ORIGINAL RESEARCH PAPER

Engineering

ANALYSIS OF VERTICAL AND HORIZONTAL VELOCITY PROFILES FOR FLOW RATE MEASUREMENT IN SMALL HYDROPOWER PLANTS

KEY WORDS: Open channel, propeller current meters, flow rate, velocity profiles.

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ABSTRACT

The efficiency measurement of hydropower stations requires the measurement of the absolute value of the discharge, Q. The IEC60041 issued guidelines and methods on how accurately it can be measured. For open channels, it stipulated a simultaneous measurement of the flow rate at a minimum of 25 measurement points at the intersection of 5 vertical and 5 horizontal lines in the channel cross-section. These measurement points cause disturbances to the flow, especially in narrow channels. This work studied the vertical and horizontal profiles of the flow in open channels in order to analyze the possibilities of minimizing the number of measurement points, reducing the disturbance and easing the task of the measurement. The theoretical models of the velocity profiles are simulated using MATLAB for a variety of site parameters and the results are discussed.

INTRODUCTION

The most challenging aspect of the unit efficiency measurement of hydro turbines is the measurement of flow rate, Q. The IEC Standard 60041 (Field Acceptance Test to Determine the Hydraulic Performance of Hydraulic Turbines, Storage Pumps, and Pump-Turbines) specifies some conditions for the measurement to be satisfactory and acceptable. However, these conditions are frequently not met due to limitations imposed by the plant's design, the cost of installing special equipment, and the limitations imposed by the plant's operating conditions. As a result, the measurement of discharge needs to be done in an open channel either at the intake upstream (head race) or the download stream (tail race) by the principle of the velocity-area method using propeller-type current meters or Pitot tube.

Mathematical models of the velocity profile in circular pipes are available (Salami, 1972) and have been used for determining the most appropriate locations of current meters and ultrasonic transducers for enhancing the accuracy of discharge measurement in circular penstocks. However, such mathematical models are not available for open channels (Gandhi et al, 2016). In open channel flow, the velocity distribution in the vertical direction is theoretically represented by logarithmic function of the depth of flow (Peck, 1992). The mean velocity along each vertical can be obtained by integration over the depth and is also called unit width discharge, being the discharge through a unit width of the section at a given vertical (ISO-748, 1997). The total discharge through the channel cross section may be determined as,

$$Q = \int q_i db = \sum \bar{v}_i \, d_i \Delta B \qquad \text{Eqn (1)}$$

where Q = total discharge (m^3/s),
V_i = average velocity in a segment (m/s),
di = depth of segment (m) and
ΔB is incremental width (m).

It is very difficult to describe the transverse velocity distribution theoretically. However, Bogle (1997) have proposed empirical equations for transverse velocity distribution based on experimental and field data. Salami (1972) developed about 23 profile models used for the analysis of asymmetric profiles. Sooky (1969) developed a transverse velocity distribution model based on experimental and field data. Seo and Baek (2004) have used a beta probability density function to cover complete spectrum of properties of the transverse velocity distribution for natural streams for flow away from the walls of the channels. The function is described as:

where α and β are real-number parameters and the gamma function

$$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha).\Gamma(\beta)} x^{(\alpha-1)}(1-x)^{(\beta-1)} \qquad \text{Eqn (2)}$$

The normalized transverse velocity distribution over the center of the walls of the channel width is given by:

$$\Gamma(\theta) = \int x^{\theta-1} e^{-x} dx, \quad \theta > 0 \qquad \text{Eqn (3)}$$

The normalized transverse velocity distribution over the center of the walls of the channel width is given by:

$$\frac{v}{V} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha).\Gamma(\beta)} y^{(\alpha-1)}(1-y)^{(\beta-1)}, \qquad 0 < y < 1 \qquad \text{Eqn (4)}$$

The velocity area method for open channels requires a number of propeller-type current meters located at specified points in a suitable cross-section of the open channel. Simultaneous measurements of a local mean velocity with the meters are integrated over the gauging section to provide the discharge (IEC60041, 1991).

Through the knowledge of the velocity profiles, it will be possible to ascertain the points of maximum discharge and those points that may not contribute much to the measurement. This will minimize the uncertainties in the method of measurement.

METHODOLOGY

The application of theoretical profiles has been used for the study of point velocity measurement techniques. The advantage of this is that the actual flow can be calculated.

Vertical Velocity Profile Model

In the open channel flow, the velocity distribution along the vertical direction (depth) is theoretically represented by a logarithmic function of the depth of flow (Schlichting, 1979).

Gandhi et al (2016) have characterized the vertical velocity profile of a rectangular open channel using five (5) parameters. These parameters are:

- Smoothness of the bed, represented by constant m3
- Nature of the flow: turbulence or laminar flow, represented by constant k3Air friction- represented by constants p1 and p2

- Von Karman constant, KV

The mathematical model for the vertical velocity profile is given as:

$$v(y) = \frac{1}{K_v}\log[m_3 k_3 h(y)] + h(y)[1 - p_1 h(y)^{p_2}] \qquad \text{Eqn (5)}$$

Where

v(y) represents the vertical velocity at point y along the depth of the channel.

h(y) is the normalized depth of point y.

