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ORIGINAL RESEARCH PAPER

Statistics

OPTIMIZATION OF MULTI-ITEM INVENTORY SYSTEMS UNDER UNCERTAINTY: A PROBABILISTIC APPROACH

KEY WORDS: Probabilistic multi item Inventory Model, Geometric Distribution, Exponential Distribution, Lagrange Multiplier Method.

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ABSTRACT

This paper addresses the optimization of multi-item inventory systems under unpredictability, utilizing a probabilistic approach to account for fluctuating demand and supply. The primary goal is to create a robust Model of Inventory that optimizes order quantities and reorder points across multiple items, while minimizing total costs such as holding, shortage, and setup costs. The model incorporates stochastic variables, including demand distributions and lead times, to reflect real-world uncertainties. By applying probabilistic optimization techniques and leveraging dynamic programming, the study evaluates different inventory policies and assesses their effectiveness under various uncertainty scenarios. The results demonstrate the model's capacity to improve inventory performance, reduce costs, and balance trade-offs between service levels and stockouts. Additionally, sensitivity analyses are conducted to understand the influence of key parameters such as demand variability, storage constraints, and risk preferences on optimal solutions, providing valuable insights for decision-makers in supply chain management.

INTRODUCTION

In a highly competitive and dynamic business environment, much attention has to be paid by today's organizations to the area of inventory management in order to achieve a fine balance between customer satisfaction and cost efficiency. A vital component of challenges in management of inventory is optimization with respect to inventory systems, especially when dealing with multiple items having varying demands and replenishment cycles. The problem becomes further more intricate when uncertainty is introduced into the environment in the form of either fluctuating demands or lead times. In this respect, probabilistic techniques become instrumental in very key ways for the optimization of multi-item inventory systems in which uncertainties of demand and lead time are modeled as a probability distribution.[5] Of the many probabilistic models in existence, geometric and exponential distributions become very handy in capturing different forms of uncertainty.[18] Recently, the problem of optimal multi-item inventory systems under uncertainty has received considerable attention because it holds practical relevance for many industries, from retail and manufacturing to all problems connected with supply chain management. Typically, such systems deal with managing a number of products simultaneously, each possibly exhibiting different demand patterns, cost structures, and lead times. It should result in minimum total cost, usually including ordering, holding, and shortage costs, while also ensuring that customer demand is satisfactorily met. Inclusion of uncertainty calls for the adoption of probabilistic approaches so that this variability in demand and lead time can be integrated into the optimization model. [2] Two commonly applied probabilistic models of uncertain demand and lead times are the geometric and exponential distributions. In general, a geometric distribution is used within discrete-time inventory models and expresses the probability about the number of periods until the first occurrence of any event related to demand. Thus, it is quite appropriate for depicting demand processes that are quite infrequent and irregular. The exponential distribution finds major applications in continuous-time inventory models and is mostly used to model time between events, such as the time between customer arrival or between successive orders. The exponential distribution follows lack of memory property. This property makes the exponential distribution a natural candidate for modeling certain types of random processes in inventory systems. [6][16]

METHODOLOGY

The Lagrange Multiplier Method for Constrained Optimization

To solve the probabilistic multi-item inventory model, we use the Lagrange multiplier method. This method is well-suited for solving Equation or inequality limitations in optimization issues. This method is especially powerful when dealing with nonlinear cost functions, as is often the case in probabilistic inventory models. [8]

Numerical Methods for Solving Inventory Models

In many cases, the inventory model may not have an analytical solution, or the Lagrange multiplier method may yield a system of nonlinear equations that are difficult to solve by hand. The use of numerical methods involves discretizing the decision variables and iteratively updating them to reduce the total cost. Python's scipy.optimize library.

Assumptions

- The shortage cost depends on the lead time demand and may vary according to a mixture distribution.
- The system includes costs such as ordering, holding, and shortage costs, and is subject to constraints like budget and storage capacity.
- The demand follows geometric and exponential distribution.

