ORIGINAL RESEARCH PAPER

Mathematics

A STUDY ON LINEAR EQUATIONS, SOLUTIONS OF LINEAR EQUATIONS.

KEY WORDS: System of equations, rank, augmented matrix.

Anil. S.C* Assistant Professor, Department of Mathematics, JNNCE, Shivamoga, Karnataka, India. *Corresponding Author

Shaila. S. Bhat Assistant Professor, Department of Mathematics, JNNCE, Shivamoga, Karnataka, India.

Dr.Gurupadavva Ingalahalli Assistant Professor, Department of Mathematics, JNNCE, Shivamoga, Karnataka, India.

ABSTRACT

This Paper is the study of a system of linear equations in n unknowns and finding solution by various methods, Gauss elimination method, Gauss Jordan method, Gauss Seidel method. A System of Linear Equations is two or more linear equations together. The solution to a system of linear equations is the point at which the lines representing the linear equations intersect. When there is no solution then the system of linear equations is referred as "inconsistent". If there is one or infinitely many solutions then the system of linear equations is referred as "consistent". Many problems in Engineering and applied sciences require the solution of System of equations. The system of linear equations used to solve age problems, to calculate speed, distance and time of a moving object. It is also used to calculate money and percentage problems, Work, time and wages problems. Problems on force and pressure can also be solved.

INTRODUCTION

A system of equations in which all the unknown quantities appear in the first degree is called a system of linear equations or linear system of equations.

Example: $3x+4y+3z=1, x+y+z=2, x+3y+z=3$

Suppose F is a field. A set of m linear equations in n unknowns $x_1, x_2, \dots x_n$ is

$a_{11}x_1+a_{12}x_2+\dots\dots+a_{1n}x_n=b_1$

$a_{21}x_1+a_{22}x_2+\dots\dots+a_{2n}x_n=b_2$

$\dots\dots\dots\dots\dots\dots$

$a_{m1}x_1+a_{m2}x_2+\dots\dots+a_{mn}x_n=b_n$.

where $b_1, b_2, \dots b_n$ are constants and a_{ij} are given elements of F. Any n-tuple $(x_1, x_2, \dots\dots x_n)$ of elements of F, which satisfies each of the equations is called a solution of the system of linear equations or solution set of system of linear equations.

If $b_1, b_2, \dots b_n$ all are equal to zero, then the system of equations is homogeneous system of linear equations. If atleast one of $b_1, b_2, \dots b_n$ is not equal to zero then the system of equations is non homogeneous system of linear equations.

A system of linear equations is consistent if it possesses a solutions, otherwise the system is said to be inconsistent.

Echelon Form of A Matrix:

A non zero matrix A is said to be in row echelon form if

i) all the zero rows are below non zero rows

ii) the first non zero entry in any non zero row is 1 and the entries below 1 in the same column are zero

The number of non zero rows in the echelon form of the matrix A is called rank of the matrix and is denoted by $\rho(A)$

The system of linear equations can be regarded as AX=B.

Here $A=\begin{pmatrix} a_{11}\ a_{12}\dots\dots\dots a_{1n} \\ a_{21}\ a_{22}\dots\dots\dots a_{2n} \\ a_{m1}\ a_{m2}\dots\dots\dots a_{mn} \end{pmatrix}$

Here $[A:B]=\begin{pmatrix} a_{11} & a_{12}\dots\dots\dots a_{1n}:b_1 \\ a_{21} & a_{22}\dots\dots\dots a_{2n}:b_2 \\ a_{m1} & a_{m2}\dots\dots\dots a_{mn}:b_m \end{pmatrix}$

is called Augmented matrix.

The system of equations represented by AX=B is consistent if $\rho(A)=\rho[A:B]$

Suppose $\rho(A)=\rho[A:B]=r$, then the condition for various types of solution are as follows.

I) Unique Solution: $\rho(A)=\rho[A:B]=r=n$, where n is the number of unknowns

ii) Infinite Solution: $\rho(A)=\rho[A:B]=r<n$.

If $\rho(A)\neq\rho[A:B]$, then the system of linear equations are said to be in consistent(do not possess a solution).

Gauss Elimination Method:

The Gauss elimination method is also referred as the row reduction algorithm for solving linear equations systems. It has a sequence of operations performed on the corresponding matrix of coefficients.

The general idea is to eliminate all but one variable using row operations and then back-substitute to solve for the other variables.

The steps of the Gauss elimination method are (1) Construct the Augmented matrix [A:B] corresponding to the linear system of equations AX=B matrix,

(2) By using elementary row operations Reduce the augmented matrix [A : B] to get [A' : B'].where A' is an upper triangular matrix.

(3) By the backward substitution in A'X = B', we get the solution of the given system of linear equations.

