ORIGINAL RESEARCH PAPER

Medicine

THE CHALLENGES AND PITFALLS OF LIMB-LEADS AND HEART-AXIS CALCULATIONS "THE DIFFERENCES BETWEEN THE OVERALL MAGNITUDES OF BIPOLAR AND AUGMENTED UNIPOLAR LEADS CANNOT BE EXPLAINED WITH THE OHM'S LAW"

KEY WORDS: electrocardiogram, limb leads, magnitude, resistance, heart axis, Novosel formula, Monte-Carlo-Simulation, Ohm's law.

Dragutin Novosel*	MD, Glaserbergstrasse 23, CH-4056 Basel. *Corresponding Author
P. Zuljevic	Cand. Med.
M. Alanovic	MSEE
Robert Zunac	Mr.Sci., MSEE
Tina Becic	MD, PhD, Faculty of Medicine, Split, Croatia, Department of Cardiology

ABSTRACT

Summary Our paper analyses the interdependence between the overall magnitudes and resistances of bipolar and augmented unipolar limb leads of electrocardiograms. The findings strongly indicate that the overall ratio of magnitude of the bipolar and augmented unipolar limb leads is not a constant value, whereas the overall ratio of resistance is a constant value. These results expand but partially contradict a previous study. The ratio of the magnitude of bipolar leads to augmented unipolar leads is between 1 and 4/3 with a mean value of $2/\sqrt{3}$, and this ratio is a function of the electrical axis of the heart. Our results also show that the overall ratio of resistance between bipolar and augmented unipolar limb leads approaches the value of 4/3. Conclusively, the ratio of magnitudes between limb leads is not a constant value, the ratio of resistances can be assumed as a constant value and therefore the difference of magnitudes between limb leads cannot be explained with the Ohm's law.

INTRODUCTION

There is no doubt that evaluating the recordings of electrocardiography devices is a basic skill that medical professionals must have (1, 2). However, the underlying physics, recording machine features, theoretical concepts and backgrounds of mathematical equations are challenging (3, 4). In fact, they are the subject of discussions within the scientific community (5, 6).

The most common standard electrocardiography device usually records and prints six limb leads of electrocardiograms (ECGs), and all of these limbs have certain diagnostic values (no reference is being made to precordial leads here). Further, according to lead theory (3), limb leads are interconnected in various-but exactly defined-ways (3, 7, 8).

The basic equations and underlying theory related to this subject are complex (3, 5, 9); yet, such knowledge may not have a major role in daily practice.

Indeed, we may have to accept and acknowledge a neglected value with respect to daily medical care: the concept of limb leads, which is grounded in Einthoven theory (3, 8), is relevant from educational and methodological points of view (1, 2, 7, 10). Understanding these concepts is crucial for developing insights into the heart's electrophysiology and is a part of basic and advanced medical training (1, 2, 5).

Despite the use of ECGs for more than a century (11, 12), numerous publications on the topic and the continuous development of guidelines (13, 14), some misunderstandings regarding basic concepts have arisen (7, 8). It has been claimed (15), that the overall magnitude of limb leads I, II and III is higher than that of aVR, aVL and AVF and that the ratio of their resistance is 4/3 due to Ohm's law. We consider these statements to be insufficiently proven.

Thus, we present a theoretical basis for relationships between the magnitudes and resistances of limb leads. This paper presents equations, calculations and data that enable researchers to calculate and compare the magnitude of bipolar and augmented unipolar leads. It also analyses the resistances of such leads and outlines equations of its relationships. Finally, it reflects on the significance of these findings with respect to a clinical setting.

1. METHODS AND MATERIAL

The subjects of this paper's analysis are proportions; therefore, it ignores units for the discussed values (e.g., volt or degree), and the distinctions between scalar/vectoral (3, 8, 10) characteristics have no relevance to the current study.

Furthermore, as this study uses several geometrical and mathematical equations involving numerous terms and ECG nomenclature, the following definitions are provided for the purpose of clarity. Thus, some repetitions are unavoidable.

As we are using cross-references to present equations, we will number the sections and equations. The reference number of the equation is on the left; the right part is a reference to other definitions and/or equations used for our delineations.

Terms 'amplitude' and 'magnitude' will be used as synonyms and are the sum of QRS peaks, as used for the calculation of the mean electrical axis of the heart (EA) and described elsewhere (7, 8, 9) and correspond to the amplitude / magnitude of the corresponding lead.

