(2\pi)^{n/2}\, 2\pi\Gamma d\,(n-1)}$$

$A_1 = \frac{\mu_1}{\sigma_1^2} + \frac{S_{m3}}{\sigma^2} \qquad B_1 = \frac{1}{\sigma_1^2} + \frac{S_{m1}}{\sigma^2}$

$A_2 = \frac{\mu_2}{\sigma_2^2} + \frac{S_{m4}}{\sigma^2} \qquad B_2 = \frac{1}{\sigma_2^2} + \frac{S_{n1}-S_{m1}}{\sigma^2}$

$A = \sum_{i=1}^{n} y_i^2 \qquad B = \beta_1^2 S_{m1} + 2\beta_1 S_{m3} + \beta_2^2(S_{n1} - S_{m1}) + 2\beta_2 S_{m4}$ **(7)**

$h_1(Z)$ is the marginal density of z given by,

$$\boldsymbol{h_1(Z)} = \sum_{m=1}^{n-1}\int_0^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} L(\beta_1, \beta_2, \sigma^{-2}, m|Z)\,.\,g(\beta_1, \beta_2, \sigma^{-2}, m)\,d\beta_1 d\beta_2 d\sigma^{-2}$$

$$= \sum_{m=1}^{n-1}\int_{-\infty}^{\infty} e^{\left[-\frac{1}{2}\beta_1^2 B_1 + \beta_1 A_1\right]} d\beta_1 \int_{-\infty}^{\infty} e^{\left[-\frac{1}{2}\beta_2^2 B_2 + \beta_2 A_2\right]} d\beta_2 \int_0^{\infty} e^{-\frac{1}{\sigma^2}\left(\frac{A+B}{2}+c\right)} \sigma^{-2\left(\frac{n}{2}+d-1\right)} d\sigma^{-2}$$

(8)

Marginal posterior of $\beta_1, \beta_2, \sigma^{-2}$ and $'m'$ respectively will be:

$$\boldsymbol{g(\beta_1|Z)} = \frac{\left[\sum_{m=1}^{n-1} e^{\left[-\frac{1}{2}\beta_1^2 B_1 + \beta_1 A_1\right]} \int_{-\infty}^{\infty} e^{\left[-\frac{1}{2}\beta_2^2 B_2 + \beta_2 A_2\right]} d\beta_2 \int_0^{\infty} e^{-\frac{1}{\sigma^2}\left(\frac{A+B}{2}+c\right)} \sigma^{-2\left(\frac{n}{2}+d-1\right)} d\sigma^{-2}\right]}{h_1(Z)} \qquad \textbf{(9)}$$

$$\boldsymbol{g(\beta_2|Z)} = \frac{\left[\sum_{m=1}^{n-1} e^{\left[-\frac{1}{2}\beta_2^2 B_2 + \beta_2 A_2\right]} \int_{-\infty}^{\infty} e^{\left[-\frac{1}{2}\beta_1^2 B_1 + \beta_1 A_1\right]} d\beta_1 \int_0^{\infty} e^{-\frac{1}{\sigma^2}\left(\frac{A+B}{2}+c\right)} \sigma^{-2\left(\frac{n}{2}+d-1\right)} d\sigma^{-2}\right]}{h_1(Z)} \qquad \textbf{(10)}$$

$$\boldsymbol{g(\sigma^{-2}|Z)} = \frac{\left[\sum_{m=1}^{n-1} e^{-\frac{1}{\sigma^2}\left(\frac{A+B}{2}+c\right)} \sigma^{-2\left(\frac{n}{2}+d-1\right)} \int_{-\infty}^{\infty} e^{\left[-\frac{1}{2}\beta_1^2 B_1 + \beta_1 A_1\right]} d\beta_1 \int_{-\infty}^{\infty} e^{\left[-\frac{1}{2}\beta_2^2 B_2 + \beta_2 A_2\right]} d\beta_2\right]}{h_1(Z)} \qquad \textbf{(11)}$$

$$\boldsymbol{g(m|Z)} = \frac{\left[\int_{-\infty}^{\infty} e^{\left[-\frac{1}{2}\beta_1^2 B_1 + \beta_1 A_1\right]} d\beta_1 \int_{-\infty}^{\infty} e^{\left[-\frac{1}{2}\beta_2^2 B_2 + \beta_2 A_2\right]} d\beta_2 \int_0^{\infty} e^{-\frac{1}{\sigma^2}\left(\frac{A+B}{2}+c\right)} \sigma^{-2\left(\frac{n}{2}+d-1\right)} d\sigma^{-2}\right]}{h_1(Z)} \qquad \textbf{(12)}$$

