ORIGINAL RESEARCH PAPER

Mathematics

ON THE INFLUENCE OF RADIATING FLUID, HEAT GENER TION AND VISCOUS DISSIPATION ON MHD FREE CONVECTION FLOW ALONG A STRETCHING SHEET.

KEY WORDS: Radiating Fluid, Heat Generation, Viscous Dissipation, Convection Flow, Magneto - hydrodynamics

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ABSTRACT

Boundary layer flow and heat transfer over a linearly stretched surface has received considerable attention in the recent years. This is because of the various possible engineering and metallurgical application, such as hot rolling and wire drawing. The radiative and viscous dissipation effects on a steady two-dimensional magneto hydrodynamics free convection flow along stretching sheet with heat generation is analyzed. The non - linear partial differential equations governing the flow field under consideration have been transformed by a similarity transformation into a system of nonlinear ordinary differential equations and then solved numerically by shooting iteration with the aid of mathematical software MAPLE18. The resulting non dimensional velocity, temperature and concentration profiles are then presented graphically for different values of the parameters. Conclusively, it was drawn that radiating fluid can increase the rate of temperature, because with increase in the radiating fluid the temperature tends to increase

INTRODUCTION

The magneto-hydrodynamics (MHD) of an electrically conducting fluid is encountered in many problems in geophysics, astrophysics, engineering applications and other industrial areas. MHD free convection flow has a great significance for the applications in the fields of stellar and planetary magneto-spheres, aeronautics.

Areo, Olaleye and Gbadegesin (2011) studied the radiative effect on velocity, magnetic and temperature fields of a magneto hydrodynamics oscillatory flow past a limiting surface with variable suction. They examine the effect of heat transfer through radiation on velocity, magnetic and temperature fields in the case of two-dimensional hydro magnetic oscillatory flow of a viscous incompressible and electrically conducting fluid past a porous limiting surface, subjected to variable suction and moving impulsively with a constant velocity in the presence of transverse magnetic field using perturbation techniques.

Kumar *et al.* (2002) studied MHD flow and heat transfer on a continuously moving vertical plate. A theoretical solution of hydro magnetic convection over a continuously moving vertical surface with uniform suction is obtained. A flow of this type is representing a new class of boundary-layer flow at a surface of finite length.

The study of heat generation or absorption in a moving fluid is important in problems dealing with chemical reactions and these concerned with dissociating fluids. Possible heat generation effects may alter the temperature distribution; consequently, the particle deposition rate in nuclear reactors, electronic chips and semi conductors' wafers.

Vajravelu and Hadjincalaous (1993) studied the heat transfer characteristics over a stretching surface with viscous dissipation in the presence of internal heat generation or absorption. An analysis is carried out to study the heat transfer characteristics in the laminar boundary layer of a viscous fluid over a linearly stretching, continuous surface with variable wall temperature subject to suction or blowing. The study examines the effects of frictional heating (viscous dissipation) and internal generation of absorption. Two cases were studied, for cases one; the surface with prescribed surface temperature (PST-case) and case two; the surface with prescribed wall heat flux (PHF-case). The solution is obtained in terms of Kummer's functions.

Samad and Mohebujjaman (2009) investigated the case along a vertical stretching sheet in the presence of magnetic field and heat generation. The steady two-dimensional heat and mass transfer free convection flow of a viscous incompressible fluid near an isothermal linearly stretching sheet in the presence of uniform magnetic field with heat generation is considered. Nachtsheim-Swigert shooting iteration along with sixth order Runge-Kutta integration scheme is employed. At high operating temperature, the effect of radiation on MHD flow can be quite significant. Nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering areas.

Ghaly (2002) considered radiation effects on a steady flow. Free convection heat and mass transfer due to the simultaneous action of buoyancy, radiation and transverse magnetic field is investigated near an isothermal sheet. A parametric study is performed to illustrate the influence of radiation parameter, magnetic parameter, Prandtl number, Grashof number and Schmidt number.

Raptis and Massalas (1998) studied the MHD flow past a plate by the presence of radiation. An analysis of the unsteady MHD flow of a viscous and electrically conducting fluid past a plate by the presence of radiation were considered. The Rosseland approximation is used to described the radiative heat flux in the energy equation.

El-Aziz (2009) analyzed the unsteady flow. An analysis of the effect of radiation on the heat and fluid flow over a steady stretching surface were considered. The problem is solved numerically for some values of the unsteadiness parameter A, the radiation parameter R and Prandtl number Pr and it was observed with the increase in R the heat rate increased.

Shateyi *et al*. (2007) studied magneto-hydrodynamics flow past a vertical plate with radiative heat transfer. The problem of steady laminar MHD flow past a semi-infinite plate is studied. The study focuses primarily on the effects of thermal radiative heat transfer, magnetic fields strength and hall currents on the flow properties. The problems were solved numerically.