1.1 Definitions and Terminology

1.1.1 Definitions and Terminology

The fundamentals of the limb-lead relationship are grounded in the concept introduced by Einthoven and are graphically presented in Figure 1 as the Einthoven triangle (3, 8, 10). Most of the listed equations are established and well known. We list them for the purpose of completeness, comprehensibility and congruency of the text.

1.1.1Body parts where the electrodes are connected are noted as follows (16):

[1] *RA=right arm*
[2] *LA=left arm*
[3] *LF = left foot*

1.1.2 Potentials

[4] *ER=Potential at the right arm*
[5] *EL=Potential at the left arm*
[6] *EF= Potential at the left foot*

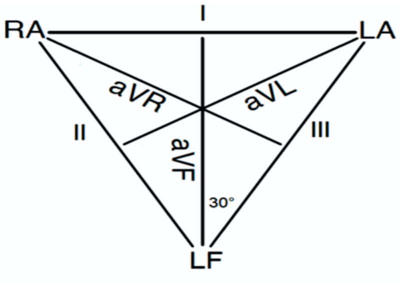

Figure 1: Einthoven triangle [1,2,3,4,5,6].

1.1.3 Terminals

[7] $Central\ terminal = \frac{1}{3}(ER + EL + EF)$ [4,5,6]

[8] $Modified\ central\ terminal\ (RL) = \frac{1}{2}(ER + EL)$ [4,5]

[9] $Modified\ central\ terminal\ (RF) = \frac{1}{2}(ER + EF)$ [4,6]

[10] $Modified\ central\ terminal\ (LF) = \frac{1}{2}(EL + EF)$ [5,6]

1.1.4 Resistances

[11] *RI= Resistance between the left arm and the right arm* [1,2]
[12] *RII= Resistance between the right arm and the left* foot [1,3]
[13] *RIII= Resistance between the left arm and the left* foot [2,3]
[14] *RaVR= Resistance between the right arm and the* modified central terminal (*LF*) [1,10]
[15] *RaVL= Resistance between the left arm and the* modified central terminal (*RF*) [2,9]
[16] *RaVF= Resistance between the left* foot *and the* modified central terminal (*RL*) [3,8]

1.1.5 Bipolar Limb Leads

Bipolar limb leads (I, II and III) are differences in the potentials between the left and right arm and the left foot as follows (see Figure 1).

[17] *I=EL-ER* [4,5]
[18] *II=EF-ER* [4,6]
[19] *III=EF-EL* [5,6]

1.1.6 Unipolar Limb Leads

Unipolar limb leads (VR, VL and VF) are the differences in the potentials between the right arm, the left arm and the left foot (separately) to a point which is the summed potential of the potentials of the aforementioned point and equals to the central terminal (the definition is in the section 1.1.3 [7]). This method was introduced by Wilson (17).

[20] $VR = ER - Central\ terminal = ER - \frac{1}{3}(ER + EL + EF)$ [4,7]

[21] $VL = EL - Central\ terminal = EL - \frac{1}{3}(ER + EL + EF)$ [5,7]

[22] $VF = EF - Central\ terminal = EF - \frac{1}{3}(ER + EL + EF)$ [6,7]

1.1.7 Augmented Unipolar Limb Leads

Augmented unipolar limb leads (aVR, aVL and aVF) are the differences in the potentials between the right arm, the left arm and the left foot (separately) and the sum of two other potentials (see section 1.1.3 [8, 9, 10] and Figure 1). This concept was introduced by Goldberg (18).

[23] $aVR = ER - Modified\ central\ terminal\ (LF) = ER - \frac{1}{2}(EL + EF)$ [4,10]

[24] $aVL = EL - Modified\ central\ terminal\ (RF) = EL - \frac{1}{2}(ER + EF)$ [5,9]

[25] $aVF = EF - Modified\ central\ terminal\ (RL) = EF - \frac{1}{2}(EL + ER)$ [6,8]

1.1.8 Mean Electrical Axis of the Heart (EA): relationships between the cardiac vector (A), EA and the I and aVF limb leads amplitude.

In the Figure 2 we present the relationships / calculations of the EA/A/I/aVF in the righ-down quadrant of the Einthoven triangle as described elswhere (7,8,9).