Now, the Bayes estimator of any function of parameter α, say $g(\alpha)$ under the squared loss function will be:

$$\boldsymbol{E_{\alpha|z}\big(g(\alpha|Z)\big) = \int_0^{\infty} \alpha\big(g(\alpha|Z)\big)\, d\alpha} \qquad \textbf{(13)}$$

where, ***g(α|Z)*** is marginal posterior density of $\boldsymbol{\alpha}$. It is little complicated to compute the equation **(13)** analytically in this case. Therefore, we propose to use Gibbs sampling and MCMC methods to find the Bayes estimators of $\boldsymbol{\beta_1}$, $\boldsymbol{\beta_2}$, $\boldsymbol{\sigma^2}$ *and* ***'m'***.

ALGORITHM USING GIBBS SAMPLING:

The Gibbs sampling procedure is implemented and therefore we re-write **(9)** as full conditional of $\boldsymbol{\beta_1}$ by fixing all other parameters, i.e. $\boldsymbol{\beta_2}$, $\boldsymbol{\sigma^{-2}}$ and ***'m'***. Hence, full conditional density of $\boldsymbol{\beta_1}$ given $\boldsymbol{\beta_2}$, $\boldsymbol{\sigma^{-2}}$ **and *'m'*** will be as follows:

$$g(\beta_1 \mid \beta_2, \sigma^{-2}, m, Z) \propto N\left(\frac{A_1}{B_1}, \left(\frac{1}{\sqrt{B_1}}\right)^2\right) \qquad \textbf{(14)}$$

where $\boldsymbol{A_1}$ and $\boldsymbol{B_1}$ are given in equation **(7)**.

Further, we also re-write **(10)** as full conditional density of $\boldsymbol{\beta_2}$ and by fixing all other parameters $\boldsymbol{\beta_1}$, $\boldsymbol{\sigma^{-2}}$ and '**m**', we get the full conditional density of $\boldsymbol{\beta_2}$ given $\boldsymbol{\beta_1}$, $\boldsymbol{\sigma^{-2}}$ and '**m**' as follows:

$$g(\beta_2 \mid \beta_1, \sigma^{-2}, m, Z) \propto N\left(\frac{A_2}{B_2}, \left(\frac{1}{\sqrt{B_2}}\right)^2\right) \qquad \textbf{(15)}$$

where A_2 and B_2 are given in equation **(7)**.

Now, we re-write **(11)** as full conditional density of σ^{-2} and by fixing parameters β_1, β_2 *and 'm'*, we get the full conditional density of σ^{-2} given **β1, β2** and **'m'** is as follows,

$$g(\sigma^{-2} | \beta_1, \beta_2, m, Z) \propto gamma\left(\frac{n}{2} + d\,, \frac{1}{c+\frac{A+B}{2}}\right) \qquad \textbf{(16)}$$

where, values of ***A*** and ***B*** are given in equation **(7)**.

We apply Gibbs sampling to generate sample from the full conditional density of $\boldsymbol{\beta_1}, \boldsymbol{\beta_2}$ and $\boldsymbol{\sigma^{-2}}$ given respectively in **(14), (15)** and **(16)**. In order to estimate the parameters $\boldsymbol{\beta_1}, \boldsymbol{\beta_2}$ and $\boldsymbol{\sigma^{-2}}$ we use following algorithm:

ALGORITHM:

Initialize $\boldsymbol{\beta_1} = \boldsymbol{\beta_{10}},\ \boldsymbol{\beta_2} = \boldsymbol{\beta_{20}}, \boldsymbol{\sigma^{-2}} = \boldsymbol{\sigma_0^{-2}}$ and $\boldsymbol{m} = \boldsymbol{m_0}$ then,

STEP 1: Generate $\boldsymbol{\beta_1} \sim \boldsymbol{N}\left(\frac{A_1}{B_1}, \left(\frac{1}{\sqrt{B_1}}\right)^2\right)$, using Gibbs Sampling.

STEP 2: Generate $\boldsymbol{\beta_2} \sim \boldsymbol{N}\left(\frac{A_2}{B_2}, \left(\frac{1}{\sqrt{B_2}}\right)^2\right)$, using Gibbs Sampling.

STEP 3: Generate $\boldsymbol{\sigma^{-2}} \sim \boldsymbol{gamma}\left(\frac{n}{2} + \boldsymbol{d}\,, \frac{1}{c+\frac{A+B}{2}}\right)$, using Gibbs Sampling.

STEP 4: Repeat the above steps.