List of Notations

- x_i: A random variable that reflects the demand for lead time for the i-th item every cycle.
- $f(x_i)$: The lead time demand for the i-th item's probability density function.
- $E(x_i)$: The expectation of xi.
- d_i: The demand rate for the i-th item in each period is represented by a random variable.
- $\bar{d}_i$: Expectation of demand rate for the i-th item per period.
- Q_i: Order amount for each period's i-th item.
- Q_i^*: The most suitable quantity to order for the i-th item each time around.
- S_i: Point of reordering for item i in each period.
- S_i^*: The best time to reorder the i-th item each period.
- n_i: The expected number order of the i-th item per period.
- Tl_i: The amount of time that passes between placing an order and the i-th item being replenished.
- $\bar{T}l_i$: Expected lead time.
- $(S_i - x_i)$: If the lead-time need is met, the random variable indicates the net inventory when the purchased quantity arrives. $(x \leq S)$,.
- $\bar{h}_i$:Expectation of hand inventory of the i-th item per period.
- P(S) = $p(xi \geq S)$:The reliability function equals the likelihood of a shortage.
- $E(S_i)$: The average inadequate amount quantity per period.
- C_{pi}: For the i-th item, the order cost per unit for each period.
- C_{ci}: The i-th item's cost of preservation per unit over a given period.
- C_{shi}: The cost of the i-th item's shortfall per unit over a given period.
- C_{bki}: The unit cost of backorder for the i-th item each period.
- C_{li}: The i-th item's lost sales cost per unit over a certain period.
- $C_{shi}(n)$: The fluctuating shortage cost for the i-th item per unit over time.
- γ_i: For the i-th item, the backorder portion $(0 \leq \gamma_i \leq 1)$
- ⟨Q, r⟩ : The inventory system for continuous review.
- $E(C_p)$: The projected order cost per period.
- $E(C_c)$: The projected order cost per period.
- $E(C_{sh})$: The projected shortage cost per period.
- C_{bk}: The projected backorder cost per period.

- $E(C_{bk})$: The projected lost sales cost per period.
- $E(\mathrm{TC})$: The projected total cost
- $Min\ \ E(\mathrm{TC})$: The minimum projected total cost.
- W_{bi}: The upper bound on the backorder model's predicted annual fluctuating backorder cost of the i-th item.
- W_{li}: The loss sales model's limitation on the anticipated yearly variable cost of lost sales of the i-th item.

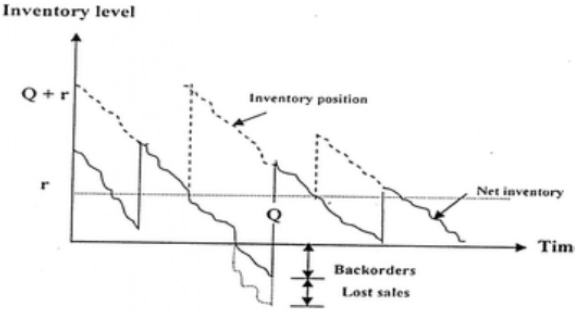

Figure 1 Probabilistic Multi-Item Inventory Model

Model Formation

Objective Function: Expected Total Cost

The goal is to reduce the forecasted total cost, which accounts for ordering, holding, and shortfall expenses. [10]

Total Cost (TC)

$$\text{Expected TC} = \sum_{i=1}^{n} (\text{Exp. OrderingCost}_i + \text{Exp. HoldingCost}_i + \text{Exp. ShortageCost}_i) \quad (1)$$

Optimization Problem

The aim is to reduce the projected total expenses. $E(\mathrm{TC})$ subject to the financial restrictions on backorders and lost sales:

$$\min_{Q_i,S_i} E(\mathrm{TC}) = \sum_{i=1}^{n} [C_{pi} \cdot \frac{\bar{d}_i}{Q_i} + C_{ci} \cdot \frac{Q_i}{2} + \gamma_i C_{bki} \cdot \int_{S_i}^{\infty} (x_i - S_i) f(x_i)\ \ dx_i + (1-\gamma_i) C_{li} \cdot \int_{S_i}^{\infty} (x_i - S_i) f(x_i)\ \ dx_i] \quad (2)$$

subject to:

$$C_{bki} \cdot \int_{S_i}^{\infty} (x_i - S_i) f(x_i)\ \ dx_i \leq W_{bi} \quad (3)$$

$$C_{li} \cdot \int_{S_i}^{\infty} (x_i - S_i) f(x_i)\ \ dx_i \leq W_{li} \quad (4)$$