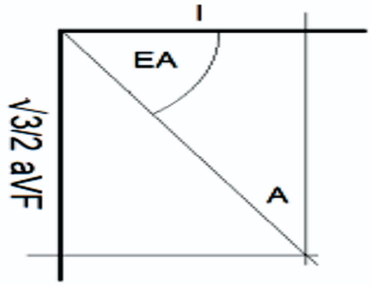

Figure 2: Trigonometrical relations of leads I, aVF, EA and A in the right-down quadrant. Equivalent figures can be made for any combination of limbs leads. Please note that for the numerical and/or geometrical calculations which include one of the bipolar and one of the augmented unipolar leads the corrections schoud be performed: when calculating the EA from the leads I and aVF the amplitude of the aVF should be multiplied with 2/√3; whereas if the aVF is delineated from the cardiac vector the value should be multiplied wirh √3/2, as described elsewhere (7,8,9).

The EA can be calculated from the combination of any of two leamb leads (3). As delineated before (9) and described elsewhere (7, 8) the relationships between the amplitudes of the limb leads amplitude and the EA can be delineated:

[26] $I = A \cdot \cos(EA)$

[27] $II = A \cdot \cos(EA - 60°)$

[28] $III = A \cdot \cos(120° - EA)$

[29] $aVR = \frac{-\sqrt{3}}{2} \cdot A \cdot \cos(30° - EA)$

[30] $aVL = \frac{\sqrt{3}}{2} \cdot A \cdot \cos(EA + 30°)$

[31] $aVF = \frac{\sqrt{3}}{2} \cdot A \cdot \sin(EA)$

1.1.9 Resistances

Es reported elsewhere (19) the avarage resistance of the body parts relevant to the ECG are:

Table 1: Average resistance of the body

	Avarage /Ohm
Resistance of the right wrist	250
Resistance of the right elbow	150
Resistance of the right shoulder	100
Resistance of the left wrist	250
Resistance of the left elbow	150
Resistance of the left shoulder	100
Resistance of the torso	100
Resistance of the hip	50
Resistance of the knee	100
Resistance of the ankle	250

1.3 Simulations

1.3.1 Simulation of the amplitudes of the QRS

In our study we used the Monte Carlo method (20) and the pseudorandom generator feature of OpenOffice (21) to generate data samples. First we produced random values of the cardiac vector with the range between 0.2V and 1.2V approximate to values as described elsewhere (22,23). As the second step we generate random values of the EA between 0° and 360° for each value of the cardiac vector separately. With the use of these two values we calculate the amplitudes of all limb leads with the use of the equations as described in section 1.1.8 [26 .. 31]. The magnitudes of bipolar and augmented unipolar limb leads were calculated as the sums of absolute values (ignoring the minus and plus values) of the amplitudes of I, II, III, aVR, aVL and aVF respectively (explanation is provided in a behind sections 2.1.).

1.4 Recorded ECG Signals

As additional data (real ECG recordings) we use data available online(24). This data-set include recordings of leads I and II which we used for further calculations. We calculate the EA from leads I and II, as described elsewhere (7,8,9) and afterwards we calculate the cardiac vector (A) from lead I and EA (section 1.1.8; [26]). Leads III, aVR, aVL and aVF were calculated with the use of equations from the section 1.1.8 [28..31]. The ratio of the bipolar and augmented unipolar leads magnitudes was calculated as described in section 2.1.

1.5 Statistical analysis

Numerical data are reported as a minimum, maximum, mean and standard error of the mean (SEM) of the data set. When the statistical analysis were performed, we will report the confidence intervals (at the level of 95%). Analysis of differences were estimated with the one-sample T-test. The values of P<0.05 are considered statistically significant. The P values>0.05 are noted as n.s. and considered as statistically not-significant difference.

2. RESULTS

2.1 The Sum and Relationsiphs of Unipolar Leads

2.1.1 The sum of augmented unipolar limb leads

Here we refer to section 1.1.7 and delineate the sum of the amplitudes of aVR, aVL and aVF.

[32]

$$aVR + aVL + aVF$$

$$= ER - \frac{1}{2}(EL + EF) + EL - \frac{1}{2}(ER + EF) + EF - \frac{1}{2}(EL + ER) \qquad [23, 24, 25]$$

$$= ER + EL + EF - \frac{1}{2}EL - \frac{1}{2}EF - \frac{1}{2}ER - \frac{1}{2}EF - \frac{1}{2}EL - \frac{1}{2}ER$$

$$= ER + EL + EF - EL - EF - ER$$

$$= 0$$

Consequently, it makes no sense to calculate the sum of the magnitudes of augmented unipolar leads as it results in a value of zero. Therefore, for the comparison of amplitudes of bipolar and augmented unipolar limb leads we will use the absolute values of the magnitudes of them.