APPLICATION OF MCMC TECHNIQUES:

There is no closed form of posterior distribution of change point **(12)**. That is why we need to propose the use of MCMC techniques to generate the samples from the posterior distribution. To implement the MCMC techniques, we re-write **(12)** as target function of ***'m'***, by fixing all other parameters i.e. $\boldsymbol{\beta_1}, \boldsymbol{\beta_2}$, **and** $\boldsymbol{\sigma^{-2}}$**.** Hence full target function of ***'m'*** given $\boldsymbol{\beta_1}, \boldsymbol{\beta_2}$, **and** $\boldsymbol{\sigma^{-2}}$ will be as follows:

$$g(m \mid \sigma^{-2}, \beta_1, \beta_2, Z) \propto e^{-\frac{1}{\sigma^2}(c+\frac{A+B}{2})}\,\sigma^{-2(\frac{n}{2}+d-1)}\,G_{1m}\,G_{2m}$$

$$G_{1m} = \int_{-\infty}^{\infty} e^{\left[-\frac{1}{2}\beta_1^2\left(\frac{S_{m1}}{\sigma^2}+\frac{1}{\sigma_1^2}\right)+\beta_1\left(\frac{S_{m3}}{\sigma^2}\right)\right]} d\beta_1 = \int_{-\infty}^{\infty} e^{\left[-\frac{1}{2}\beta_1^2 B_1 + \beta_1 A_1\right]} d\beta_1 = \frac{e^{\frac{A_2^2}{2B_1}}\sqrt{2\pi}}{\sqrt{B_1}}$$

$$G_{2m} = \int_{-\infty}^{\infty} e^{\left[-\frac{1}{2}\beta_2^2\left(\frac{S_{n1}-S_{m1}}{\sigma^2}+\frac{1}{\sigma_2^2}\right)+\beta_2\left(\frac{S_{m4}}{\sigma^2}\right)\right]} d\beta_2 = \int_{-\infty}^{\infty} e^{\left[-\frac{1}{2}\beta_2^2 B_2 + \beta_2 A_2\right]} d\beta_2 = \frac{e^{\frac{A_3^2}{2B_2}}\sqrt{2\pi}}{\sqrt{B_2}}$$

$$g(m \mid \sigma^{-2}, \beta_1, \beta_2, Z) \propto e^{-\frac{1}{\sigma^2}(c+\frac{A+B}{2})}\,\sigma^{-2(\frac{n}{2}+d-1)}\,\frac{e^{\frac{A_1^2}{2B_1}}\sqrt{2\pi}}{\sqrt{B_1}}\,\frac{e^{\frac{A_2^2}{2B_2}}\sqrt{2\pi}}{\sqrt{B_2}} \qquad \textbf{(17)}$$

where ***A, B, A₁, B₁, A₂ and B₂*** are given in equation **(7)**.

SOLUTION OF A NUMERICAL EXAMPLE:

The two phase linear regression model is assumed as under:

$$\boldsymbol{y_t} = \begin{cases} \boldsymbol{3x_t + \epsilon_t}, & \mathbf{t = 1, 2, 3, 4} \\ \boldsymbol{3.5x_t + \epsilon_t}, & \mathbf{t = 5, 6, \ldots, 15} \end{cases}$$

Here, the independently and identically distributed random errors which follow Normal Distribution **N (0, 1)** and are denoted by $\boldsymbol{\epsilon_t}$. The generated observations are precisely given in **TABLE 1**. $\boldsymbol{\beta_1}$ and $\boldsymbol{\beta_2}$ themselves were random observations from standard normal distribution with $\boldsymbol{\mu_1}$**=0** and standard deviation $\boldsymbol{\sigma}$ **=*1*** and precision $\frac{\mathbf{1}}{\boldsymbol{\sigma^2}}$ was from gamma distribution with (prior mean $\boldsymbol{\mu_3} = \mathbf{1}$ and $\emptyset^2 = \mathbf{0.02}$). Also ***c* = *50*** and ***d* = *50***.

TABLE 1

DATA GENERATED FROM THE TPLR MODEL

T	x_t	y_t	T	x_t	y_t	t	x_t	y_t	T	x_t	y_t	t	x_t	y_t
1	1.7	4.6	4	3.3	8.0	7	4.2	13.90	10	4.8	16.40	13	6.0	17.99
2	2.2	6.4	5	3.9	13.75	8	4.5	14.18	11	5.1	16.90	14	6.2	18.92
3	2.7	6.9	6	4.1	11.27	9	4.7	15.45	12	5.2	17.32	15	6.5	19.36