Example:

$x+y+z=9$,

$x-2y+3z=8$,

$2x+y-z=3$

$$[A:B]=\begin{pmatrix} 1 & 1 & 1 & : & 9 \\ 1 & -2 & 3 & : & 8 \\ 2 & 1 & -1 & : & 3 \end{pmatrix}$$

By using elementary row operations the matrix [A:B] is reduced to

$$\sim\begin{pmatrix} 1 & 1 & 1 & : & 9 \\ 0 & -3 & 2 & : & -1 \\ 0 & 0 & -11 & : & -44 \end{pmatrix}$$

$-11z=-44$ hence $z=4$,

$-3y+2z=-1$ hence $y=3$,

$x+y+z=9$ hence $x=2$,

Soln (2,3,4)

Gauss Jordan Elimination Method:

This method is similar to Gauss elimination method. The steps of Gauss elimination method are

(1) Construct the Augmented matrix [A:B] corresponding to the linear system of equations AX=B matrix,

(2) By using elementary row operations Reduce the augmented matrix [A : B] to get [A' : B'].where A' is a diagonal matrix.

(3) By the backward substitution in A'X = B', we get the solution of the given system of linear equations.

Example:

$$2x + 5y + 7z = 52,$$

$$2x + y - z = 0,$$

$$x + y + z = 9$$

$$R_2 \to R_2 - 2R_1, R_3 \to R_3 - 2R_1$$

$$\sim \left(\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -1 & -3 & -18 \\ 0 & 3 & 5 & 34 \end{array}\right)$$

$$R_1 \to R_1 + R_2, R_3 \to R_3 + 3R_2$$

$$\sim \left(\begin{array}{ccc:c} 1 & 0 & -2 & -9 \\ 0 & -1 & -3 & -18 \\ 0 & 0 & -4 & -20 \end{array}\right)$$

$$R_3 \to \frac{R_3}{-4}$$

$$\sim \left(\begin{array}{ccc:c} 1 & 0 & -2 & -9 \\ 0 & -1 & -3 & -18 \\ 0 & 0 & 1 & 5 \end{array}\right)$$

$$R_1 \to R_1 + 2R_3, R_2 \to R_2 + 3R_3$$

$$\sim \left(\begin{array}{ccc:c} 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & -3 \\ 0 & 0 & 1 & 5 \end{array}\right)$$

Soln (1,3,5)

Gauss-Seidel Iteration Metod:

Gauss–Seidel method, is also referred as the method of successive displacement. It is an iterative method used to solve a system of linear equations. It can be applied to any matrix with non-zero elements on the diagonals, and the matrix is strictly diagonally dominant. To start with, an initial solution is to be assumed. One of the equations is then used to obtain the revised value of a particular variable by substituting in it the present values of the remaining variables. The process is to be continued for all the variables, this completes one iteration. The iterative process is then repeated till the solution converges within prescribed accuracy.

Example:

10x+y+z=12,

$$x + 10y + z = 12,$$

$$x + y + 10z = 12$$

$$x = \frac{1}{10}[12 - y - z],$$

$$y = \frac{1}{10}[12 - x - z],$$

$$z = \frac{1}{10}[12 - x - y]$$

start with the trial soln (0,0,0)

Table-1

	I iteration	II iteration
x	1.2	0.9948
y	1.08	1.00332
z	0.972	1.000188

	III iteration	IV iteration
x	0.99965	0.99999≈1
y	1.00002	0.99999≈1
z	1.00003	1

Soln (1,1,1)

CONCLUSIONS:

Gauss elimination method and Gauss-Jordan method look like similar but the difference is, in Gauss elimination method the matrix is reduced to an upper triangular matrix and in the Gauss-Jordan method, the matrix is reduced to a diagonal matrix. For large systems, Gauss-Jordan method is preferred over Gauss elimination method. Gauss Seidel method is a iterative method. It is one of the simplest iterate method. In this method calculations are simple and so the programming is simple. The memory requirement is also less. Appropriate for small systems.

REFERENCES:

[1] Khushbu kumari and R K Poonia (2021) 'A Study of solving system of linear equation using different methods and its real life applications' Journal of University of Shanghai for Science and Technology, Vol 23, Issue 7,pp 723-733.

[2] Dr. Udai Bhan Trivedi, Mr. Santosh Kumar Sharma,Mr. Vishok Kumar Singh(2017)'Gauss Elimination, Gauss-Jordan Elimination and Gauss Seidel Iteration Method: Performance Comparison using MAT LAB' International Journal of Engineering Technology Science and Research, Vol 4, Issue 10, pp 753-763.

[3] Ghulam Omer, Abdul Manan(2021) 'The Different Solving Methods of Linear Systems' International Journal of Innovative Science and Research Technology,Vol6, Issue 2, pp.688-691.

[4] Shastry.S.S. 'Introductory methods of Numerical Analysis', 5th edition.

[5] Alok Kumar Maurya, Atendra Singh Yadav, Ashish Kumar, Kishan Kumar Kushwaha (2017) 'An Overview on Numerical Solution of System of Linear Equations through Direct Methods' International Journal of Innovative Research in Science, Engineering and Technology, Vol 6, Issue 5, pp 9476-9481.

[6] Dr. Yogeesh N(2016)' A Study of Solving Linear System of Equations by GAUSS-JORDAN Matrix Method-An Algorithmic Approach' Journal of Emerging Technologies and Innovative Research, Vol 3, Issue 5, pp.314-320