2.1.2 Augmented Unipolar Leads vs. Unipolar Leads

As described in the section terminology, the unipolar leads and the augmented unipolar leads are not the same (sections 1.1.6 and 1.1.7) and the confusion should be avoided. Certainly, in a daily praxis the aVR, aVL and aVF are mostly called unipolar leads but from the methodological point of view this disctintion should be made. The amplitude of the augmented unipolar leads is always 50% higher than that of the unipolar leads.

[33]

$$aVR = ER - \frac{1}{2}(EL + EF) \qquad [23]$$

$$aVR = ER + \frac{1}{2}ER - \frac{1}{2}ER - \frac{1}{2}EL - \frac{1}{2}EF$$

$$aVR = \frac{3}{2}ER - \frac{1}{2}(ER + EL + EF)$$

$$\frac{2}{3}aVR = ER - \frac{1}{3}(ER + EL + EF)$$

$$VR = ER - \frac{1}{3}(ER + EL + EF) \qquad [7, 20]$$

$$\frac{2}{3}aVR = VR$$

$$aVR = 1.5\,VR$$

Current ECG devices in use are producing / showing / printing the augmented unipolar leads (aVR, aVL and aVF) and not unipolar limb leads (VR, VL and VF) as introduced by Goldberg (18).

2.2 Relationship of the magnitude of bipolar and augmented unipolar limb leads

2.2.1 Monte Carlo Simulation of the magnitude of bipolar and augmented unipolar limb leads in the range of EA between 0° and 360°.

(N = 1000)	Min	Max	Mean	SEM
Cardiac Vector	0.201	1.199	0.706	0.009
EA	0.220	359.978	177.938	3.364
Ratio (I, II, III / aVR, aVL, aVF)	1.001	1.333	1.156	0.003

Table 2: Descriptive statistics of the results of the Monte-Carlo-Method of the simulation of the cardiac vector in the range of 0.2 to 1.2 and the EA between 0° and 360°. The data generation of the cardiac vector, EA and the calculation of the limb leads and of the ratio of magnitudes of the sum of bipolar and augmented unipolar leads are described in section 1.1.8 [26..31].

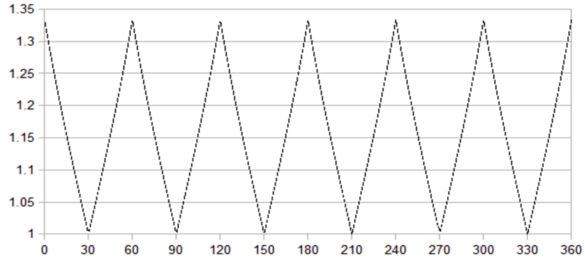

Figure3: The relationship between the EA and the quotient of the magnitudes of bipolar and augmented unipolar leads in the range of the EA between 0° and 360°. The horizontal axis is the mean EA of the heart and the vertical axis is the quotient of the magnitudes. The quotient of the sums of absolute amplitudes of bipolar limb leads to the sum of the absolute values of augmented unipolar leads is a function of the EA.

The unilateral T-test between observed (calculated) and the expected value of 1.1547 (see section 2.2.3 [41] below) of the mean value of the ratio as presented in Table 2 shows no significant difference (P=0.557=n.s., confidence intervals of the difference are -0.0041 / +0.0077).

2.2.2 Monte Carlo Simulation of the magnitude of bipolar and augmented unipolar limb leads in the range of EA between 0° and 30°.

(N = 91)	Min	Max	Mean	SEM
Cardiac Vector	0.20	1.19	0.74	0.03
EA	0.22	29.41	14.28	0.92
Ratio (I, II, III / aVR, aVL, aVF)	1.01	1.33	1.17	0.01

Table 3: Descriptive statistics of the Monte-Carlo-Method of the simulation with the cardiac vector between 0° and 30° (a subset of data from the sample described in section 2.2.1 and Table 2).