The vertical velocity profile consists of two functions: Logarithmic and Parabolic functions The logarithmic function represents the lower region which is affected by the roughness of the bed while the parabolic function represents the upper region of the vertical velocity profile and is affected by the air friction.

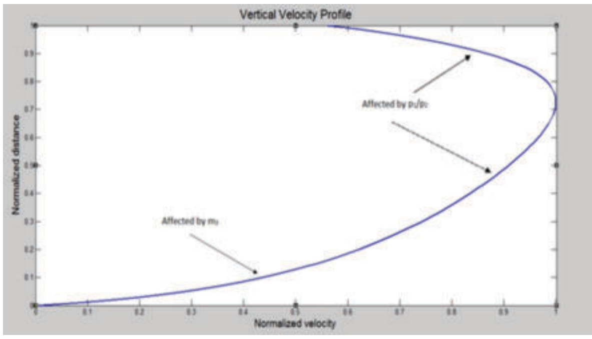

Figure 1: Vertical Velocity Profile

The vertical velocity profile shows that the maximum discharge is not uniform throughout the height of the channel. The maximum velocity occurs somewhere close to the top layer. It is affected by the air friction at the top and by the roughness of the bed at the bottom.

Unit-width Discharge (horizontal Velocity Profile) Model

The unit-width discharge represents the discharge of a unit length x along the width of the channel. According to Gandhi et al (2016), the model consists of three components: two logarithmic functions representing the flow on the left and right sides; the third component is a beta distribution function representing the flow throughout the width of the channel except the portions near the walls. The mathematical model consists of the following parameters:

- Smoothness of the wall surfaces represented by constants m1 and m2 for the left and right bank sides respectively.
- Nature of the water: turbulence or laminar, which is represented by the parameters k1 and k2 for both sides of the bank.
- The constants representing the channel geometry such as convergence, divergence and bends in the channel, represented by α and β.

The unit width discharge is represented by the mathematical model:

$$q_i(x) = \log[m_1 k_1 w(x)] + \log[m_2 k_2(1 - w(x))] \pm \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}[w(x)]^{\alpha-1}[1 - w(x)]^{\beta-1} \qquad \text{Eqn (6)}$$

Where
qi(x) represents the unit width discharge at point x.

w(x) is the normalized distance of the point x from the left side of the wall of the channel.

Adding the beta function produces single peak curves, whereas subtracting the beta function produces double peak curves.

Figure 2 shows the unit-width discharge profile with a single peak. It is symmetrical when the parameters for both walls are the same and α = β. The velocity is steeper at the two ends and the value is reduced to zero on the walls. The area of maximum discharge appeared to be at the center of the channel.

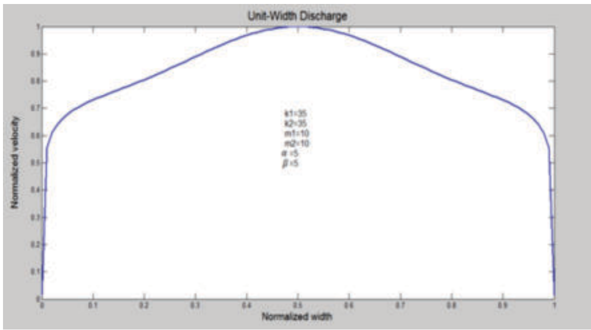

Figure 2: Unit-width discharge for a rectangular open channel

The effect of the skewness coefficients on the profile is shown in Figure 3. When α is less than β, the profile is skewed to the left of the channel. Similarly, when β is less than α the profile is skewed to the right. These are the cases in rectangular curved channels.

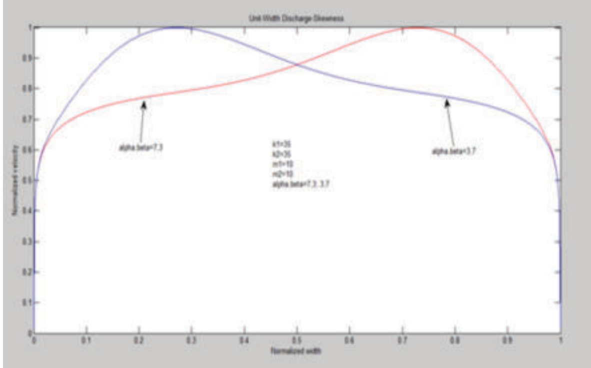

Figure 3: Effect of skewness exhibiting rectangular curved channels

Simulations And Discussions

Variations Of Bed Smoothness On The Vertical Velocity Profile

The profile is affected at the top by the air friction coefficients p1/p2 and by the smoothness coefficient m3 at the bottom. Figure 4 shows that the smoothness coefficient of the bed does not affect the region of maximum discharge. As the smoothness of the bed m3 increases, reasonable velocity can be obtained down the depth of the channel.