Mahmoud (2007) investigated variable viscosity effects on MHD flow in the presence of radiation. The work presents a study of the flow and heat transfer of an incompressible viscous electrically conducting fluid over a continuously moving vertical infinite plate with uniform suction and heat flux in the presence of radiation taking into account the effects of variable viscosity. It was observed that the velocity increase as the viscosity of air and magnetic parameter decreases.

Gebhart (1962) was the first who studied the problem taking into account the viscous dissipation. Viscous dissipation is considered in this study for vertical surfaces subjected to both isothermal and uniform-flux surface conditions using a perturbation method.

The effect of viscous dissipation in natural convection is appreciable when the induced kinetic energy become appreciable compared to the amount of heat transferred. This occurs when either the equivalent body force is large or when the convection region is extensive.

Capiello and Fabbri (2008) studied the effects of viscous dissipation on the heat transfer in a sinusoidal profile finned dissipater. The efficiency of finned heat dissipaters cooled by laminar flow is studied; the analysis is carried out by varying certain sizing parameter in correspondence with different viscous dissipation conditions. The velocity distribution in the fluid and the temperature distribution in the dissipater and in the fluid are determined by means of a finite element method.

Alam *et al.* (2007) considered the effect of viscous dissipation in natural convection over a sphere. A study was considered on the viscous dissipation effects on magneto-hydrodynamics natural convection flow over a sphere in the presence of heat generation. The problem is solved numerically using finite-difference method together with Keller-box scheme.

Pantokratoras (2005) studied the effect of viscous dissipation in a new way. The steady laminar boundary layer flow along a vertical stationary heated plate is studied taking into account the viscous dissipation of the fluid. The results are obtained with the numerical solution of the boundary layer equation. Both the upward and downward flow is considered for the isothermal and uniform flux case. It is observed that the interaction between the viscous heating and the buoyancy force has a strong influence on the results.

Ishrat Zahan and M. A Samad (2013) carried out an analysis to investigate the effects of radiation and chemical reaction on a steady two-dimensional magneto-hydrodynamic heat and mass transfer free convection flow of a viscous incompressible fluid along a stretching sheet with heat generation along with effect of viscous dissipation.

Mohammad Medhi Rashidi *et.al*. (2014) a homotopy analysis method was carried out to examine the free convective heat and mass transfer in a steady two-dimensional magneto-hydrodynamic fluid flow over a stretching vertical surface in a porous medium. The thermal radiation and non-uniform magnetic field are taken into consideration.

Tania S. Khaleque and M A Samad (2010) studied the effect of radiation, heat generation and viscous dissipation on magneto-hydrodynamics free convection flow along a stretching sheet. The radiation and viscous dissipation effects on a steady two-dimensional magneto-hydrodynamics free convection flow along a stretching sheet with heat generation is analyzed, using similarity transformation to transform the sets of differential equation and then solved numerically by applying the Nachtscheim-Swigert shooting iteration technique together with the sixth order Rungo-Kutta integration scheme.

Nalivela *et. al*. (2021) studied the chemical reaction impact on magneto-hydrodynamics natural convection flow through a porous medium past an exponentially stretching sheet in the presence of heat source/sink and viscous dissipation. By using the similarity transformation to convert the sets of differential equation then the problem is solved using the Keller-box method

In this paper, the radiative and viscous dissipation effects on a steady two-dimensional magneto hydrodynamics free convection flow along stretching sheet with heat generation is studied.

Mathematical Formulation

A steady two-dimensional MHD free convection laminar boundary layer flow of a viscous incompressible and electrically conducting fluid along a vertical stretching sheet with heat generating under the influence of a radiative fluid and viscous dissipation is considered in the figure below.

Introducing Cartesian coordinate system, the x-axis is taken along the stretching sheet in the vertically upward direction and the y-axis is taken normal to the sheet. Two equal and opposite forces are introduced along the x-axis, so that the sheet is stretched keeping the origin fixed. The plate is maintained at a constant temperature T_w and the concentration is maintained at constant value C_w. The ambient temperature of the flow is ∞T and the concentration of the uniform flow is ∞C. The fluid is considered to be gray, absorbing emitting radiation but non-scattering medium and the Rosseland approximation is used to describe the radioactive heat flux in the energy equation. The radioactive heat flux in the X-direction is considered in comparison to the y-direction. The concentration is assumed to be non-reactive.

A strong magnetic field is applied in the y-direction. Here, we can neglect the effect of the induced magnetic field in comparison to the applied magnetic field. The electric current flowing in the fluid gives rise to an induced magnetic if the fluid were an electrical insulator, but here we have taken the fluid to be electrically conducting. Hence, only the magnetic field gives rise to magnetic force.