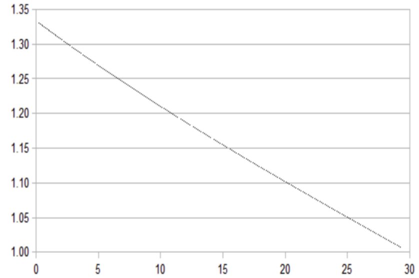

Figure 4: The relationship between the EA and the ratio of the magnitudes of bipolar and augmented unipolar leads in the range of 0° and 30° (subset of data from the Table 2 / Figure

3). The horizontal axis is the mean EA of the heart and the vertical axis is the ratio of the magnitudes. The ratio of the sums of absolute amplitudes of bipolar limb leads to the sum of the absolute magnitudes of augmented unipolar leads is a function of the EA. The unilateral T-test between observed (calculated) and the expected value of 1.1547 (see section 2.2.3 below) of the mean value of the ratio as presented in the Table 3 shows no significant difference (P=0.243=n.s., confidence intervals of the difference are -0.01 / +0.03).

2.2.3 Mathematical Calculation of the Magnitude of Bipolar and Augmented Unipolar Limb Leads in the Range of EA between 0° and 30°.

For the EA range of 0° to 30°, the amplitudes of I, II, aVL and aVF are positive; hence, the amplitudes of III and aVR are negative and the sum of limb leads can be reduced to the calculation for this particular range of the EA as follows.

The sum of I, II and III will be calculated as follows:
[34] $Dmagnitude = I + II - III$
The sum of aVR, aVL and aVF will be calculated as:

[35] $Amagnitude = -aVR + aVL + aVF$

The quotient for the magnitudes of bipolar and augmented unipolar leads between 0° and 30° is as follows:

[36] $$DAratio = \frac{Dmagnitude}{Amagnitude} = \frac{I + II - III}{-aVR + aVL + aVF}$$ [34,35]

After introducing the equations from the section 1.1.8 [26..31] and simplifying the equation we will become the following equation:

[37] $$DAratio = \frac{4}{\sqrt{(3)}} \cdot \frac{\cos(EA)}{\sqrt{(3)} \cdot \cos(EA) + \sin(EA)}$$ [26..31]

Additionally, we can express the same equation as the subject of the magnitudes of potentials:

$$Dmagnitude = EL - ER + EF - ER - EF + EL = 2 \cdot (EL - ER)$$ [17, 18, 19, 34]

[38] $$Dmagnitude = 2 \cdot (EL - ER) = 2 \cdot I$$ [17]

$$Amagnitude = -ER + \frac{1}{2} \cdot (EL + EF) + EL - \frac{1}{2} \cdot (ER + EF) + EF - \frac{1}{2} \cdot (EL + EF)$$ [23, 24, 25, 35]

$$Amagnitude = -2 \cdot ER + EL + EF$$

$$Amagnitude = (EF - ER) + (EL - ER)$$ [17, 18]

[39] $$Amagnitude = II + I$$

[40] $$DAratio = \frac{Dmagnitude}{Amagnitude} = \frac{I + II - III}{-aVR + aVL + aVF} = \frac{(2 \cdot I)}{(I + II)}$$ [36, 38, 39]

As shown in Tables 2, 3, 4 and Figure 3, 4, 5 the ratio is a function of the EA and is not a constant value.

Consequently, the overall magnitude of I, II and III are higher than of aVR, aVL and aVF, but this value is not a constant. This value range is from 1 to 4/3 with a mean value of 2/√3.

2.2.4 Analysis of the ECG recordings.

(N = 100)	Min	Max	Mean	SEM
Cardiac Vector	0.43	1.81	0.94	0.03
EA	6.13	80.91	51.34	1.89
Ratio (I, II, III / aVR, aVL, aVF)	1.00	1.33	1.18	0.01

Table 4: Descriptive statistics of the publicly available ECG recordings obtained as described in section 1.3. The method for the calculation is described in sections 1.1.7 and 1.2.1.

The unilateral T-test between observed (calculated) and the expected value of 1.1547 (see section 2.2.3) of the mean value of the ratio as presented in the Table 4 shows a statisticaly significant difference (P=0.024, confidence intervals of the difference are -0.00 / +0.04).