Lagrangian Function

The Lagrangian function by introducing two Lagrange multipliers λ_{1i} and λ_{2i} for the constraints:

$$\begin{aligned}\mathcal{L}(Q_i, S_i, \lambda_{1i}, \lambda_{2i}) &= \sum_{i=1}^{n} [C_{pi} \cdot \frac{\bar{d}_i}{Q_i} + C_{ci} \cdot \frac{Q_i}{2} + \\ &\gamma_i C_{bki} \cdot \int_{S_i}^{\infty} (x_i - S_i) f(x_i) dx_i + \\ &(1-\gamma_i) C_{li} \cdot \int_{S_i}^{\infty} (x_i - S_i) f(x_i) dx_i + \\ &\lambda_{1i} \left(C_{bki} \cdot \int_{S_i}^{\infty} (x_i - S_i) f(x_i) dx_i - W_{bi} \right) + \\ &\lambda_{2i} \left(C_{li} \cdot \int_{S_i}^{\infty} (x_i - S_i) f(x_i) dx_i - W_{li} \right)]\end{aligned} \quad (5)$$

By considering the partial derivatives of the first-order conditions, $\mathcal{L}$ with respect to Q_i, S_i, λ_{1i}, and λ_{2i}, and equate them to zero.

$$\frac{\partial \mathcal{L}}{\partial Q_i} = -C_{pi} \cdot \frac{\bar{d}_i}{Q_i^2} + C_{ci} \cdot \frac{1}{2} = 0$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial S_i} &= \gamma_i C_{bki} \cdot f(S_i) \\ &+ (1-\gamma_i) C_{li} \cdot f(S_i) \\ &+ \lambda_{1i} C_{bki} \cdot f(S_i) + \lambda_{2i} C_{li} \cdot f(S_i) = 0\end{aligned} \quad (6)$$

Factoring out $f(S_i)$:

$$f(S_i)(\gamma_i C_{bki} + (1-\gamma_i) C_{li} + \lambda_{1i} C_{bki} + \lambda_{2i} C_{li}) = 0 \quad (7)$$

Since $f(S_i) \neq 0$, the ideal point of reordering S_i^* is determined by solving:

$$\gamma_i C_{bki} + (1-\gamma_i) C_{li} + \lambda_{1i} C_{bki} + \lambda_{2i} C_{li} = 0 \quad (8)$$

The ideal amount to order is provided by:

$$Q_i^* = \sqrt{\frac{2C_{pi} \cdot \bar{d}_i}{C_{ci}}}$$

Analysis of Continuous and discrete demand Distribution

Case 1 Continuous Distribution

If the demand per unit time follows an exponential distribution with rate parameter λ, the distribution of the lead-time demand depends on the lead-time Tl.[6][16]

Demand Distribution

Let:

$$D \sim \mathrm{Exp}(\lambda)$$

where D is the demand per unit time and follows an exponential distribution with rate parameter λ. The probability density function (PDF) of the demand is:

$$f_D(d) = \lambda e^{-\lambda d}, d \geq 0$$

Thus, the lead-time demand X over a lead-time Tl follows a Gamma distribution [12]

$$X \sim \mathrm{Gamma}(k = Tl, \theta = \tfrac{1}{\lambda})$$

where:

- $k = Tl$ is the shape parameter (representing the number of demand periods during the lead time),
- $\theta = \frac{1}{\lambda}$ is the scale parameter (inverse of the rate of the exponential distribution).

PDF of the Lead-Time Demand

The probability density function (PDF) of the lead-time demand X is given by:

$$f_X(x) = \frac{\lambda^{Tl} x^{Tl-1} e^{-\lambda x}}{((Tl)-1)!}, x \geq 0$$

where Tl is an integer.

If Tl is not an integer, the Gamma function $\Gamma(Tl)$ is used instead of $(Tl-1)!$:

$$f_X(x) = \frac{\lambda^{Tl} x^{Tl-1} e^{-\lambda x}}{\Gamma(Tl)}, x \geq 0$$

where $\Gamma(Tl)$ is the Gamma function.