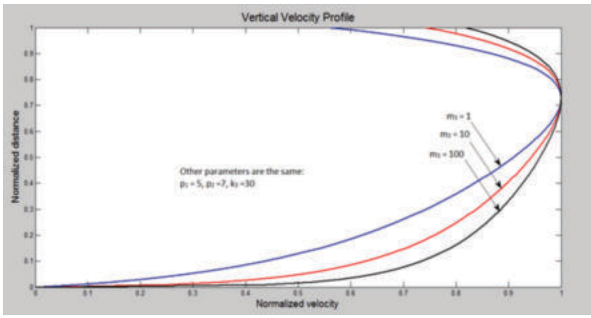

Figure 4: Effect of bed smoothness coefficient m3 on vertical velocity profile

Variation Of Air Friction On The Vertical Velocity Profile

The variation of the vertical velocity profile with the air friction is seen in Figure 3. It can be seen that the region where the velocity is highest along the depth varies as the ratio of air friction coefficients p1/p2 varies. The region shifts down in depth as the ratio increases. When there is no air friction, p1/p2 = 0, the region of highest velocity is found in the uppermost layer.

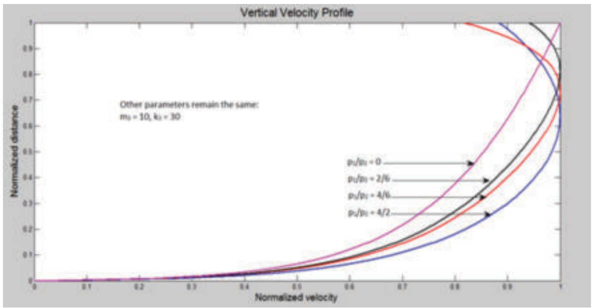

Figure 5: Variation of Air Friction Coefficient on the Vertical Velocity Profile

Effect Of Wall Smoothness On The Unit Width Discharge

The effect of the wall smoothness on the unit discharge is shown in Figure 6. The smoother the walls' surfaces, the higher the velocity around the walls and as the value of m1 and/or m2 increases, so does the velocity. This means the smoother the walls, the greater the area of higher discharge. The smoothness coefficients for both walls, m1 and m2, produced the same effect as would turbulence coefficients k1 and k2.

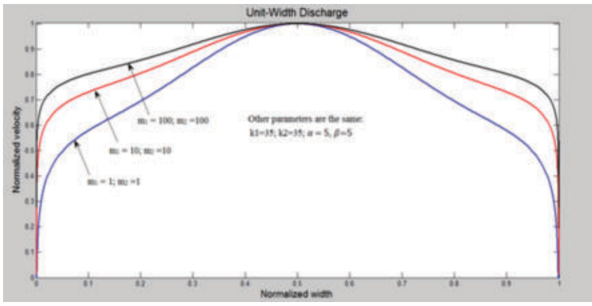

Figure 6: Effect of smoothness coefficients m1 and m2 on the unit-width discharge

Effect Of Beta Distribution Function On The Unit Width Discharge

The subtraction of the beta distribution function is to model the irregularities of the channel, such as the presence of obstructions or irregular beds. Figure 7 shows three different profiles with double peaks. It can be observed that when α is equal to β the profile is symmetrical along the width of the channel. When α is less than β the profile skewed to the left and it skewed to the right if β is less than α. These are the cases for the rectangular curved channels.

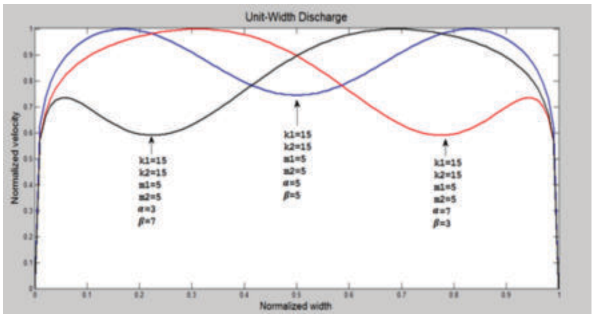

Figure 7: Unit-width discharge profile for a rectangular open channel with obstruction and bends

CONCLUSION

Having simulated the vertical velocity profile and the unit-width discharge of the rectangular open channels, it was observed that the portion where the velocity is higher depends on the parameters of the channel. For a straight channel with no obstruction, the intersection between the portion around 0.73 of the normalized height from the bed and the center of the channel along the width is the point of maximum discharge.

A further study on the topic will help towards minimizing the number and determining the best positions of the propeller current meters during the flow rate measurement in open channels for the purpose of unit efficiency test of hydroturbines using the velocity area method.

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