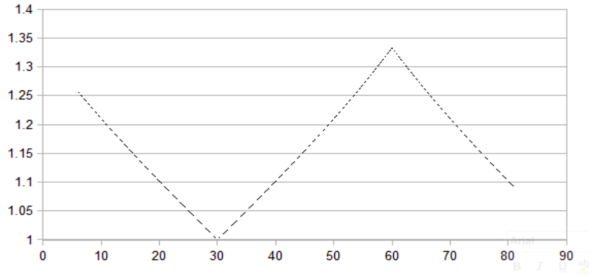

Figure 5: Relationship of the ratio of magnitudes od bipolar and unipolar limb leads from the data set publicly available.

2.3 Relationships of Resistance

2.3.1 Resistances of limbs leads.

The resistances of the bipolar limbs leads are calculated as the sum of body parts as described elsewhere (19):

The resistances of the limbs are:
R-RA = right wrist + right elbow + right shoulder
R-LA = left wrist + left elbow + left shoulder
R-LF = torso + hip + knee + ankle
It follows from this:
[41] *RI=R-RA+R-LA*
[42] *RII=R-RA+R-LF*
[43] *RIII=R-LA+R-LF*

For the calculation of the resistance of the augmented unipolar leads the definition of the corresponding modified central terminal must be considered.

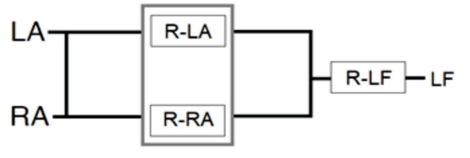

Figure 6: Schematic representation of the resistance of the aVF lead. The left part corresponds to the modified central terminal.

Accordingly, the calculations for the resistance of the augmented unipolar leads are as follows:

$$RaVR = \frac{R\text{-}LA * R\text{-}LF}{R\text{-}LA + R\text{-}LF} + R\text{-}RA$$

$$RaVL = \frac{R\text{-}RA * R\text{-}LF}{R\text{-}RA + R\text{-}LF} + R\text{-}LA$$

$$RaVF = \frac{R\text{-}LA * R\text{-}RA}{R\text{-}LA + R\text{-}RA} + R\text{-}LF$$

For the ideal case, where the resistances have the same value (e. g. R=R-LA), the ratio would be as follows:

$$R\text{-}D = RI + RII + R\,III$$

$$= R\text{-}LA + R\text{-}RA + R\text{-}RA + R\text{-}LF + R\text{-}LA + R\text{-}FL$$

$$= 2*(R\text{-}RA + R\text{-}LA + R\text{-}LF) \qquad (\text{if } R\text{-}RA = R\text{-}LA = R\text{-}LF = R)$$

$$= 6 \cdot R$$

$$R\text{-}A = R\text{-}aVR + R\text{-}aVL + R\text{-}aVFR =$$

$$+ (R-LA \cdot R-LF)/(R-LA + R-LF)$$

$$+ (R-RA \cdot R-LF)/(R-RA + R-LF)$$

$$+ (R-LA \cdot R-RA)/(R-LA + R-RA)$$

$$+ {+R\text{-}RA} + R\text{-}LA + R\text{-}LF \qquad \text{(if } R\text{-}RA = R\text{-}LA = R\text{-}LF = R\text{)}$$

$$= \frac{3}{2}R + 3R = \frac{9}{2}R$$

$$\frac{R\_D}{R\_A} = \frac{(6 \cdot R)}{(\frac{9}{2} \cdot R)}$$

$$Rratio = \frac{R\text{-}D}{R\text{-}A} = \frac{4}{3} \qquad [41]$$

DISCUSSION

Understanding the principles of the ECG is still a challenging topic in medical education (1, 4, 6) as it requires knowledge of the heart electrophysiology, concepts of leads and a bit of mathematics (2, 5).

Using the basics of Einthoven's triangle (3, 7, 8, 10), this paper delineated the ratio of sums for the magnitudes of bipolar limb leads by comparing it to the magnitudes of the augmented unipolar limb leads. According to the calculations and an analysis of the data gained by the Monte Carlo method (20) and real ECG recordings (24), the ratio of the magnitude is not a constant value; rather, it lies in the range of 1 to 4/3 and has an average value of $2/\sqrt{3}$. Furthermore, the ratio is a function of the electrical axis of the heart.

Trying to be precise, we propose the following definition: The ratio of the magnitudes of bipolar to augmented unipolar leads depends on the electrical activity of the heart. If the activity is expressed through the mean electrical axis of the heart, the ratio is the function of the electrical axis of the heart.