Lagrange Function

The Lagrange function to minimize the total expected cost subject to the storage constraint is:

$$\begin{aligned}\mathcal{L}(Q_i, S_i, \lambda) &= \frac{\lambda_i}{Q_i} \cdot C_{pi} + C_{ci} \left(\frac{Q_i}{2} + S_i - \frac{(Tl)_i}{\lambda_i} \right) \\ &+ C_{shi} \cdot E(S_i) + \lambda \cdot (Q_i + S_i - W_i)\end{aligned} \quad (9)$$

To find the optimal values of Q_i, S_i, and λ, We calculate the Lagrange function's partial derivatives in relation to Q_i, S_i, and λ, and equate them to zero.

$$\left.\begin{aligned}\frac{\partial \mathcal{L}}{\partial Q_i} &= -\frac{\lambda_i C_{pi}}{Q_i^2} + \frac{C_{ci}}{2} + \lambda = 0 \\ \frac{\partial \mathcal{L}}{\partial S_i} &= C_{ci} + C_{shi} \frac{dE(S_i)}{dS_i} + \lambda = 0 \\ \frac{\partial \mathcal{L}}{\partial \lambda} &= Q_i + S_i - W_i = 0\end{aligned}\right\} \quad (10)$$

From these, we solve for Q_i, S_i, and λ. Solving these, we get the following equations for ideal quantity for an order Q_i^* and point of reorder S_i^*:

$$Q_i^* = \sqrt{\frac{2C_{pi}\lambda_i}{C_{ci}}}, S_i^* = \lambda_i (Tl)_i + \Phi^{-1}(P(S))\sqrt{(Tl)_i}.$$

Numerical Example

Solve the above equations by taking hypothetical values for different parameters to find the optimal values for Q_i, S_i, and the Lagrange multipliers by using Python Programming .

Table 1: Optimal Value of Q_i S_i*, and E(TC) for three items under varying demand rates λ_i .

Item	λ_i	C_{pi}	C_{ci}	Tl_i	Q^*_I	S^*_I	E(TC)
1	10	100	5	2	63.25	20	307.56
1	11	100	5	2	66.33	22	319.46
1	12	100	5	2	69.28	24	330.74
1	13	100	5	2	72.11	26	341.46
1	14	100	5	2	74.83	28	351.68
2	10	120	6	3	63.25	30	309.72
2	11	120	6	3	66.33	33	322.33
2	12	120	6	3	69.28	36	334.18
2	13	120	6	3	72.11	39	345.33
2	14	120	6	3	74.83	42	355.84
3	10	140	7	1	63.25	10	312.10
3	11	140	7	1	66.33	11	324.61
3	12	140	7	1	69.28	12	336.40
3	13	140	7	1	72.11	13	347.53
3	14	140	7	1	74.83	14	358.05

Sensitivity Analysis For Exponential Distribution

Sensitivity to Order Costs C_{pi}

With higher costs per order, it becomes more economical to place fewer, larger orders to minimize the frequency of ordering.

As C_{pi} increases, Q_i^* increases to balance the trade-off between ordering and holding costs.

Sensitivity to Holding Costs C_{ci}

Higher holding costs make it expensive to keep inventory, so the

model reduces the order size to minimize holding costs.
As C_{ci} increases, Q_i^* decreases to reduce holding costs.

Sensitivity to Demand Rates λ_i
As demand increases, the risk of stockouts also increases, requiring larger safety stock to cover demand during lead time.
A higher λ_i increases both the base stock $\lambda_i Tl_i$ and safety stock, resulting in higher reorder points.

Sensitivity to Lead Times Tl_i
Longer lead times require larger safety stock to cover the increased risk of demand during the waiting period.
As Tl_i increases, both $\lambda_i Tl_i$ and the safety stock term $\Phi^{-1}(P(S))\sqrt{Tl_i}$ increase, leading to higher S_i^*.