Mathematical Analysis

The governing boundary layer equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$U\frac{\partial u}{\partial x} + V\frac{\partial v}{\partial y} \quad \nu\frac{\partial^2 u}{\partial y^2} + \beta(T - T_\infty) - \frac{\sigma B_0^2 u}{\rho_\infty}) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho_\infty c_p}\frac{\partial^2 T}{\partial y^2} + \frac{Q_0(T - T_\infty)}{\rho c_p} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} + \frac{v}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 - \tag{3}$$

$$4(T - T_\infty)\int_0^\infty K_{\lambda w}\left(\frac{\partial L_b \lambda}{\partial T_w}\right)^2 dk$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y} \tag{4}$$

Subject to the corresponding boundary conditions

$$u = D_x, \quad v = V_w \quad T = T_w \quad C = C_w \quad at \; y = 0 \; and \; u = o$$

$$T = T_\infty \quad C = C_\infty \quad as \quad y \to 0 \qquad (5)$$

The governing equations (1) – (4) can be expressed in a simpler form by introducing the following similarity transformations:

$$u = \frac{\partial \psi}{\partial y} \; \text{and} \; v = -\frac{\partial \psi}{\partial x} \qquad (6)$$

Introducing the following dimensionless variables

$$\eta = \frac{y}{\delta} = y\sqrt{\frac{D}{v}}, \quad U = \frac{\partial \psi}{\partial y} = D_x f'(\eta), \quad \psi(x,y) = \sqrt{Dv}\, f(v)$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \varphi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \; V = \frac{\partial \psi}{\partial x} = -\sqrt{Dv}\, f(\eta) \qquad (7)$$

Where ψ is the stream function, θ and φ are the non dimensional, temperature and concentration parameters. Substituting (6) and (7) into (2) – (4) produces the following differential equations

$$f''' + \lambda\theta - Mf' - (f')^2 + ff'' = 0 \qquad (8)$$

$$f''' + ff'' - (f')^2 - Mf' + \lambda\theta = 0 \qquad (9)$$

$$\theta'' + \left(\frac{3N}{3N+4}\right)\Pr f\theta' + \left(\frac{3N}{3N+4}\right)\Pr Q\theta +$$

$$\left(\frac{3N}{3N+4}\right)\Pr Ec(f'')^2 \quad \left(\frac{3N}{3N+4}\right)\Pr C\theta = 0$$

$$\varphi'' + Scf\varphi' = 0 \qquad (11)$$

And the boundary condition (6) becomes

$$f(\eta) = \pm F_w \quad f'(\eta) = 1$$

$$\theta(\eta) = 1, \quad g(\eta) = 0, \quad \varphi(\eta) = 1 \quad at\, \eta = 0$$

$$F'(\eta) \to 0, \quad \theta(\eta) \to 0,$$

$$\varphi(\eta) \to 0, \quad g(\eta) \to 0 \; as \; \eta \to \infty \qquad (12)$$

Where $Sc = \frac{v}{D_m}, \quad \lambda = \frac{g\beta(T_w - T_\infty)}{D^2 x} \quad and \quad M = \frac{\sigma B_0^2}{\rho D}$

$\lambda = \frac{gB(T_w - T_\infty)}{D^2 x}$ is the buoyancy parameter.

$M = \frac{\sigma B_0^2}{\rho D}$ is the magnetic field parameter

$\Pr = \frac{\mu_{cp}}{k}$ is the Prandtl number.

$N = \frac{kk_1}{4\sigma_1 T_\infty^3}$ is the radiation parameter

$Q = \frac{Q_0}{\rho c_p D}$ is the heat source parameter.

$Sc = \frac{v}{Dm}$ is the Schmidt number.

$Ec = \frac{D^2 x^2}{c_p(T_w - T_\infty)}$ is the Eckert number

$C = \frac{4}{D}\left(\int_0^\infty K_{\lambda w}\left(\frac{\partial I_b \lambda}{\partial T_w}\right)\right)dk$ is the Radiative fluid.

RESULTS AND DISCUSSION

The problem of the MHD flow of a radiating electrically conducting fluid on free convection flow along a stretching sheet was formulated and solved numerically by shooting iteration with the aid of mathematical software MAPLE 18 the numerically computed and graphical results are also presented.