The Monte Carlo method we used is suitable for problems when the relationship between variables is complex and / or complicated to solve mathematically.

In our study, we are calculating the sum of absolute values of several valiables - what is usually inextricable. Therefore, our justification for the use of the Monte Carlo method is that the derivations of formulas of absolute sums of several variables require extensive and in our study dispensable mathematical delineations. Nevertheless, based on simulated data we were able to perform the ensuing calculations of the relationships and afterward exactly calculate the ratio.

As the distrubution of the EA in our simulated data set is uniform (all values of the EA have the same frequency) it cannot be expected that in the daily praxis the avarage value of the ratio will be $2/\sqrt{3}$. In the normal population, the frequency distribution of the EA is not uniform (22, 23): the most recorded ECGs diagnosed as normal findings will have the EA about 60°. Therefore the data from real ECG recordings (Table 4) is different from the simulated one.

The ratio for the sum of the resistances of bipolar and augmented unipolar limb leads follows a different relationship and depends on the differences between the resistances of the left arm, the right arm and the left foot. As shown in equation **[41]** in a ideal case (wherein the resistances have the same value), the ratio equals 4/3. Additionally, the ECG recodring devices have a high input resistance (actually impedance) which annulates the differences in resistance between limb leads. It does not mean that the body constitution should be ignored in the interpretation of the ECGs.

As the consequence of the different relationships of magnitudes and resistance the ratio of them can not be simply explained with Ohm's law. Otherwise the ratios would follow the same distributions and relationships. Further, the explanation of the ratio of magnitudes lies in the relationships between limb leads.

It is doubtful that all of this information's about the differences and relationships between overall amplitudes and/or resistances between bipolar and augmented unipolar leads may play any role in daily clinical practice. Indeed, all of the indices commonly used (Sokolow, Morris, Cornell, Lewis, Cabrera, Gertsh) use the separate elevations of the QRS and only the Lewis index uses only limb leads (but not the overall magnitude).

However, the discussed subject does have a role in basic medical education-and the presented equations may be useful when monitoring the quality of ECG recordings as a structural and procedural quality-control parameter with respect to error detection. Future research should investigate this potential use.

Limitations of the study

There are several points which we would like to bring the readers' attention. We are using the Monte-Carlo-Simulation with a pseudorandom generator. The produced values are per the definition not random. Additionally, the distribution of the generated values was uniform and certainly did not reflect the nature of the electrical activity of the heart, as mentioned before.

As we used recorded ECG values available online, we did not have information about the validity of the data. An additional source of errors / variability and problems with the methodology stems from manual outreading of the ECG signal, which will always result in variability, reflecting measurement error.

Contrasting these, we can argue that the data we used supports our theoretical hypothesis, and therefore the validity of our results should be met with cautious optimism.

Funding: no funding

Conflict of Interests: none

Ethical considerations: not applicable

REFERENCES

1 Cook, David & Oh, So-Young & Pusic, Martin. (2020). Accuracy of Physicians' Electrocardiogram Interpretations: A Systematic Review and Meta-analysis. JAMA internal medicine. 180. 10.1001/jamainternmed.2020.3989.

2 Kadish, Alan & Buxton, Alfred & Kennedy, Harold & Knight, Bradley & Mason, Jay & Schuger, Claudio & Tracy, Cynthia & Winters, William & Boone, Alan & Elnicki, Michael & Hirshfeld, John & Lorell, Beverly & Rodgers, George & Weitz, Howard. (2001). ACC/AHA Clinical competence statement on electrocardiography and ambulatory electrocardiography. Journal of the American College of Cardiology. 38. 10.1016/S0735-1097(01)01680-1.

3 JAKOB HOHL and Stanley Rush. THE COMPLETE HEART-LEAD RELATIONSHIP IN THE EINTHOVEN TRIANGLE. BULLETIN OF MATHEMATICAL BIOPHYSICS VOLUME 30, 1968. 615-623.

4 Trauschke, Mathias. (2019). A qualitative study on the development and rectification of advanced medical students' misconceptions about the physiological electrocardiogram (ECG). GMS Zeitschrift für medizinische Ausbildung. 36. 10.3205/zma001280.

5 Anderson, J & Dicarlo, Stephen. (2000). "Virtual" experiment for understanding the electrocardiogram and the mean electrical axis. Advances in physiology education. 23. 1-17. 10.1152/advances.2000.23.1.S1.