From the above table, it is quite clear that a random sample of size ***n* = *100*** is generated from $\boldsymbol{g(m \mid \sigma^{-2}, Z)}$ using the **Random Walk Metropolis - Hasting (RWM-H) algorithm** for **5000 times** each. Further, we observe that the selected proposal is **U (1, 14)** which is same as the prior and it is symmetric around **7.5**. Also, we note from the above table that the initial distribution is chosen as **U (1, 14)** since the target function is bounded. Moreover, the initial distribution is truncated and then we obtain the integer value of the Bayes Estimate of change point ***'m'*** as **4** when selected proposal is **U (1, 14)** and initial distribution is **U (3, 14)**. The data given in **TABLE 1** has its results in **TABLE 2** when given values are taken as $\boldsymbol{\beta_1}$ **= 2,** $\boldsymbol{\beta_2}$ **= 3 and** $\boldsymbol{\sigma^2}$ **= 2.**

TABLE 2

BAYES ESTIMATES OF CHANGE POINT *'m'* USING RWM-H ALGORITHM UNDER SQUARED ERROR LOSS FUNCTION

Selected Proposal	***Initial Distribution***	***Bayes Estimate of Change Point 'm'***	***Integer Value of Bayes Estimate of Change Point 'm'***
U (1,14)	U (1,14)	1.66	2
U (1,14)	U (2,14)	2.65	3

U (1,14)	U (2,14)	2.65	3
U (1,14)	**U (3,14)**	**3.55**	**4**

The results where the Bayes Estimates of ***'m'*** are computed using **RWM-H algorithm** for the different prior under consideration for the data given in **TABLE 1** are shown in following **TABLE 3**.

TABLE 3

BAYES ESTIMATES OF CHANGE POINT *'m'* USING RWM-H ALGORITHM UNDER SQUARED ERROR LOSS FUNCTION FOR DIFFERENT PRIOR UNDER CONSIDERATION

Serial Number	μ_1	μ_2	$\sigma_1{}^2$	$\sigma_2{}^2$	*Bayes Estimate of Change Point 'm' (Posterior Mean)*
1	0	0	1000	12000	4
2	2	3	1000	12000	4
3	2	3	4	9	4
4	2	3	1	9	4
5	2	3	0.09	0.09	4
6	**2**	**3**	**0.01**	**0.09**	**4**
7	20	30	0.01	0.09	4
8	0.2	0.3	0.01	0.09	4

Here, we have used **Gibbs Sampling Technique and MCMC algorithm** for the different prior under consideration for the data given in **TABLE 1** where we have computed the Bayes Estimates of $\boldsymbol{\beta_1}$ and $\boldsymbol{\beta_2}$ (when given value of $\boldsymbol{\beta_2}$ **= 3, *m* = 4 and** $\boldsymbol{\sigma^2}$ **= 2**). The results are shown below in **TABLE 4**.

TABLE 4

BAYES ESTIMATES OF β_1 AND β_2 USING GIBBS SAMPLING TECHNIQUE AND MCMC ALGORITHM UNDER SQUARED ERROR LOSS FUNCTION FOR DIFFERENT PRIOR UNDER CONSIDERATION

Serial Number	μ_1	μ_2	$\sigma_1{}^2$	$\sigma_2{}^2$	*Bayes Estimates of*			*Standard Deviation of Bayes Estimates of*		
					β_1	β_2	σ^2	β_1	β_2	σ^2
1	0	0	1000	12,000	2.573	3.168	2.348	0.237	0.112	0.022
2	2	3	1000	12,000	2.573	3.168	2.348	0.237	0.112	0.022
3	2	3	4	9	2.562	3.168	2.348	0.235	0.112	0.022
4	2	3	1	9	2.568	3.168	2.348	0.236	0.112	0.022
5	2	3	0.09	0.09	2.309	3.156	2.348	0.175	0.108	0.022
6	**2**	**3**	**0.01**	**0.09**	**2.065**	**3.156**	**2.348**	**0.081**	**0.108**	**0.022**
7	20	30	0.01	0.09	17.963	5.074	2.348	0.081	0.108	0.022
8	0.2	0.3	0.01	0.09	0.475	2.964	2.348	0.081	0.108	0.022

Here also we have used **Gibbs Sampling Technique and MCMC algorithm** for different prior under consideration for the data given in **TABLE 1** and computed the Bayes Estimates of $\boldsymbol{\sigma^2}$ when given value of $\boldsymbol{\beta_1}$ **= 2,** $\boldsymbol{\beta_2}$ **= 3 and *m* = 4**. The results are shown below in **TABLE 5**.