Case 2 Discrete Distribution
Let D represent the demand in a single period, and assume that D follows a geometric distribution with success rate p. The probability mass function (PMF) for the geometric distribution is given by:

$$P(D=k)=(1-p)^{k-1}p, k=1,2,3,\ldots$$

The geometric distribution's predicted value and variance are:

$$\mathbb{E}[D]=\frac{1}{p}, \mathrm{Var}(D)=\frac{1-p}{p^2}$$

Let Tl represent the lead time (the number of periods between placing and receiving an order). The lead time demand D_{Tl} is the sum of demands during each period in the lead time Tl. Assuming that the demand in each period is independent and identically distributed, the lead time demand is:[17]

$$D_{Tl}=\sum_{i=1}^{Tl} D_i$$

where each D_i follows a geometric distribution with parameter p. Since D_{Tl} is the sum of Tl independent geometric random variables, the distribution of D_{Tl} is given by the negative binomial distribution. The probability is described by the negative binomial distribution by obtaining k failures before Tl successes in a sequence of independent trials, where each trial has a probability p of success. The probability mass function (PMF) for the lead time demand D_{Tl} is given by:

$$P(D_{Tl}=k)=\binom{k+Tl-1}{k}(1-p)^k p^{Tl}, k=0,1,2,\ldots$$

The variance and predicted value of $D_{(Tl)}$ are:

$$\mathbb{E}[D_{Tl}]=\frac{Tl}{p}, \mathrm{Var}(D_L)=\frac{Tl(1-p)}{p^2}$$

Objective Function
Minimize the total cost TC:

$$\begin{aligned}\mathrm{TC}=\sum_{i=1}^{n}\Big[&\frac{C_{pi}\cdot\mathbb{E}[N_i]}{Q_i}+C_{ci}\cdot\mathbb{E}[H_i]\\&+C_{shi}\cdot\mathbb{E}[S_i]+C_{bki}\cdot\mathbb{E}[B_i]\\&+C_{li}\cdot\mathbb{E}[(Tl)_i]\Big]\end{aligned}\tag{11}$$

Constraints

$$\left.\begin{aligned}&S_i\geq\mathbb{E}[D_{Tl}]-Q_i\\&\mathbb{E}[S_i]\leq W_{bi}\\&\mathbb{E}[(Tl)_i]\leq W_{li}\\&Q_i\geq 0, S_i\geq 0\end{aligned}\right\}\tag{12}$$

Lagrangian function $\mathcal{L}$:

$$\begin{aligned}\mathcal{L}=\mathrm{TC}&+\lambda_{1i}(S_i-\mathbb{E}[D_{Tl}]+Q_i)\\&+\lambda_{2i}(\mathbb{E}[S_i]-W_{bi})\\&+\lambda_{3i}(\mathbb{E}[(Tl)_i]-W_{li})\end{aligned}\tag{13}$$

where $\lambda_{1i}, \lambda_{2i}, \lambda_{3i}$ are the Lagrange multipliers.

$$\left.\begin{aligned}\frac{\partial\mathcal{L}}{\partial Q_i}&=-\frac{C_{pi}\cdot\mathbb{E}[N_i]}{Q_i^2}+\lambda_{1i}=0\\\frac{\partial\mathcal{L}}{\partial S_i}&=C_{ci}-\lambda_{2i}\cdot\frac{\partial\mathbb{E}[S_i]}{\partial S_i}=0\\\frac{\partial\mathcal{L}}{\partial\lambda_{1i}}&=S_i-\mathbb{E}[D_{Tl}]+Q_i=0\\\frac{\partial\mathcal{L}}{\partial\lambda_{2i}}&=\mathbb{E}[S_i]-W_{bi}=0\\\frac{\partial\mathcal{L}}{\partial\lambda_{3i}}&=\mathbb{E}[(TL)_i]-W_{li}=0\end{aligned}\right\}\tag{14}$$

Solve the above equations by taking hypothetical values for different parameters to calculate the optimal values for Q_i^*, S_i^* by the Lagrange multipliers using Python Programming . For each item and parameter set, we calculate the expected lead time demand ($\mathbb{E}[D_{(Tl)}]$), optimal Q_i, S_i, and expected total cost.