6 Viljoen, C.A., Millar, R.S., Manning, K. et al. Effectiveness of blended learning versus lectures alone on ECG analysis and interpretation by medical students. BMC Med Educ 20, 488 (2020). https://doi.org/10.1186/s12909-020-02403-y

7 Dragutin Novosel, Matija Alanovi , Robert unac and Tina Be i . "Visualising the Novosel Formula: Comments on Dahl and Berg's A formula for the mean electrical axis of the heart". Physiology News /Autumn 2021 / Issue 123: 7-8. https://doi.org/10.36866/pn.123.8

8 Dahl R and Berg R (2000). Trigonometry of the ECG. A formula for the mean electrical axis of the heart. Physiology News 120, 25-27.

9 Novosel D et al. (1999). Corrected formula for the calculation of the electrical heart axis. Croatian Medical Journal, 40, 77-79.

10 Samuel, Rajini. (2018). What to the world Einthoven talking about?. JOURNAL OF PHARMACEUTICAL AND BIOMEDICAL SCIENCES, 23 (7), 1-7.

11 AlGhatrif, Majd & Lindsay, Joseph. (2012). A brief review: history to understand fundamentals of electrocardiography. Journal of community hospital internal medicine perspectives. 2. 10.3402/jchimp.v2i1.14383.

12 Luna, Antoni & Fiol Sala, Miquel & Bayes-Genis, Antoni & Baranchuk, Adrian &

Elosua, Roberto & Martinez Selles, Manuel. (2021). The History of Electrocardiography. 10.1002/9781119536475.ch2.

13 Kligfield P et al. (2007). Recommendations for the Standardization and Interpretation of the Electrocardiogram Part I: The Electrocardiogram and Its Technologyt. Circulation 115, 1306-1324.

14 Mason JW et al. (2007). Recommendations for the Standardization and Interpretation of the Electrocardiogram Part II: Electrocardiography Diagnostic Statement List. Circulation 115, 1325-1332.

15 Samuel, Rajini. (2021). APPLICATION OF CARDIAC VECTOR THEORY IN ECG INTERPRETATION. PARIPEX-INDIAN JOURNAL OF RESEARCH. 10.

16 Hadjiantoni, Alexandros. (2020). Is the Correct Anatomical Placement of the Electrocardiogram (ECG) Electrodes Essential to Diagnosis in the Clinical Setting A Systematic Review. 10.21203/rs.3.rs-74147/v1.

17 Frank N. Wilson M.D. Franklin D. Johnston M.D. A. Garrard Macleod M.D. Paul S. Barker M.D. Electrocardiograms that represent the potential variations of asingle electrode. American Heart Journal. Volume 9, Issue 4, April 1934, Pages 447-458

18 Emanuel Goldberger M.D. A simple, indifferent, electrocardiographic electrode of zeropotential and a technique of obtaining augmented, unipolar,extremity leads. American Heart Journal. Volume 23, Issue 4, April 1942, Pages 483-492

19 Nikolaos D. Katopodes. (2019). Skin Resistance. Free-Surface Flow 2019.

20 Nicholas Metropolis; S. Ulam. (1949). The Monte Carlo Method. Journal of the American Statistical Association, 44 (247) 335-341.

21 https://www.openoffice.org/

22 CAPT. ROLAND G. HISS, MC, USAF, LAWRENCE E. LAMB, M.D., F.A.C.C. And MARGARET F. ALLEN, MS. Electrocardiographic Findings in 67,375 Asymptomatic Subjects X. Normal Values. THE AMERICAN JOURNAL OF CARDIOLOGY. JUNE 1960. Page 200-231

23 Peter R. Rijnbeek, Gerard van Herpen, Michiel L. Bots, Sumche Man, Niek Verweij, Albert Hofman, Hans Hillege, Matthijs E. Numans, Cees A. Swenne, Jacqueline C.M. Witteman, Jan A. Kors. (2014). Normal values of theelectrocardiogram for ages 16–90years, Journal of Electrocardiology. 47 (6), 914-921, doi.org/10.1016/j.jelectrocard.2014.07.022.

24 Atakora-Amaniampong, Larry. (2021). Electrocardiogram (QRS amplitude, duration and axis). DOI: 10.17632/3pj4nnhng5.1 Downloaded on 2021/10/31 from:https://www.researchgate.net/publication/351082888_Electrocardiogram_QRS_amplitude_duration_and_axis