



ORIGINAL RESEARCH PAPER

Mathematics

EIGEN VALUE DECOMPOSITION AND ITS APPLICATIONS IN SCIENCE AND ENGINEERING

KEY WORDS: Eigenvalue decomposition; Principal component analysis; Signal processing; Structural engineering; Stability analysis

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ABSTRACT	Eigenvalue decomposition (EVD) is a key method for breaking down matrices in linear algebra. It is widely used in engineering, data science, and applied mathematics. This study looks at the theoretical basics and various applications of EVD. It highlights the benefits of its computation and the insights it can provide.
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INTRODUCTION

Eigenvalue decomposition (EVD) is a fundamental tool in linear algebra. It allows us to analyze the basic properties of matrices by breaking them down into eigenvalues and eigenvectors. This process simplifies complex calculations and gives important insights into the structure and behavior of systems represented by matrices [1].

EVD is used in engineering, computer science, and natural sciences, where linear transformations model systems. In fields like signal processing, EVD helps with data compression and noise reduction. In control theory, it is essential for stability analysis. In machine learning, it supports methods such as principal component analysis (PCA) [2,3].

This paper aims to explore the theory and practical uses of EVD and provide a view from multiple disciplines.

MATERIALS AND METHODS

This research adopted a two-fold methodology:

- Mathematical Framework:** Formal definitions, properties, and theorems related to EVD were revisited to establish a rigorous foundation.
- Systematic Literature Review:** Peer-reviewed articles indexed in Scopus, IEEE, Springer, and Elsevier databases were reviewed to map applications of EVD. Case studies from engineering, data science, and applied mathematics were analyzed.

The EVD of a square matrix A is defined as:

$$A = V \Lambda V^{-1}$$

where V is the eigenvector matrix and Λ is the diagonal matrix of eigenvalues.

RESULTS AND DISCUSSION

Signal Processing And Data Compression

Eigenvalue decomposition allows for Principal Component Analysis (PCA), which reduces the number of dimensions in high-dimensional datasets. PCA is commonly used in image compression and face recognition systems [4].

Control Theory And Stability Analysis

In control engineering, the eigenvalues of a system matrix determine system stability. Negative real parts of eigenvalues indicate stability. Positive values suggest instability [5].

Structural Engineering And Vibration Analysis

EVD helps with modal analysis of structures. It identifies natural frequencies and vibration modes. This is important for earthquake engineering and aerospace applications [6].

Machine Learning And AI

In machine learning, spectral clustering and feature reduction depend on eigenvalue calculations. EVD is also essential in kernel methods and recommender systems [7].

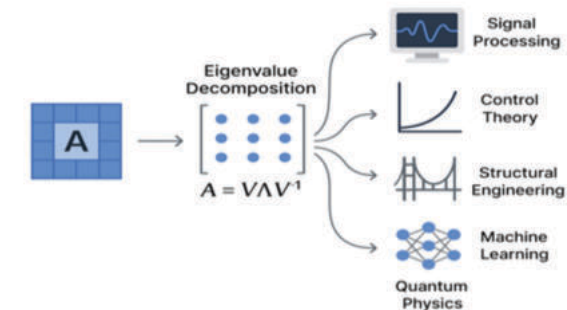
Quantum Mechanics And Physics

In quantum mechanics, operators like the Hamiltonian are examined with eigenvalues and eigenvectors. These represent measurable physical quantities, such as energy states [8].

Figures And Tables

Table 1. Applications Of Eigenvalue Decomposition Across Domains.

Domain	Application	Outcome
Signal Processing	PCA, Image compression	Reduced dimensionality
Control Engineering	Stability analysis	System behavior prediction
Structural Engg.	Modal analysis	Natural frequency identification
Machine Learning	Spectral clustering, recommender systems	Feature extraction
Physics	Quantum operator analysis	Energy state determination



Graphical Abstract

CONCLUSION
 This study highlights the key role of eigenvalue

decomposition in various fields. By providing computational efficiency and clarity, EVD allows for dimensionality reduction, stability analysis, structural monitoring, and improved modeling in physics. Future research may focus on better algorithms for large-scale eigenvalue problems, especially in big data and quantum computing. The study shows how EVD improves performance in real-world applications like image compression, facial recognition, wireless communication, structural health monitoring, and cryptography. Eigenvalue decomposition acts as a unifying framework in mathematics and applied sciences. Its ability to break down complex systems into understandable parts makes it essential for future research in computational and applied areas.

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