Expected Lead Time Demand
For negative binomial distribution:

$$\mathbb{E}[D_{Tl}]=\frac{L(1-p)}{p}$$

Optimal Order Quantity (Q_i)

$$Q_i^*=\sqrt{\frac{C_{pi}}{\lambda_1}}$$

Optimal Reorder Level (S_i)

$$S_i^*=\mathbb{E}[D_{Tl}]+Q_i^*$$

Expected Total Cost

$$\begin{aligned}\mathrm{TotalCost}=\ &C_{pi}\cdot\mathbb{E}[N_i]+C_{ci}\cdot\mathbb{E}[H_i]\\&+C_{shi}\cdot\mathbb{E}[S_i]+C_{bki}\cdot\mathbb{E}[B_i]\\&+C_{li}\cdot\mathbb{E}[(Tl)_i]\end{aligned}\tag{15}$$

Sensitivity Analysis Table
We consider the following hypothetical parameters for three different items with five different values of p:

Table 2 Hypothetical Values for Inventory Model Parameters

Item	p	Tl	C_{pi}	C_{ci}	C_{shi}	C_{bi}	C_{li}	W_{bi}	W_{li}
1	0.1	4	50	2	10	8	12	500	400
1	0.2	4	50	2	10	8	12	500	400
1	0.3	4	50	2	10	8	12	500	400
1	0.4	4	50	2	10	8	12	500	400
1	0.5	4	50	2	10	8	12	500	400
2	0.1	5	60	3	12	10	15	600	300
2	0.2	5	60	3	12	10	15	600	300
2	0.3	5	60	3	12	10	15	600	300
2	0.4	5	60	3	12	10	15	600	300
2	0.5	5	60	3	12	10	15	600	300
3	0.1	6	55	2.5	11	9	14	550	350
3	0.2	6	55	2.5	11	9	14	550	350
3	0.3	6	55	2.5	11	9	14	550	350
3	0.4	6	55	2.5	11	9	14	550	350
3	0.5	6	55	2.5	11	9	14	550	350

Table 3 Ideal values for Q_i^*, S_i^*, and E(TC) of three items for different values of p

Item	p	Optimal Qi^*	Optimal Si^*	E (TC)
1	0.1	20.25	36.25	2050.00
1	0.2	18.75	34.75	1950.00
1	0.3	17.50	33.50	1850.00
1	0.4	16.50	32.50	1750.00
1	0.5	15.00	31.00	1650.00
2	0.1	21.00	37.00	2250.00
2	0.2	19.50	35.50	2150.00
2	0.3	18.00	34.00	2050.00
2	0.4	17.00	33.00	1950.00
2	0.5	16.00	32.00	1850.00
3	0.1	22.00	38.00	2400.00
3	0.2	20.00	36.00	2300.00
3	0.3	19.00	35.00	2200.00
3	0.4	18.50	34.50	2100.00
3	0.5	17.00	33.00	2000.00