TABLE 5

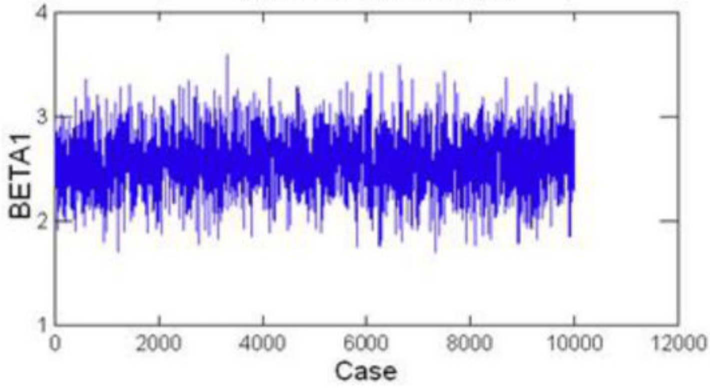
BAYES ESTIMATES OF σ^2 USING GIBBS SAMPLING TECHNIQUE AND MCMC ALGORITHM UNDER SQUARED ERROR LOSS FUNCTION FOR DIFFERENT PRIOR UNDER CONSIDERATION

Serial Number	μ_3	φ^2	*c*	*D*	*Bayes Estimates of* σ^2	*Standard Deviation of Bayes estimates of* σ^2
1	**1**	**0.0002**	**5000**	**5000**	**2.34**	**0.032**
2	0.1	0.002	5000	500	1.44	0.020
3	0.5	0.002	1000	500	7.18	0.231
4	5	0.002	100	500	67.72	6.727
5	50	0.0002	100	5000	109.86	10.913
6	5	0.0002	1000	5000	11.66	0.374
7	2	0.0002	2500	5000	4.68	0.090
8	1.67	0.0002	3000	5000	3.90	0.069
9	0.6	0.000333	5000	3000	1.94	0.027
10	0.5	0.0004	5000	2500	1.84	0.026
11	0.1	0.002	5000	500	1.44	0.020

GRAPHS AND INTERPRETATIONS

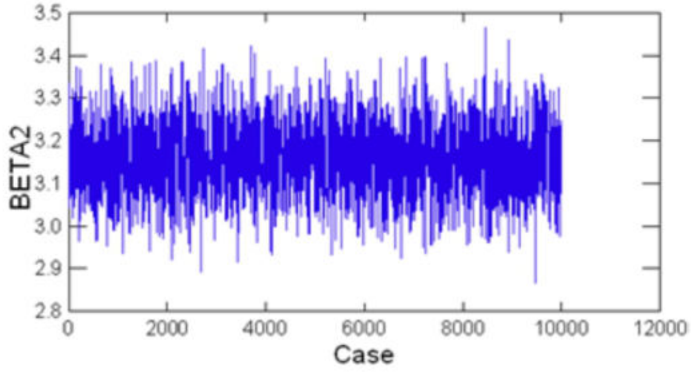
Here, **GRAPH 1** shows the full conditional of β_1 when a sample of size *10,000* was generated from $g(\beta_1 \mid \beta_2, \sigma^{-2}, m, Z)$. Also, Gibbs Sampling with MCMC algorithm showed the results $\beta_2 = 3, \sigma^2 = 2, m = 4$

Figure 1: Full Conditional of β_1



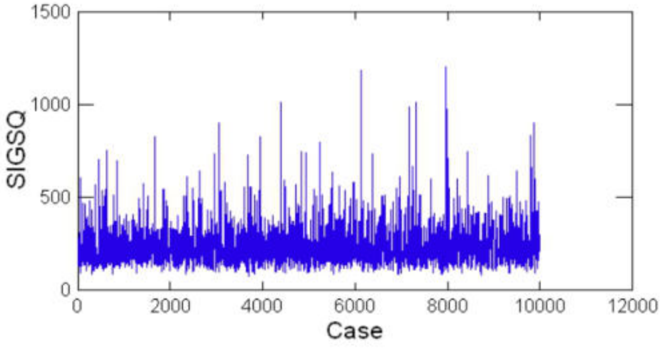
Here, **GRAPH 2** shows the full conditional of β_2 when a sample of size *10,000* was generated from $g(\beta_2 \mid \beta_1, \sigma^{-2}, m, Z)$. Also, Gibbs Sampling with MCMC algorithm showed the results $\beta_1 = 2, \sigma^2 = 2, m = 4$

Figure 2: Full Conditional of β_2



Here, **GRAPH 3** shows the full conditional of σ^{-2} when a sample of size *10,000* was generated from $g(\sigma^{-2} \mid \beta_1, \beta_2, m, Z)$. Also, Gibbs Sampling with MCMC algorithm showed the results $\beta_1 = 2, \beta_2 = 3, m = 4$

Figure 3: Full Conditional of σ^{-2}



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