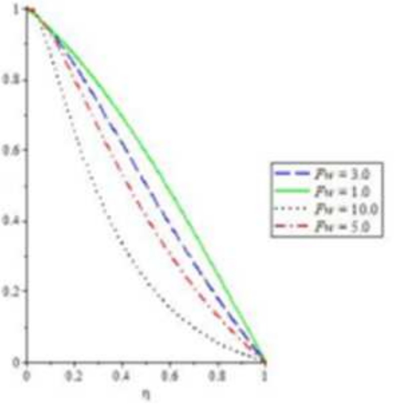

Figure 1: Section parameter (Fw) effect on Temperature Profile

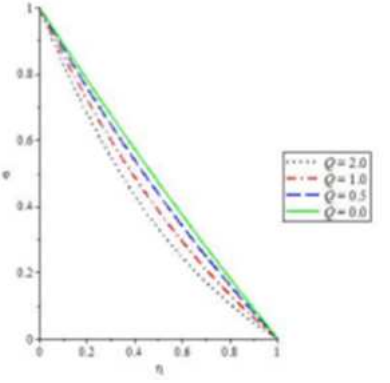

Figure 2: Heat Source Parameter (Q) effect on Concentration Profile

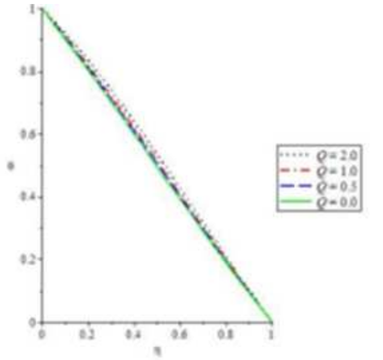

Figure 3: Heat Source Parameter (Q) effect on Temperature Profile

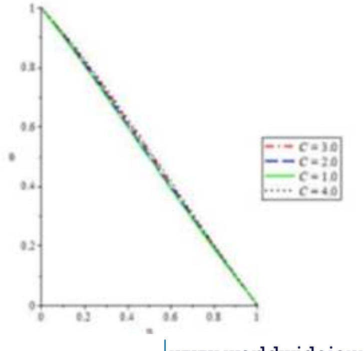

Figure 4: Radiating Fluid Parameter (C) effect on Temperature Profile

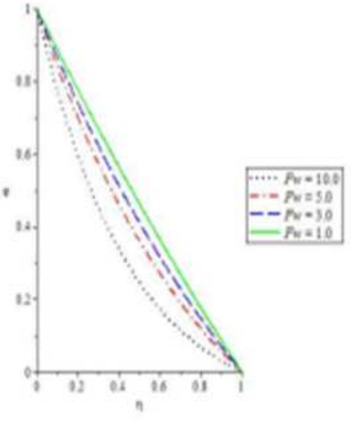

Figure 5: Section Parameter (Fw) effect on Concentration Profile

DISCUSSION OF RESULTS

In order to discuss the problem under consideration, the results of numerical calculations are presented in the form of non-dimensional velocity, temperature and concentration profiles.

Figure 1: It shows that the temperature profile decreases with increase in the suction parameter Fw. The temperature reduces with the suction parameter value increase due to increase in the hot fluid which is taken away from the boundary layer. The dimensionless velocity, temperature and concentration boundary layers have plotted respectively showing the effect of suction parameter F_w For=1.0 It was noticed that the velocity and temperature profiles increase near the surface and then start to decrease. However, for other values of the suction parameter, the velocity and the temperature profiles start to decrease monotonically from the very beginning. Thus sucking the decelerated fluid particles reduces the growth of the fluid boundary layer as well as thermal and velocity, temperature and concentration boundary layers. In Figure 2 the effect of heat source parameter Q on the velocity, temperature concentration boundary layers are displayed. It was observed that the velocity and temperature profile increases as heat source parameter Q increases. On the other hand, the concentration profiles decrease with the increase of heat source parameter Q. Figure 3: It shows that the temperature profile increase slightly with increase in the heat source parameter

Q. Heat source physically implies generation of heat from the surface this is due to the constant temperature greater than ambient temperature which increases the temperature in the flow field. Therefore, as the heat source parameter increased, the temperature increases gradually from the surface.

Figure 4: It shows that the temperature profile increases slightly with increase in the radiating fluid parameter C. It is observed that as the values of the radiating fluid parameter increases, the temperature increases.

Figure 5: It can be observed that the concentration profile decreases with increase in the suction parameter Fw. For Fw =1.0 increase and then start to decrease as the values of suction parameter increases.

CONCLUSION

This study examined the effect of radiating fluid and viscous dissipation on a steady two-dimensional magneto hydrodynamics free convection flow along stretching sheet with heat generation. The governing equations were reduced to dimensionless equations by various values of dimensionless parameters the problem which is then solved analytically. Comparison with previously published work (Tania et al 2010), it agrees with the results obtained which presented graphically.

From the results, conclusion is drawn that radiating fluid can increase the rate of temperature, because with increase in the radiating fluid the temperature tends to increase.

Large value of heat source parameter Q has significant effect on the temperature distributions whereas it causes reduction in the concentration distribution in the boundary layer.

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