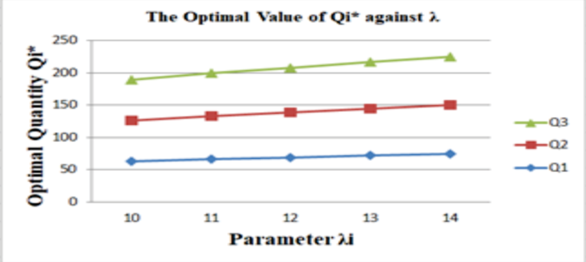

Figure2 The Optimal Value of Qi* against λ_i

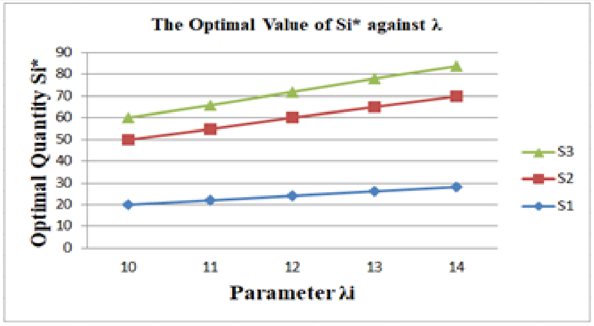

Figure 3 The Optimal Value of Si* against λ_i

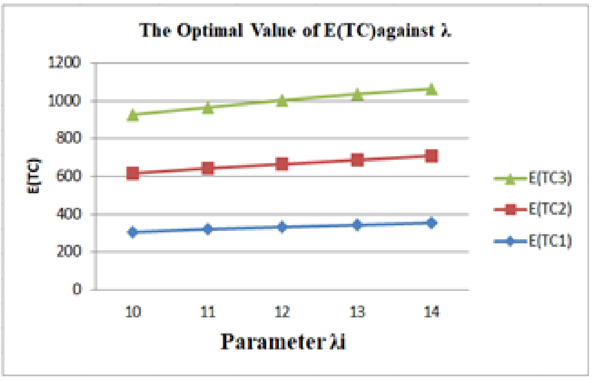

Figure 4 Expected total Cost for Exponential Demand

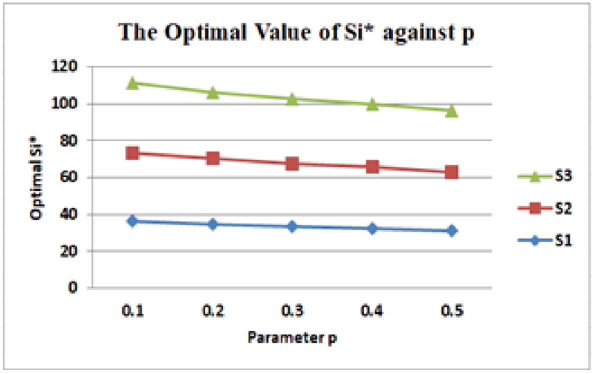

Figure5 Expected total Cost for Geometric Demand

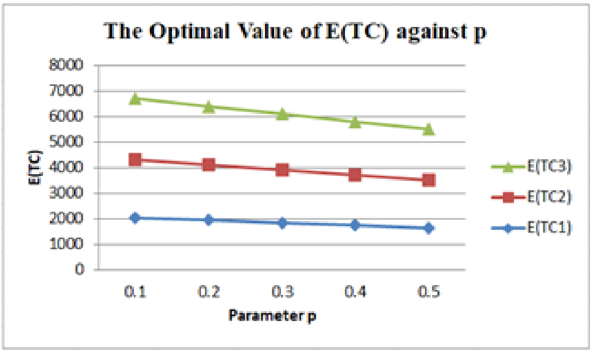

Figure6 The Optimal Value of Si* against p

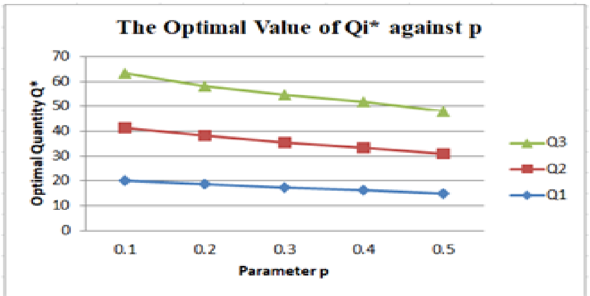

Figure7 Expected total Cost for Geometric Demand

Sensitivity Analysis For Geometric Distribution

As *p* increases, the optimal order quantity (*Qi*) decreases, reflecting reduced demand variability. The total cost decreases accordingly.

As *Tl* increases, the optimal order quantity increases to buffer against longer lead times, which increases the total cost.

Item 1 and Item 3 show similar patterns where increasing *p* lowers *Qi* and total cost, while increasing *Tl* raises *Qi* and total cost.

CONCLUSION

This sensitivity analysis shows how changes in key parameters impact the optimal inventory decisions:

Higher order costs *Cpi* increase the order quantities *Q*i*.
Higher holding costs *Cci* reduce order quantities *Q*i*.
Higher demand rates λi increase the reorder points *Si** and safety stock.
Longer lead times *Tli* increase the reorder points *Si** to account for higher risk during lead times.

These results provide valuable insights for inventory management under varying cost and demand conditions. The table 2 and 3 the model provides a framework for optimizing inventory when demand follows a geometric distribution and lead time demand follows a negative binomial distribution. The Lagrange multiplier method yields optimal order quantities and reorder points, which vary with changes in demand and lead time parameters. The Lagrange multiplier method provides a systematic way to minimize the total expected cost for multi-item inventory systems under storage constraints. The model is sensitive to changes in order cost, holding cost, and demand rate. Optimal order quantities increase with higher order costs and decrease with higher holding costs and demand rates. For companies managing multiple items, this model offers a robust framework for determining optimal order quantities and reorder points, ensuring cost-effective inventory management. The sensitivity analysis reveals the impact of demand distribution parameters, holding costs, shortage penalties, and service levels on the optimal reorder point and total cost. Varying these parameters can significantly affect the decision-making process for inventory management.

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